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VILNIUS GEDIMINAS TECHNICAL UNIVERSITY

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THE INVESTIGATION OF RESONANT INTERACTION OF SOME MATHEMATICAL MODEL OF NONLINEAR WAVES

SUMMARY OF DOCTORAL DISSERTATION
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VILNIAUS GEDIMINO TECHNIKOS UNIVERSITETAS

Olga LAVCEL-BUDKO

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Introduction

Problem Formulation

The dissertation focuses on weak nonlinear oscillations described by partial differential equations when nonlinear members included in the equations are proportional to small parameter ε. Under the value of small parameter $\varepsilon = 0$, the examined differential equations turn into linear form and are solved employing the well-known methods. However, in case $\varepsilon > 0$, complex expressions of interactions, the impact of which grows along with time, occur. For application purposes, the process remains important up to the values of time $t \sim \varepsilon^{-1}$ – in the so-called long interval of time ($t \in [0, O(\varepsilon^{-1})]$) that usually includes the classical solution of the examined tasks. It is considered that, when $\varepsilon \to 0$, solving such tasks with the help of numerical methods decreases the efficiency of the latter ones due to the fact that a long interval requires a greater number of the points of difference schemes. On the other hand, asymptotic approximations, if constructed for a long interval of time, are the closer to the exact estimates of the approached tasks the lower the values ε are.

To simulate the interaction of weak nonlinear hyperbolic oscillations, the small-parameter method is traditionally used. Generally, in case of small parameter ε direct expansions have the so-called secular members εt (Nayfen, 1981); for this reason, asymptotic approximations are applied in short intervals of time $t \ll \varepsilon^{-1}$ only. When $t \sim \varepsilon^{-1}$, for making a uniformly valid asymptotic approximation, specific methods of asymptotic analysis are necessary, the most important of which are those of averaging and the multiple scaling. A study on the periodic problem is compounded by the emergence of resonances, and therefore the systems must be averaged using a specific method along to characteristics (sometimes referred to as internal averaging (Krylov, 1989)), when undetected functions - the estimates of the task are averaged. This means that for constructing asymptotic, a new problem – the averaged system that may prove to be more complex than the original one is to be solved. The value of the above presented averaged system in asymptotic analysis does not include small parameter ε and thus it does not encounter integration problems. Frequently, such systems are left unsolved as a separate object. When undetected functions are averaged, internal averaging sometimes applied indirectly coping with specific asymptotic problems. To apply asymptotic approximations to the used tasks, the faced problems must be worked out with the help of numerical methods. In this particular case, asymptotic analysis, i. e. numerical asymptotic methods (Bakhvalov, Panasenko, Shtaras 1987), can be considered. This is somewhat contrary to the traditions of employing asymptotic methods when simple engineering formulas are obtained. In general, a possibility of discerning a certain problem of asymptotic analysis arises, i. e. for finding asymptotics, a new object, which requires separate non-trivial research, is designed.

Topicality of the Research Work

Despite the fact that averaged system

$$\frac{\partial v_j}{\partial \tau} = \lim_{T\to\infty} \frac{1}{T} \int_0^T f_j\left(s, y_j + \lambda_j s, v_1(\tau, y_j + (\lambda_j - \lambda_1)s\right), \ldots, \tag{1}$$

$$\frac{\partial v_n}{\partial y_n}(\tau, y_j + (\lambda_j - \lambda_n)s))ds$$

appeared in literature a long time ago (Kalyakin 1989; Krylovas, Shtaras 1984), the object still needs more detailed investigation. Generally, when functions f_j depend on partial derivatives, no existence of the estimate and proximity to the exact estimate in the case of quasi-linear hyperbolic systems were proved, which is an analogue of the N. N. Bogoliubov theorem. Up to now, this has been applied to semi-linear hyperbolic systems, i. e. functions f_j in system do not depend on partial derivatives $\frac{\partial u_i}{\partial x}$.

Research Object

The exact object of the thesis is the averaged system of equations (1) with periodical initial conditions.

The Aim of the Work

To analyse the averaged hyperbolic system, which allows re-writing it in a more acceptable form of research and making specific asymptotic approximations.

Tasks of the Work

1. To investigate the method of differential equations with a small parameter and specific methods of asymptotic analysis embracing multiple scaling and averaging.
2. To substantiate the method of averaging along the characteristics in the case of quasi-linear hyperbolic systems.
3. To apply the internal averaging method to solving practical problems.
4. To develop and employ the algorithm the Maple software intended for making the approximations of a special form.

Applied Methods

The dissertation has applied to the method of differential equations with a small parameter and specific methods of asymptotic analysis covering multiple scaling and averaging. For conducting experiments, algorithms for making approximations the Maple software have been implemented.

Scientific Novelty

The obtained results presented in the dissertation are new material. The proved lemmas allowed simplifying the averaged differential system of equations and recording it in a form analogue to the semi-linear hyperbolic system recorded in Riemann invariants. This admitted proving the presence of the estimate of quasi-linear hyperbolic systems and its proximity to the exact estimate.

The investigated specific tasks take the developed asymptotic approximations that are more in line with the traditions of asymptotic analysis and appear as more acceptable than the numerical ones used for practical testing of models.

As regards differential equations for gas dynamics, a uniformly valid asymptotic approximation in the long time interval has been made. The conditions, under which, compared to ideal gas, resonance does not occur in Van der Waals gas have been established.

Practical Value of the Work Results

The internal averaging method allows dealing with the applied problems in gas and hydro-dynamics, the theory of elasticity, plasma physics, optics and mechanics. The problems solved using the above presented method are expressed in a form acceptable to asymptotic approximations (engineering formulas), the precision of which can be easily monitored.

Statements Presented for Defence

1. The presence of the estimate of quasi-linear hyperbolic systems and proximity to the exact estimate have been validated, which is an analogue of the Bogoliubov theorem.
2. The accepted lemmas allow proving and assist in re-writing the system in a more suitable form.
3. The asymptotic approximations to the examined tasks are received in an acceptable form, and therefore precision can be easily monitored.
4. The conditions, under which, compared to ideal gas, resonance does not occur in van der Waals gas have been established.

Approval of the Work Results

The results of the dissertation are published in 5 publications: 5 of those in the reviewed scientific journals. The results were presented at 14 Lithuanian and international conferences and 3 seminars at the Department of Mathematical Modelling of Vilnius Gediminas Technical University.

The Scope of the Scientific Work

The dissertation consist of an introduction, 3 chapters, main conclusion, bibliography, the list of the author,s publications and 4 addends. The total scope of the dissertation 100 pages, 318 mathematical expressions, 9 items and 12 tables.

1. The Ideas and Methods of Asymptotic Analysis

The first chapter of the dissertation examines the systems of differential equations with a small parameter and increasingly focuses on resonant processes.

