Baltic J. Modern Computing, Vol. 10 (2032), No. 2, pp. 105–120
https://doi.org/10.22364/bjmc.2022.10.2.01

Sensitivity Study of TOPSIS and COPRAS Methods with Respect to Normalization Techniques

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Abstract. In the most MADM (Multiple Attribute Decision Making) methods the attribute values are provided to transform into one dimension. This restructuring is carried out in accordance with appropriate normalization techniques. It has been observed that some of MADM methods are used for different normalization techniques. In practical calculations it was noted, that using different normalization techniques in the same MADM method, different alternative priority lines, with the same data, are produced. Research object of this paper is the group of the normalization techniques that are used in the multiple attribute decision making methods of TOPSIS and COPRAS. The aim of this paper is to analyze the normalization techniques influence the ranking of alternatives and to find out which mentioned multiple attributes decision making method is sensitive with respect to the normalization techniques.

Keywords: multiple attribute decision-making methods, normalization, statistical analysis, monotonic transformation, sensitivity.

1 Introduction

In most of the multiple attribute decision-making (MADM) methods, the values of the attributes are converted into non-dimensional sizes. This conversion shall be carried out in accordance with the relevant normalization techniques. In the literature analysis, it has been observed that the same MADM methods sometimes use different normalization techniques to determine which alternative is dominant. There are cases where scientists propose to apply a new normalization techniques in classic MADM methods.

One of the simplest and most commonly used MADM method is the Simple Additive Weighting (SAW) method (Hwang and Yoon, 1981). The SAW method provides for linear scale normalization of initial data, when the individual formulas normalize, minimized and maximized attributes. The same normalization formulas are used in the SECA (Simultaneous Evaluation of Criteria and Alternatives) method, introduced by Keshavarz-Ghorabaee et al. (2018). Australian Monash university scientists Chakraborty and Yen (2007) analyzed the influence of normalization techniques on the results of the SAW method. They found out that linear scale transformation Max and vector normalization (Vect) yield similar alternative ranking results compared to linear scale transformations: Sum and Max-Min. The normalization of the minimized attributes used in the SAW method was assigned to non-linear normalization by Yu et al. (2009), because the exponential function is used for the normalization of the minimized attribute when the degree indicator is -1.

In addition to linear scale normalization formulas, non-linear normalization formulas are also proposed to be applied. In 2008, logarithmic normalization formulas were proposed that were applied to the techniques for calculating optimal strategies in a games theory (Zavadskas and Turskis, 2008). In this work, the authors compare the influence of vector, linear scale (Min-Max and Korth-Juttler) and non-linear normalization Peldschus (Peldschus, 2007) and logarithmic formulas on the alternative priority line. The authors recommend applying logarithmic normalization in case of significant differences in the values of the attributes. There are works where several normalizations are applied to the same methods, and then the results are compared. In 2014, linear scale normalization formulas were used in the TOPSIS (Technique for Order Preference by An Ideal Solution) method: Min-Max, Max, Sum and vector normalization (Celen, 2014). The results of this study have shown that vector normalization is the best one for consistent results. In 2015, different normalization formulas were applied to the following MADM methods: WSM (The weighted sum model), WPM (The weighted product model), TOPSIS and PROMETHEE II (The Preference Ranking Organization Method for Enrichment Evaluation) (Kaftanowicz and Krzeminski, 2015). Different types of normalization techniques were applied for these methods: linear scale normalization (Street (Manhattan), Max, Min-Max, Korth-Juttler) non-linear (Peldschus and logarithmic), and vector normalization. The results of the study showed that PROMETHEE II was the least sensitive to the influence of normalization. While WSM, WPM and TOPSIS methods are very sensitive to the selection of normalization techniques.

Jahan and Edvards (2015) analysed linear and non-linear normalization techniques for benefit, cost and target attributes. The authors noted that the choice of normalization techniques influenced the quality of multiple attribute decisions.

The analysis of the related work has shown that the question of selecting normalization techniques for multiple attribute decision-making methods is examined by many authors. In most cases, the influence of normalization techniques on the results of multiple attribute decision-making methods is analysed only in the case of a specific example without simulation modelling. In 2016, the influence of normalization techniques (vector, Sum, Max, Min-Max and logarithmic) on the results of the TOPSIS method was performed. The results of the study showed that the alternative rankings distribution obtained by the TOPSIS method, using vector normalization, coincide with the alternative ranking distributions obtained using linear scale normalization Sum and do not coincide with alternative ranking distribution using Min-Max normalization (Simanavičienė, 2016).

The effects of different normalization techniques on the multi-criteria (MCDM) methods SAW (Simple Additive Weighting), Weighted Average (WA) (Vafaei et al., 2018) and Analytical Hierarchy Process (AHP) (Vafaei et al., 2016) are evaluated. These researches are assessing which are the most appropriate normalization techniques in decision problems for the multi-criteria methods. Study of Jafaryeganeh et al. (2020) is focused on the effect of the four normalization technique Linear normalization, Vector normalization, Linear max-min normalization and Logarithmic normalization for the cost and benefit criteria in multi-criteria decision making methods: WPM, WSM, TOPSIS and ELECTRE on the final design selection from a Pareto.The authors point out that the applied normalization techniques in the MCDM methods need to be considered carefully, because the variation of these techniques may lead to different rankings for the designs. Before using the normalization techniques to introduce the input of MCDM methods, a comparison is recommended between the dominance order of original and normalized values of alternatives (Jafaryeganeh et al., 2020). There are authors who, while researching normalization techniques and methods for aggregating normalized values, propose new multicriteria methods to overcome the shortcomings of existing MCDM methods (Wen et al, 2020). Chatterjee and Chakraborty (2014) applied five different normalization techniques within MCDM methods: PROMETHEE, Grey Relation Analysis (GRA) and TOPSIS for solving a flexible manufacturing system (FMS) selection problem. They wanted to assess the effect of the normalization techniques for the sensitivity of MCDM methods.