The Small Parameter Method. The principle of Multiple Scaling. Averaging. When all coefficients of system

$$\frac{\partial u_j}{\partial t} + \lambda_j \frac{\partial u_j}{\partial x} = \varepsilon f_j \left(t, x, u_1, \ldots, u_n, \frac{\partial u_1}{\partial x}, \ldots, \frac{\partial u_n}{\partial x} \right) \tag{2}$$

with periodical initial conditions

$$u_j(0, x; \varepsilon) = u_{0j}(x) \equiv u_{0j}(x + 2\pi). \tag{3}$$

$\lambda_j = 0$, o and spatial variable x is treated as a parameter (analogically, if $\lambda_1 = \lambda_2 = \cdots \lambda_n = \lambda$ and change variable $y = x - \lambda t$) according to N.N.Bogoliubov, a standard system of ordinary differential equations:

$$\frac{du_j}{dt} = \varepsilon f_j \left(t, u_1, u_2, \ldots, u_n \right), \; j = 1, 2, \ldots, n, \tag{4}$$

is received under the formulated initial conditions

$$u_j \left(0; \varepsilon \right) = u_{j0} + \varepsilon u_{j1} + \varepsilon^2 u_{j2} + \cdots . \tag{5}$$

If we accept that $f_j \left(t + 2\pi, u \right) \equiv f_j \left(t, u \right)$, $u = \left(u_1, \ldots, u_n \right)$ expand f_j with the help of Fourier series:

$$f_j \left(t, u \right) = \sum_{k=-\infty}^{+\infty} f_{jk} \left(u \right) e^{\mathbf{i}kt}, \mathbf{i} = \sqrt{-1}. \tag{6}$$

A direct (4) expansion of the estimate of the system, with reference to the degrees of small parameter ε

$$u_j \left(t; \varepsilon \right) = u_j^{(0)} \left(t \right) + \varepsilon u_j^{(1)} \left(t \right) + \varepsilon^2 u_j^{(2)} \left(t \right) + \cdots \tag{7}$$

requires dealing with Cauchy problems

$$\frac{du_j^{(s+1)}}{dt} = f_j^{(s)} \left(t, u^{(s)} \right), \; u^{(s+1)} \left(0 \right) = \left(u_{1,s+1}, u_{2,s+1}, \ldots, u_{n,s+1} \right), \tag{8}$$

where $f_j^{(s)}$ – coefficients of the Taylor formulas for the functions f_j received replacing functions u_j with appropriate expressions (7). When $s = 0$, $u^{(0)}(t) \equiv u_{j0}$. With reference to (8) and (6) is obtained:

$$u_j^{(1)}(t) = u_{j1} + t f_{j0}(u_{10}, \ldots, u_{n0}) + \sum_{k \neq 0} f_{jk}(u_{10}, \ldots, u_{n0}) \frac{e^{\mathbf{i}kt} - 1}{\mathbf{i}k}. \quad (9)$$

It is supposed that the function of expansion (7) is not bound, when $u_j^{(1)}(t)$ and when $t \to +\infty$, due to secular members $t f_{j0}$ is not a correct approximation $u_j(t; \varepsilon) \approx u_j^{(0)}(t),\ t \sim \varepsilon^{-1}$.

In order to receive a uniformly valid asymptotic approximation in interval $\left[0; \frac{t^0}{\varepsilon}\right]$ specific methods of asymptotic analysis, including multiple scaling and averaging, have to be applied.

Concerning that $f_{j0}(u) = \lim_{T \to +\infty} \frac{1}{T} \int_0^T f_j(t, u)\, dt \equiv \langle f_j \rangle (u)$ we can observe it is purposeful to replace the functions of expansion (7) with more general functions for finding which $v_j^{(s)}(t; \varepsilon)$ new tasks are approached. To search for functions v_j^0 the averaged system:

$$\frac{dv_j^{(0)}}{dt} = \varepsilon \langle f_j \rangle \left(v^{(0)}\right). \quad (10)$$

Let use note the slow time $\tau = \varepsilon t$ (the principle of double scaling) and $v^{(0)}(t; \varepsilon) = v(\tau)$ it is obtained that $\frac{dv}{dt} = \varepsilon \frac{dv}{d\tau}$ and therefore small parameter ε is eliminated from the averaged system:

$$\frac{dv_j}{d\tau} = \langle f_j \rangle (v). \quad (11)$$

The classical result of asymptotic analysis is establishing conditions guaranteeing that functions $u_j(t; \varepsilon)$ are close to functions $v_j(\varepsilon t)$, when $t \in \left[0, O\left(\varepsilon^{-1}\right)\right]$ and $\varepsilon \to 0$, which can be referred to as the Bogoliubov theorem.

With reference to the elementary example, the first chapter of the thesis has described a decrease in the difference between exact and uniformly valid estimates in the long interval of time (obtained from the averaged system) when ε, decreases and the difference between them is bound by certain constant $S(\varepsilon) = O(\varepsilon)$, when $\varepsilon \to 0$. Direct expansion having secular members is close to the exact estimate in the short interval of time $t \in \left[0, \frac{t^0}{\sqrt{\varepsilon}}\right]$, and the error is $O(\sqrt{\varepsilon})$. The difference between the exact estimate and approximation does not decrease when $\varepsilon \to 0$, but remains in the long interval of the same order.

Models for Oscillation Problems. The dissertation investigates models for resonant wave interaction described using partial differential equations. However, inherent problems can be faced through the examination of oscillations in the material points of the systems defined by ordinary differential equations:

$$u_j'' + \omega_j^2 u_j = \varepsilon f_j \left(u, u'\right), \ j = 1, 2, \ldots, n. \tag{12}$$

When $\varepsilon = 0$, (12) the estimate of the system is functions $u_j\left(t\right) = a_j \cos\left(\omega_j t + \varphi_j\right)$. The idea of the averaging method is to replace constants a_j, φ_j with undetected functions $a_j\left(t; \varepsilon\right)$, $\varphi_j\left(t; \varepsilon\right)$ in this formula and demand that the first derivative of function $u_j\left(t\right)$ should be equal to:

$$u_j'\left(t\right) = -a_j \omega_j \sin\left(\omega_j t + \varphi_j\right). \tag{13}$$

To ensure requirement (13), the following equation is worked out:

$$a_j' \cos\left(\omega_j t + \varphi_j\right) - a_j \varphi_j' \sin\left(\omega_j t + \varphi_j\right) = 0. \tag{14}$$

Next, having inserted $u_j\left(t\right)$ into (12), the second-order derivative is not observed, and therefore the second equation is received:

$$-a_j' \omega_j \sin\left(\omega_j t + \varphi_j\right) - a_j \varphi_j' \cos\left(\omega_j t + \varphi_j\right) = \varepsilon f_j\left(\cdots\right). \tag{15}$$

With reference to (14), (15), the first-order standard (4) system is obtained:

$$\begin{cases} a_j' = -\frac{\varepsilon}{\omega_j} f_j\left(\cdots\right) \sin\left(\omega_j t + \varphi_j\right), \\ \varphi_j' = -\frac{\varepsilon}{a_j \omega_j} f_j\left(\cdots\right) \cos\left(\omega_j t + \varphi_j\right). \end{cases} \tag{16}$$

Let use note $y_j = \omega_j t + \varphi_j$. Then, system (16) can be rewritten as follows (i. e. to get a standard system applying the averaging method):

$$u_j' = \varepsilon F_j\left(y_1, y_2, \ldots, y_n, u\right), \tag{17}$$

where F_j are periodical functions with 2π – period. If functions F_j are sufficiently continuously differentiated, they can be expanded using converged Fourier series

$$F_j\left(y, u\right) = \sum_{\mathbf{k} \in Z^n} F_{j\mathbf{k}}\left(u\right) e^{\mathbf{i}(\mathbf{k}, \mathbf{y})}, \tag{18}$$

where

$$\begin{array}{c} F_{j\mathbf{k}}\left(u\right) = \frac{1}{(2\pi)^n} \int_0^{2\pi} \cdots \int_0^{2\pi} F_j\left(y, u\right) e^{-\mathbf{i}(\mathbf{k}, \mathbf{y})} \, dy_1 dy_2 \ldots dy_n, \\ (\mathbf{k}, \mathbf{y}) = k_1 y_1 + k_2 y_2 + \cdots + k_n y_n. \end{array} \tag{19}$$

Let use note a set of resonance vectors as $\mathcal{R}_\omega = \left\{\mathbf{k} \in Z^n : (\mathbf{k}, \omega) = 0\right\}$.

Then, the average of function F_j according to variable t, is:

$$< F_j >= \lim_{T\to+\infty} \frac{1}{T} \int_0^T F_j\left(\omega_1 t + \varphi_1, \omega_2 t + \varphi_2, \ldots, \omega_n t + \varphi_n, u\right) dt = \sum_{\mathbf{k}\in\mathcal{R}_\omega} F_{j\mathbf{k}}\left(u\right) e^{\mathbf{i}(\mathbf{k},\varphi)}. \tag{20}$$

It is noticeable that the averaged system has the same form (10). However, in the case of resonance (20), the average is not equal to average $\frac{1}{(2\pi)^n} \int_0^{2\pi} \cdots \int_0^{2\pi} F_j\left(y, u\right) dy_1 dy_2 \ldots dy_n = F_{j\mathbf{0}}\left(u\right)$, as then, set $\mathcal{R}_\omega$ has not only element $\mathbf{0} = (0, 0, \ldots, 0)$.

Mathematical Model of an Elastic Pendulum. The third chapter focuses on a classical task – a gravitational pendulum on an elastic thread. Although the problem has been worked out, it was not fully resolved in the study by Magnus (1982). The model of an elastic mathematical pendulum takes the assumptions of weightless suspension and absolute elasticity, however, the assumption of inflexibility is ignored.

$$\begin{cases} m\ddot{L} - mL\dot{\varphi}^2 - mg\cos\varphi + k(L - L_0) = 0\,, \\ L\ddot{\varphi} + 2\dot{L}\dot{\varphi} + g\sin\varphi = 0\,. \end{cases} \tag{21}$$

Next, system (21) is examined in the general case. Equations of motion (21) switch to dimensionless variables and functions:

$$l = \frac{L}{L_0}\,, \quad \omega_x^2 = \frac{k}{m}\,, \quad \omega_\varphi^2 = \frac{g}{L_0}\,, \quad \omega^2 = \frac{\omega_x^2}{\omega_\varphi^2}\,, \quad \tau = \omega_\varphi t\,. \tag{22}$$

Then, equations of motion (21) take a dimensionless form like:

$$\begin{cases} \ddot{l} - l\dot{\varphi}^2 - \cos\varphi + \omega^2(l - 1) = 0\,, \\ l\ddot{\varphi} + 2\dot{l}\dot{\varphi} + \sin\varphi = 0\,. \end{cases} \tag{23}$$

Under assumptions that phase fluctuations are small and the length of the pendulum changes to a small extent

$$\varphi(t, \varepsilon) = \varepsilon\tilde{\varphi}(t), l(t, \varepsilon) = l_0 + \varepsilon\tilde{l}(t), 0 < \varepsilon \ll 1,$$

the system of two second-order ordinary differential equations is obtained:

$$\begin{cases} \ddot{\tilde{l}} + \omega_l^2\tilde{l} = \varepsilon(l_0\dot{\tilde{\varphi}}^2 - \frac{\tilde{\varphi}^2}{2}), \\ \ddot{\tilde{\varphi}} + \omega_\varphi^2\tilde{\varphi} = \varepsilon(-\omega_\varphi^2\tilde{l}\ddot{\tilde{\varphi}} - 2\omega_\varphi^2\dot{\tilde{l}}\dot{\tilde{\varphi}}). \end{cases} \tag{24}$$

where ω_l – length frequency, ω_φ – oscillation frequency.

For solving this problem, the methodology described in the first two chapters has been applied. An asymptotic approximation in the uniformly valid the long time interval is received and compared to the estimate obtained employing the method of direct development. Thus, under a decrease in parameter ε in the short interval of time, the error also gets smaller and series $\sqrt{\varepsilon}$ appear.

2. Averaging Along the Characteristics of Quasilinear Hyperbolic Systems

This chapter explores the systems of quasilinear hyperbolic first-order differential equations with partial derivatives.

Resonances and Internal Averaging. Analysis of the Averaged System. When functions $f_j,\ u_{0j}$ are differentiated sufficiently continuously, Cauchy problem (2) has a classical estimate in the long interval of time $t \in \left[0, O(\varepsilon^{-1})\right]$, (Rozhdestvenskii, Yanenko 1978). The method of internal averaging allows receiving a uniformly valid estimate in this particular interval. It should be remembered that the main idea of the above introduced method is that functions $f_j(t, x, v_1(\tau, y_1), \ldots, v_n(\tau, y_n))$ in system (2), (3) are averaged not only taking into account fast variables t, x, but also averaging to undetected functions $v_1(\tau, y_1), \ldots, v_n(\tau, y_n)$ and with averaging to along characteristics of the discontinued system as in system (1). Let use note the right-hand side of the system as $M_j\left[f_j\right]$. Then, the asymptotic approximation $u_j(tx;\, \varepsilon) \approx v_j(\varepsilon t,\, x - \lambda_j t) + O(\varepsilon)$ of system (2), (3) is the estimate of the averaged system

$$\frac{\partial v_j}{\partial \tau} = M_j\left[\ldots, v_i(\tau, y_j), \ldots\right],\ v_j(0,\, y_j) = u_{0j}(y_j). \tag{25}$$

Chikwendu, Kevorkian (1972) used internal averaging according to characteristics $x \pm t = const$ for solving certain equations for oscillations and waves, and Eckhaus (1975) showed how asymptotics investigated by Chikwendu and Kevorkian differed from the exact estimate in interval $t \in \left[0,\, O(\varepsilon^{-1})\right]$. Krylovas (1983) and Štaras (1978) researched the averaged systems with constant coefficients and periodic initial conditions. Later, this methodology was generalized (Krylovas, Štaras, 1984) for the case when coefficients depend on slow time $\lambda_j = \lambda_j(\tau)$. Similar results were received by Kalyakin (1989). Krylovas (1987) generalized the method of internal averaging when coefficients depend on fast and slow variables and small parameter ε: $\lambda_j = \lambda_j(t, x, \tau, \xi, \varepsilon)$.

The systems of internal averaging (1) were also examined by Hunter, Keller (1988), Maslov (1987), Majda, Rosales (1988). Most frequently such systems used to be left refusing further investigation and presented in literature as a certain result of asymptotic analysis, Arora (2008), Sharma (2004). In their papers, Krylovas (1983), Krylovas, Čiegis (2001) applied numerical methods for solving problems similar to (2). The author of the thesis and co-authors Krylovas, Lavcel-Budko, Miškinis (2010) made analytical approximations of the asymptotic estimate.

The papers by Krylovas (1989, 1990) focused on the substantiation of the averaging method (29), (30) in the case (i. e. when function (26) is equal to

$f_{ji} \equiv 0, i \neq 0$) of the semi-linear system (theorem of the asymptotic existence of estimates, uniqueness theorem and their approximation in the long interval of time $t \in \left[0, O(\varepsilon^{-1})\right]$ were proved).

Kurihara, Yano (2006) investigated nonlinear systems with the help of the double-scaling method, performed Fourier analysis and then to averaging technique. Similar schemas for internal averaging were employed by Gutierrez, Silva Dias, Raup (2011). Simpson, Weinstrein (2011), like Prelinovsky, Simpson, Weinstrein (2012), obtained an averaged system of equations and made out harmonic analysis. The above mentioned authors studied particular problems with no general theory when specific functions were observed on the right side and when transforming which asymptotics could be achieved.

The Substantiation of Averaging Quasilinear Hyperbolic Systems Along Characteristics. The following chapter of the thesis provides an separate case of problem (2), which is a system of quasilinear hyperbolic first-order partial differential equations, small positive parameter ε:

$$\frac{\partial u_j}{\partial t} + \lambda_j \frac{\partial u_j}{\partial x} = \varepsilon \left(f_{j0}(u) + \sum_{i=1}^{n} f_{ji}(u) \frac{\partial u_i}{\partial x} \right), \tag{26}$$

$$u = (u_1, u_2, \ldots, u_n), j = 1, 2, \ldots, n$$

and periodic initial conditions (3).

Letters D_j^0, F_{ji}^k, U_j^k, $(U_j^0 < D_j^0)$ are the positive constants, and $D = \left\{u \in R^n : |u_j| \leqslant D_j^0\right\}$ is the domain. All known functions of the problems (26), (3) are smoothly differentiable: $f_{ji} \in C^p(D), p \geqslant 2$, $u_{0j} \in C_{2\pi}^q(R)$, $q \geqslant 2$. The functions from (3) are periodical with the period 2π.

Let us assume that these functions and their partial derivatives are bounded:

$$\max_{x \in R} \left| \frac{d^k u_{0j}(x)}{dx^k} \right| \leqslant U_j^k, k = 0, 1, \ldots, q,$$

$$\max_{\substack{u \in D \\ \alpha_1 + \ldots + \alpha_n = k}} \left| \frac{\partial^k f_{ji}(u)}{\partial^{\alpha_1} u_1 \ldots \partial^{\alpha_n} u_n} \right| \leqslant F_{ji}^k, \; i = 0, 1, \ldots, n, \; k = 0, 1, \ldots, p. \tag{27}$$

Then, let us require that all constants λ_j of equations (26) are different. Thus, in such a case, system (26) is hyperbolic and Cauchy task has a classical solution when $t \in \left[0, \frac{t^0}{\varepsilon}\right]$. Where, t^0 is a positive constant. All the constants depend on the constants F_{ji}^k, U_j^k and do not depend on the small parameter ε.

The approximation in the general case is not close to the exact solution of (26), when $t = O(\varepsilon^{-1})$; thus, constructing an asymptotic approximation uniformly valid in the long time interval is not easy task. To find such an approximation, the double-scale method is applied and the system of averaged equations is developed.

As mentioned above, let us note slow time as $\tau = \varepsilon t$, fast characteristics variables as $y_j = x - \lambda_j t$ and the averaging operator along to the characteristic:

$$M_j\left[g(\tau, y_1, y_2, ..., y_n)\right] \equiv \lim_{T\to\infty} \frac{1}{T}\int\limits_0^T g(\tau, y_j + (\lambda_j - \lambda_1)s, \tag{28}$$

$$y_j + (\lambda_j - \lambda_2)s, ..., y_j + (\lambda_j - \lambda_n)s)ds \equiv \langle g\rangle_j(\tau, y_j).$$

Analyse the averaged system:

$$\frac{\partial v_j}{\partial \tau} = M_j\left[f_{j0}(v) + \sum_{i=1}^{n} f_{ji}(v)\frac{\partial v_i}{\partial y_i}\right], \tag{29}$$

$$v(\tau, y) = \left(v_1(\tau, y_1), v_2(\tau, y_2), \ldots, v_n(\tau, y_n)\right), j = 1, 2, ..., n$$

with the initial conditions (3), i. e.:

$$v_j(0, y_j) = u_{0j}(y_j) \equiv u_{0j}(y_j + 2\pi). \tag{30}$$

Theorem 1. *Suppose that functions* $f_{ji} \in C^2(D)$, $u_{0j} \in C^2_{2\pi}(R)$, $j = 1, 2, .., n$, $i = 0, 1, .., n$ *and restrictions* (27) *are valid. Then*

1) *there exists a positive constant* t^0 *when the problem* (26), (30) *has a unique solution* $u_j(t, x; \varepsilon) \in C^1_{2\pi}\left(\left[0, \frac{t^0}{\varepsilon}\right] \times R\right)$, $\varepsilon \in (0, \varepsilon^0)$;

2) *there exists a positive constant* τ^0 *when the problem* (29), (30) *has a unique solution* $v_j(\tau, y_j) \in C^1_{2\pi}([0, \tau^0] \times R)$;

3) $\forall\mu > 0$, $\exists\varepsilon_\mu \in (0, \varepsilon^0)$:

$$\max_{\substack{(t,x)\in[0,\frac{c}{\varepsilon}]\times R \\ j=1,2,..,n}} |u_j(t, x; \varepsilon) - v_j(\varepsilon t, x - \lambda_j t)| \leqslant \mu.$$

Here, $\varepsilon \in (0, \varepsilon_\mu]$, $c = \min\left\{t^0, \tau^0\right\}$.

The first proposition of the theorem for the hyperbolic system is well known. The second and third statements of the theorem are proved in the second chapter of the thesis, which is an equivalent for the classical Bogoliubov theorem proved applying the standard method of sequential iterations. The core element of this proof is 1, which under the development of the so-called expanded system, allowed differentiating the right sides of the averaged system along to the fast variable so that the right side should be continuously differentiated because the second-order partial derivatives cannot be observed.

Lemma 1. *Suppose, that* $f_{ji} \in C^1(D)$, $v_j(\tau, y_j) \in C^1_{2\pi}([0,\tau^0] \times R)$,

$$g_j(\tau, y_j) = M_j \left[f_{j0}(v) + \sum_{i \neq j} f_{ji}(v) \frac{\partial v_i}{\partial y_i} \right]$$

and the rule $(\forall i \neq j)\ \lambda_i \neq \lambda_j$ *are valid. Then, the function* $g \in C^1_{2\pi}([0,\tau^0] \times R^n)$, *and*

$$\frac{\partial g_j}{\partial y_j} = M_j \left[\sum_{l=1}^{n} \left(\frac{\partial f_{j0}}{\partial v_l} \frac{\partial v_l}{\partial y_l} + \sum_{i \neq j} \sum_{k \neq j} \frac{\lambda_l - \lambda_k}{\lambda_j - \lambda_k} \frac{\partial f_{ji}}{\partial v_l} \frac{\partial v_l}{\partial y_l} \frac{\partial v_k}{\partial y_k} \right) \right]. \qquad (31)$$

The above lemma allows rewriting the system using the form analogous to the semilinear system rewritten using Riemann's invariants.

Let us note $w_j(\tau, y_j) = \dfrac{\partial v_j(\tau, y_j)}{\partial y_j}$ and, applying lemma's 1 the formula (31) rewrite the problem (29), (30):

$$\frac{\partial v_j}{\partial \tau} = M_j \left[g_{0j}(v, w) \right], \qquad (32)$$

$$\frac{\partial w_j}{\partial \tau} - \langle f_{jj}(v) \rangle_j \frac{\partial w_j}{\partial y_j} = M_j \left[g_{1j}(v, w) \right], \qquad (33)$$

$$v_j(0, y_j) = u_{0j}(y_j),\ w_j(0, y_j) = u_{1j}(y_j) \equiv \frac{du_{0j}(y_j)}{dy_j}. \qquad (34)$$

Here $\quad g_{0j} = f_{j0}(v) + \sum_{i=1}^{n} f_{ji}(v) w_i,$

$$g_{1j} = \sum_{l=1}^{n} \left(\frac{\partial f_{j0}(v)}{\partial v_l} w_l + \sum_{i \neq j} \sum_{k \neq j} \frac{\lambda_l - \lambda_k}{\lambda_j - \lambda_k} \frac{\partial f_{ji}(v)}{\partial v_l} w_l w_k \right).$$

Construct the solution of the system (32)–(34) by with the help of iterations, used by Rozdestvenskij, Janenko (1978): $\overset{0}{v}_j \equiv u_{0j}$, $\overset{0}{w}_j \equiv u_{1j}$,

$$\frac{\partial \overset{m+1}{v_j}}{\partial \tau} = M_j \left[g_{0j}(\overset{m}{v}, \overset{m}{w}) \right],\ \overset{m+1}{v_j}(0, y_j) = u_{0j}(y_j), \qquad (35)$$

$$\frac{\partial \overset{m+1}{w_j}}{\partial \tau} - \left\langle f_{jj}(\overset{m}{v}) \right\rangle_j \frac{\partial \overset{m+1}{w_j}}{\partial y_j} = M_j \left[g_{1j}(\overset{m}{v}, \overset{m}{w}) \right], \qquad (36)$$

$$\overset{m+1}{w_j}(0, y_j) = u_{1j}(y_j).$$

Lemma 2. *These exists positive constant* τ^0 when (35), (36) *all functions of*

$\overset{m}{v_j}(\tau, y_j) \in D$, *i.e.* $\left|\overset{m}{v_j}\right| \leqslant D_j^0, \ |\overset{m}{w_j}| \leqslant D_j^1$.

Lemma 3. *The functions* $\overset{m}{v}, \overset{m}{w}$ *are smoothly continuous, i. e. these exist constants* C_1, C_2, C_3, C_4, *when* $(\forall\tau', \tau'', y_j', y_j'' \in [0, \tau^0] \times R^2),\ k \in N$

$$\left|\overset{m}{v_j}(\tau', y') - \overset{m}{v_j}(\tau'', y'')\right| \leqslant C_1|\tau' - \tau''| + C_2|y' - y''|,$$

$$\left|\overset{m}{w_j}(\tau', y') - \overset{m}{w_j}(\tau'', y'')\right| \leqslant C_3|\tau' - \tau''| + C_4|y' - y''|.$$

Lemma 2 assists in proving that all consistent approximations (35), (36) are bound in conjunction with their derivatives accordingly to y (analogically according to τ). Hence, all iterations $\overset{m}{v}$ of the averaged system are bound. With reference to Lemma 3, it can be proved that consistent approximations (35), (36) make a series of uniformly continuous functions $\overset{m}{v_j}(\tau, y_j)$, $\overset{m}{w_j}(\tau, y_j)$. Upon the application of the well-known Arzela–Ascoli theorem showing the existence of its converging subsequence, the boundary of the subsequence is

$$\lim_{m\to\infty} \overset{m}{v_j} = v_j$$

and is continuous and continuously differentiated function that is the estimate of system (32)–(34).

3. Examples of Mathematical Models for Resonant Wave Interactions

Asymptotics for the Model for nonlinear oscillations of the absolute elastic weightless string. The mathematical model for string nonlinear oscillations is presented. To found the asymptotic solution to the problem an averaging scheme was constructed and cited in the latest work Krylovas, Miškinis (2007).

$$\begin{gathered}
\frac{\partial R^+}{\partial \tau} + \beta \cdot \left(R^+\right)^2 \frac{\partial R^+}{\partial y^+} = \frac{\alpha}{2\pi} \int_0^{2\pi} \frac{\partial R^-(\tau, y^+ - 2s)}{\partial y^-} \times \\
\cos\left(\omega(y^+ - s)\right)\, ds - \frac{\partial R^+}{\partial y^+} \cdot \frac{\beta}{2\pi} \int_0^{2\pi} \left(R^-\right)^2 (\tau, y^+ - 2s)\, ds, \\
\frac{\partial R^-}{\partial \tau} - \beta \cdot \left(R^-\right)^2 \frac{\partial R^-}{\partial y^-} = -\frac{\alpha}{2\pi} \int_0^{2\pi} \frac{\partial R^+(\tau, y^- + 2s)}{\partial y^+} \times \\
\cos\left(\omega(y^- + s)\right)\, ds + \frac{\partial R^-}{\partial y^-} \cdot \frac{\beta}{2\pi} \int_0^{2\pi} \left(R^+\right)^2 (\tau, y^- + 2s)\, ds
\end{gathered} \tag{37}$$

with periodical initial conditions

$$R^{\pm}\left(\tau, y^{\pm}\right)\big|_{\tau=0} = R_0^{\pm}\left(y^{\pm}\right) \equiv R_0^{\pm}\left(y^{\pm} + 2\pi\right). \tag{38}$$

Here α, β, ω are constant parameters. When the functions $R_0^{\pm}$ are smoothly differentiated, there exists a positive constant τ_0 which makes the problem (37), (38) to have only one smoothly differentiated (as many times as $R_0^{\pm}$) solution $R^{\pm}\left(\tau, y^{\pm}\right)$ periodical according to $y^{\pm}$ in the domain $[0, \tau_0] \times (-\infty, +\infty)$.

An integral differential system of the averaged equations has been constructed for modelling nonlinear oscillations of the absolutely elastic weightless string. To solve this system, the Maple software has been compiled, which allows constructing the solution approximations of a special form.

$$R_{N,M}^{\pm}\left(\tau, y^{\pm}\right) = a_0^{\pm}\left(\tau\right) + \sum_{k=1}^{N} a_k^{\pm}\left(\tau\right)\cos\left(ky^{\pm}\right) + b_k^{\pm}\left(\tau\right)\sin\left(ky^{\pm}\right). \tag{39}$$

In the nonresonance case, an exact solution is possible, which this allowed testing the program. The accuracy of the approximation under construction depends on the number of harmonics and the degree of the polynomials that approximate the extension.

Calculations have been performed to show string profile variations in a long time interval for the resonance and nonresonance cases. Interestingly, the obtained asymptotic formulas, allow recalculating the values of functions for other values of ε and t .

Asymptotic Analyse of Gas Dynamics System. Consider a system of the first-order hyperbolic differential equations for gas dynamics, when the coefficients of thermal conductivity and viscosity coefficients are equal to zero. The physical sense of these equations is small perturbations influencing the interaction between acoustic waves. Asymptotic analysis allows to investigating the resonance interaction of periodic waves and ideal and non-ideal gas.

The system of differential equations is supplemented with equations of state, that are non-dependent on the properties of gas:

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho \\ \rho u \\ \rho\mathcal{E} + \rho\dfrac{u^2}{2} \end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix} \rho u \\ P + \rho u^2 \\ \rho u\mathcal{E} + Pu + \rho\dfrac{u^3}{2} \end{pmatrix} = 0, \tag{40}$$

$$P = P(\rho, \theta),\ \mathcal{E} = \mathcal{E}(\rho, \theta),$$

here $\rho-$is density, $u-$velocity, $\theta-$temperature, $P-$pressure is function of time t and space variable x and $\mathcal{E}$ – is energy. Gas state (ρ_0, u_0, θ_0) – const satisfies the system (40) and will bring in small non-negative parameter ε and investigate the waves of a small amplitude (acoustic waves):

$$\rho = \rho_0 + \varepsilon\rho_1(t, x; \varepsilon), \; u = u_0 + \varepsilon u_1(t, x; \varepsilon), \; \theta = \theta_0 + \varepsilon\theta_1(t, x; \varepsilon). \qquad (41)$$

The system rewritten using Ryman's invariants, is integrated by characteristics, and we can notice that secular terms εt are appear, and a solution is not appropriate on big domain. Using method of small parameter and averaging operators, we can construct averaged system of equations. This system can be solved using numerical or approximate methods, since there be no more problems with integration:

$$\frac{\partial V_j}{\partial \tau} = \sum_{i=1}^{3}\sum_{k=1}^{3} f_{jik} M_j \left[V_i \frac{\partial V_k}{\partial y_k} \right], \quad j = 1, 2, 3. \qquad (42)$$

After detailed investigation of averaged system noticed that:

1. At the initial time $t = 0$, density ρ_1 and temperature θ_1 of gas satisfy the condition:

$$\rho_1(0, x; \varepsilon) = \frac{\rho_0^2 \mathcal{E}_{0\theta}}{P_0 - \mathcal{E}_{0\rho}\rho_0^2}\theta_1(0, x; \varepsilon) + O(\varepsilon). \qquad (43)$$

In this case, is no resonance is observed and periodic, long and nonlinear waves are non-dependent and have a small amplitude.

2. The second equation of averaged system (42) is non-dependent and we can write its solution in the form of: $V_2(\varepsilon t, x - u_0 t) = \varphi(x - u_0 t)$.

3. Consider noticed that the coefficients are dependent on two parameters μ ir ν:

$$\frac{\partial V_j}{\partial \tau} - \mu V_j \frac{\partial V_j}{\partial y_j} = \pm M_j \left[V_2 \frac{\partial V_k}{\partial y_k} \right], \qquad (44)$$

$$\mu = \frac{1}{2d}\left(-2\alpha + d\left(h_{211}\alpha^2 - 1 + h_{213}\alpha\gamma + h_{231}\alpha\gamma + h_{233}\gamma^2\right)\right. \qquad (45)$$

$$\left. + \beta\left(\alpha h_{312} - \gamma + h_{332}\gamma\right)\right),$$

$$\nu = \frac{1}{2d}\left(\beta - d\left(h_{211}\alpha\beta + h_{231}\beta\gamma - h_{231}\alpha - h_{233}\gamma\right) + \beta\left(-\beta h_{321} + h_{332}\right)\right), \quad (46)$$

here

$\overline{P} = \dfrac{P_0 - \mathcal{E}_{0\rho}\rho_0^2}{\rho_0 \mathcal{E}_{0\theta}}, \; \alpha = \dfrac{\rho_0}{\lambda_0}, \; \beta = \dfrac{P_{0\theta}}{P_{0\rho}}, \; \gamma = \dfrac{\overline{P}}{\lambda_0}, \; d = \alpha + \beta\gamma,$

$\lambda_0 = \sqrt{\dfrac{\overline{P} P_{0\theta}}{\rho_0} + P_{0\rho}}$ $-$ is the local speed of sound.

$h_{211} = -\dfrac{P_{0\rho\rho}}{\rho_0} + \dfrac{P_{0\rho}}{\rho_0^2}$, $h_{213} = -\dfrac{P_{0\rho\theta}}{\rho_0} + \dfrac{P_{0\theta}}{\rho_0^2}$, $h_{231} = -\dfrac{P_{0\rho\theta}}{\rho_0}$, $h_{233} = -\dfrac{P_{0\theta\theta}}{\rho_0}$,

$$h_{312} = \frac{P_{0\rho} - 2\mathcal{E}_{0\rho}\rho_0 - \mathcal{E}_{0\rho\rho}\rho_0^2}{\rho_0\mathcal{E}_{0\theta}} + \frac{P_0 - \mathcal{E}_{0\rho}\rho_0^2}{\mathcal{E}_{0\theta}\rho_0^2} + \frac{P_0\mathcal{E}_{0\theta\rho}\rho_1 - \mathcal{E}_{0\rho\theta}\mathcal{E}_{0\rho}\rho_0^2}{\rho_0\mathcal{E}_{0\theta}^2},$$

$$h_{332} = \frac{P_{0\theta} - 2\mathcal{E}_{0\rho\theta}\rho_0^2}{\rho_0\mathcal{E}_{0\theta}} + \frac{P_0\mathcal{E}_{0\theta\theta} - \mathcal{E}_{0\theta\theta}\mathcal{E}_{0\rho}\rho_0^2}{\rho_0\mathcal{E}_{0\theta}^2}.$$

Also we can notice that the coefficients of the averaged equations are dependent on functions $P = P(\rho, \theta)$ and $\mathcal{E} = \mathcal{E}(\rho, \theta)$ of state equations.

Lets compare ideal polytrophic gas ($P = \mathcal{R}\rho\theta$, $\mathcal{E} = c_V\theta$) with Van der Waals polytrophic gas ($P = \dfrac{\mathcal{R}\rho\theta}{1-\rho b} - a\rho^2$, $\mathcal{E} = c_V\theta$). In case of ideal gas ν – has a steady sign. So we can make a conclusion that if condition (43), is not satisfied at the initial time then resonance will appear in ideal gas. Using formula (46) we calculate ν for Van der Waals equation, get, ν can be also equal to zero. It means that in case of Van der Waals gas there can be no resonance. It disappears under the following conditions:

for oxygen: $\theta_0 = 400$, $\rho_0 = 11885.8$; $\theta_0 = 350$, $\rho_0 = 11715.3$; $\theta_0 = 300$, $\rho_0 = 11493.4$; $\theta_0 = 250$, $\rho_0 = 11193.5$;

for nitrogen: $\theta_0 = 400$, $\rho_0 = 11885.8$; $\theta_0 = 350$, $\rho_0 = 11715.3$; $\theta_0 = 300$, $\rho_0 = 11493.4$; $\theta_0 = 250$, $\rho_0 = 11193.5$.

General Conclusions

1. Method of averaging along the characteristics has been substantiation for quasilinear hyperbolic systems, which is an analogue of the Bogoliubov theorem.

2. Solution approximations of a special form have been constructed for modelling made oscillations of the absolutely elastic weightless string. The calculations have been performed to show string profile variation in a long time interval for the resonance and nonresonance cases.

3. The asymptotic approximation of differential equations for gas dynamics has been made in the uniformly valid long interval of time. The conditions, under which, compared to ideal gas, resonance does not occur in Van der Waals gas have been established.

List of Scientific Publications on the Topic of Dissertation

In the Reviewed Scientific Journals

Krylovas, A.; Lavcel-Budko, O. 2013. Vienmačio dujų dinamikos uždavinio sprendinio asimptotinis aproksimavimas *Lietuvos matematikos rinkinys. Lietuvos matematikų draugijos darbai / Matematikos ir informatikos institutas, Lietuvos matematikų draugija, Vilniaus* *universitetas*, 54 (B): 48–53, ISSN 0132-2818.

Krylovas, A.; Lavcel-Budko, O.; Miškinis, P. 2012. Judėjimo lygčių atskyrimas dvimatės gravitacinės švytuoklės modelyje. *Lietuvos matematikos rinkinys. Lietuvos matematikų draugijos darbai / Matematikos ir informatikos institutas, Lietuvos matematikų draugija, Vilniaus universitetas*, 53(B): 31–36, ISSN 0132-2818.

Krylovas, A.; Lavcel-Budko, O. 2012. Vidurkinimo išilgai hiperbolinės sistemos charakteristikų operatoriaus savybės. *Lietuvos matematikos rinkinys. Lietuvos matematikų draugijos darbai / Matematikos ir informatikos institutas, Lietuvos matematikų draugija, Vilniaus universitetas*, 53(B): 25–30, ISSN 0132-2818.

Krylovas, A.; Lavcel-Budko, O.; Miškinis, P. 2010. Asymptotic solutions of the mathematical model of nonlinear oscillations of absolutely elastic inextensible weightless string. *Nonlinear analysis: modelling and control*, 15(3): 307–323, ISSN 1392-5113 (ISI Web of Science).

Krylovas, A.; Lavcel-Budko, O. 2009. Absoliučiai tamprios nesvarios stygos netiesinių svyravimų asimptotikų tyrimas. *Lietuvos matematikos rinkinys. Lietuvos matematikų draugijos darbai / Matematikos ir informatikos institutas, Lietuvos matematikų draugija, Vilniaus universitetas*, 50: 41–46, ISSN 0132-2818.

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NETIESINIŲ BANGŲ REZONANSINĖS SĄVEIKOS MATEMATINIŲ MODELIŲ TYRIMAS

Problemos formulavimas

Darbe nagrinėjami silpnai netiesiniai svyravimai, aprašomi diferencialinėmis lygtimis dalinėmis išvestinėmis, kai įeinantys į diferencialines lygtis netiesiniai nariai yra proporcingi mažajam parametrui ε. Esant mažojo parametro reikšmei $\varepsilon = 0$, nagrinėjamos diferencialinės lygtys tampa tiesinėmis ir sprendžiamos tiksliai gerai žinomais metodais. Tačiau, kai $\varepsilon > 0$, atsiranda sudėtingi sąveikos reiškiniai, kurių įtaka auga laikui t bėgant ir taikymams yra svarbus procesas iki laiko reikšmių $t \sim \varepsilon^{-1}$ – vadinamajame ilgajame laiko intervale ($t \in \big[0, O(\varepsilon^{-1})\big]$), nes paprastai šiame intervale egzistuoja nagrinėjamų uždavinių klasikiniai sprendiniai. Atkreiptinas dėmesys į tai, kad sprendžiant tokius uždavinius, kai $\varepsilon \to 0$ skaitiniais metodais mažėja pastarųjų efektyvumas, nes ilgajame intervale reikia imti vis daugiau skirtuminių schemų mazgų. Kita vertus, asimptotiniai artiniai, kai juos pavyksta sukonstruoti tinkamais ilgajam laiko intervalui, yra tuo artimesni tiksliesiems uždavinių sprendiniams, kuo mažesnės ε reikšmės.

Silpnai netiesinių hiperbolinių bangų sąveikai modeliuoti tradiciškai taikomas mažojo parametro metodas. Tiesioginiai skleidiniai mažojo parametro ε laipsniais bendruoju atveju turi vadinamuosius sekuliariuosius narius (Nayfeh, 1981); dėl jų asimptotiniai artiniai taikytini tik trumpuose laiko intervaluose $t \ll \varepsilon^{-1}$. Tolygiai tinkamam, kai $t \sim \varepsilon^{-1}$ asimptotiniam artiniui konstruoti reikalingi specifiniai asimptotinės analizės metodai, tarp kurių ypatingai svarbūs yra kelių mastelių ir vidurkinimo. Periodinių uždavinių tyrimą dar apsunkina rezonansų atsiradimas, dėl to būtina sistemas vidurkinti pagal charakteristikas specialiuoju būdu (kartais vadinamu vidiniu vidurkinimu), kai vidurkinamos dar nerastos funkcijos – uždavinio sprendiniai. Tai reiškia, kad asimptotikai konstruoti reikia spręsti naują uždavinį – suvidurkintąją sistemą, kuris gali pasirodyti sudėtingesniu už pradinį. Šios suvidurkintosios sistemos vertė asimptotinėje analizėje yra ta, kad į ją tiesiogiai neįeina mažasis parametras ε ir todėl ji neturi asimptotinio integravimo problemų. Dažnai tokios sistemos paliekamos neišspręstos, kaip atskiras objektas. Kartais vidinis vidurkinimas, kai vidurkinamos dar nerastos funkcijos, taikomas netiesiogiai sprendžiant konkrečius asimptotinius uždavinius. Norint pritaikyti asimptotinius artinius taikomiesiems uždaviniams, galima spręsti gaunamus uždavinius skaitiniais metodais. Čia reikėtų kalbėti apie asimptotinės analizės kryptį – skaitinius asimptotinius metodus (Bakhvalov, Panasenko, Shtaras 1987). Tai šiek tiek prieštarauja asimptotinių metodų taikymo tradicijoms, kai gaunamos paprastos inžinerinės formulės.

Kalbant bendrai, galima įžvelgti tam tikrą asimptotinės analizės problemą – asimptotikai rasti konstruojamas naujas objektas, kuris reikalauja atskiro netrivialaus tyrimo.

Darbo aktualumas

Nepaisant to, kad suvidurkintos sistemos

$$\frac{\partial v_j}{\partial \tau} = \lim_{T\to\infty} \frac{1}{T} \int_0^T f_j\left(s, y_j + \lambda_j s, v_1(\tau, y_j + (\lambda_j - \lambda_1)s)\right), \ldots, \qquad (1)$$

$$\frac{\partial v_n}{\partial y_n}(\tau, y_j + (\lambda_j - \lambda_n)s))ds$$

atsirado literatūroje gana seniai Kalyakin (1989), Krylovas, Štaras (1984), tai nėra išsamiai ištirtas objektas. Bendruoju atveju, kai funkcijos f_j priklauso nuo dalinių išvestinių, kvazitiesinių hiperbolinių sistemų atveju nebuvo įrodyta sistemos sprendinio egzistavimas ir artumas tiksliam sprendiniui. Tai yra N. N. Bogoliubovo teoremos analogas. Iki šiol tai buvo padaryta tik pusiau tiesinėms hiperbolinėms sistemoms, t.y. funkcijos f_j sistemoje nepriklauso nuo dalinių išvestinių $\frac{\partial u_i}{\partial x}$.

Tyrimų objektas

Disertacijos tyrimo objektas – suvidurkinta lygčių sistema su periodinėmis pradinėmis sąlygomis.

Darbo tikslas

Atlikti suvidurkintos hiperbolinės sistemos analizę, leidžiančią perrašyti ją patogesniu tyrimams pavidalu ir sukonstruoti specialaus pavidalo asimptotinius artinius, nagrinėjant taikymo pavyzdžius.

Darbo uždaviniai

1. Išnagrinėti diferencialinių lygčių mažojo parametro tyrimo metodą ir specifinius kelių mastelių ir vidurkinimo asimptotinės analizės metodus.
2. Pagrįsti vidurkinimo pagal charakteristikas metodą kvazitiesinių hiperbolinių sistemų atveju.
3. Pritaikyti vidinio vidurkinimo metodą kai kuriems praktiniams uždaviniams spręsti.
4. Sudaryti ir realizuoti Maple terpėje algoritmą, skirtą specialaus pavidalo artiniams konstruoti.

Tyrimų metodika

Darbe panaudoti diferencialinių lygčių mažojo parametro tyrimo metodas, specialieji asimptotinės analizės vidurkinimo ir kelių mastelių metodai, eksperimentams atlikti buvo realizuoti „Maple“ terpėje artinių konstravimo algoritmai.

Darbo mokslinis naujumas

Disertacijoje įrodytos lemos leido supaprastinti suvidurkintą diferencialinę lygčių sistemą ir užrašyti ją pavidalu, analogišku pusiau tiesinės hiperbolinės sistemos, užrašytos Rymano invariantais. Tai leido įrodyti suvidurkintos sitemos sprendinio egzistavimą ir jo artumą tiksliam sprendiniui kvazitiesinių hiperbolinių sistemų.

Nagrinėjamiems konkretiems uždaviniams konstruojami asimptotiniai artiniai, kurie labiau atitinka asimptotinės analizės tradicijas ir patogesni už skaitinius praktiniam modelių tyrimui.

Dujų dinamikos diferencialinėms lygtims sukonstruota tolygiai tinkama ilgajame laiko intervale asimptotinė aproksimacija. Nustatytos sąlygos, kurioms esant, Van der Waalso dujose, palyginus su idealiomis dujomis, neatsiranda rezonansas.

Darbo rezultatų praktinė reikšmė

Vidinio vidurkinimo metodas leidžia spręsti dujų ir hidrodinamikoje, tamprumo teorijoje, plazmos fizikoje, optikoje, mechanikoje taikomuosius uždavinius. Šiuo metodu spręstiems uždaviniams gaunami asimptotiniai artiniai patogiu pavidalu (inžinerinės formulės), kurių tikslumą galima lengvai kontroliuoti.

Ginamieji teiginiai

1. Suvidurkinta pagal charakteristikas sistema turi sprendinį, asimptotiškai artimą kvazitiesinės hiperbolinės sistemos tiksliajam sprendiniui ilgajame laiko intervale.
2. Patogesnis tyrimams suvidurkintų sistemų pavidalas gaunamas taikant įrodytas disertacijoje lemas.
3. Analizinio pavidalo asimptotiniai artiniai gaunami taikomiesiems uždaviniams taikant bendrą metodiką ir turi nesunkiai kontroliuojamą tikslumą.
4. Van der Vaalso dujose neatsiranda rezonansas, esant nustatytiems dujų parametrų sąryšiams, kurie negalioja idealioms dujoms.

Darbo rezultatų aprobavimas

Su disertacijos tema susiję rezultatai paskelbti 5 moksliniuose straipsniuose: 5 straipsniai publikuoti recenzuojamuose mokslo leidiniuose, iš jų vienas – leidinyje įtrauktame į MII *Web of Science* duomenų bazę. Disertacijos tema perskaityti 14 pranešimų Lietuvos ir tarptautinėse konferencijose. Tyrimo rezultatai buvo pristatyti 3 Vilniaus Gedimino technikos universiteto Matematinio modeliavimo katedros seminaruose.

Disertacijos struktūra

Disertaciją sudaro įvadas, trys pagrindiniai skyriai, bendros išvados, literatūros sąrašas ir keturi priedai. Pirmajame skyriuje aptartos problemos su kuriomis susiduriama sprendžiant diferencialinių lygčių dalinėmis išvestinėmis sistemas, kai įeinantys į lygtis netiesiniai nariai proporcingi mažajam parametrui ε. Antrajame skyriuje pagrįstas sprendinio egzistavimas ir artumas tiksliam sprendiniui kvazitiesinių hiperbolinių sistemos atveju. Trečiajame skyriuje pateikti rezonansinės bangų sąveikos matematinių modelių pavyzdžiai.

Bendrosios išvados

1. Įrodyta teorema, leidžia pagrįsti vidurkinimo pagal charakteristikas metodą kvazitiesinėms hiperbolinėms sistemoms (įrodytas N. N. Bogoliubovo teoremos analogas).

2. Sukonstruoti specialaus pavidalo sprendinio artiniai, absoliučiai tamprios nesvarios stygos netiesiniams svyravimams modeliuoti. Atlikti skaičiavimai rodo stygos profilio kitimą ilgajame laiko intervale rezonansiniu ir nerezonansiniu atvejais.

3. Sukonstruota tolygiai tinkama ilgajame laiko intervale asimptotinė aproksimacija dujų dinamikos diferencialinėms lygtims. Nustatytos sąlygos, kurioms esant, Van der Vaalso dujose, palyginus su idealiomis dujomis, neatsiranda rezonansas.

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THE INVESTIGATION OF RESONANT INTERACTION OF
SOME MATHEMATICAL MODEL OF NONLINEAR WAVES

Summary of Doctoral Dissertation
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MATEMATINIŲ MODELIŲ TYRIMAS

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