The result of a multiple attribute decision-making method is the priority line of alternatives. The aim of the article is to propose a method for determining the effect of normalization techniques on the results of multiple attribute decision-making methods using simulation modelling. To achieve the purpose of the article, the authors analyze the techniques of normalization commonly used in multiple attribute decision-making methods; describe the proposed algorithm for the sensitivity analysis method for assessing the sensitivity of multiple attribute decision-making methods in relation to normalization techniques; the proposed method is adapted to the sensitivity of TOPSIS or COPRAS methods in terms of normalization techniques. Comparing the sensitivity of these methods to the normalization techniques, the Kendall's rank correlation coefficient is calculated.

2 Review of Normalization Techniques

In cases where attributes are measured in different units of measure in a multiple attribute decision-making problems, some MADM methods provide for the transformation of attributes values to standardize and unify the dimensions of the attributes.

In statistics, normalization of data has several meanings. One of these is data transformation, in which the research data presented in different units of measurement are transformed into non-dimensional, comparable sizes. However, this does not necessarily mean that normalized values will have a normal distribution (Dodge, 2003). The normalization techniques of the multiple attribute decision-making methods are used for attribute values, for conversion to non-dimensional sizes. One of the key requirements for the normalization of attribute values is that the transformation must be monotonous (Edwards and Barron, 1994).

Monotonous transformation is a way to transform one set of numbers into another set of numbers, keeping the order of numbers in the set. Monotonous transformation f satisfies the condition:

$$(a_1 > a_2) \Rightarrow (f(a_1) > f(a_2)). \quad (1)$$

The techniques for the attribute normalization according to the applied monotonous transformations are also divided into linear, non-linear and vector ones. In some multiple attribute decision-making methods, the benefit and cost attributes are normalized generally, such as COPRAS, TOPSIS. In other methods, the benefit and cost attributes are normalized separately, such as SAW, VIKOR, SECA. There are works in which applying the TOPSIS method, benefit and cost attributes are normalized by different formulas (Celen, 2014).

This article will examine the sensitivity of the TOPSIS and COPRAS methods to normalization techniques, when the benefit and cost attributes are normalized generally. The article presents an analysis of the impact of five normalization techniques, the methods for the results of TOPSIS and COPRAS. The authors of TOPSIS and COPRAS methods, respectively, Hwang and Yoon (1981), Zavadskas and Kaklauskas (1996) anticipated the general technique for the normalization of benefit and cost attributes. As mentioned before, the vector normalization is default in the TOPSIS method, the linear scale normalization is default in the COPRAS method.

Let's say we have the attributes $X_1, X_2, ..., X_n$, according to which *m* of the alternatives $A_1, A_2, ..., A_m$ are evaluated. The matrix of attribute values, according to all alternatives, is created $X = \left(x_{ij}\right), (i = \overline{1,m}), (j = \overline{1,n})$. Matrix X is called a decision matrix. This article provides for techniques on the normalization of maximizing attributes. The following normalization techniques were chosen for this purpose: vector, linear scale (Sum, Max, Min-Max) and non-linear (Log) (Table 1).

Table 1: Normalization techniques for benefit (maximized) attributes

Name of a technique	Formula	Range of normalized values
Vector (Vect)	$\bar{x}_{ij} = \frac{x_{ij}}{\|X_j\|}$	$\|X_j\|$ is a norm of the vector in the *n*-dimensional Euclidean space. Range (0,1) (Hwang and Yoon, 1981).
Linear (Sum)	$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}$	Range (0,1) (Hwang and Yoon, 1981).
Non-linear (Log)	$\bar{x}_{ij} = \frac{ln(x_{ij})}{\sum_{i=1}^{m} ln(x_{ij})}$	Range (0,1) (Zavadskas and Turskis, 2008).
Linear (Max)	$\bar{x}_{ij} = \frac{x_{ij}}{\max\limits_{j} x_{ij}}$	Range (0,1] (Weitendorf, 1976).
Linear (Min-Max)	$\bar{x}_{ij} = \frac{x_{ij} - min_j x_{ij}}{\max\limits_{j} x_{ij} - min_j x_{ij}}$	Range [0,1] (Weitendorf, 1976).

For the analysis of the influence of normalization formulas, 20 pseudo-random values were generated in the interval [1, 100] applying uniform distribution. After normalizing these values using vector, linear (Sum), linear (Max), linear (MIn-Max) and non-linear (Log) normalization techniques, the dispersion characteristics (i.e., standard deviation, coefficient of variation, sample width, and sum of values) were obtained to show that those normalization techniques produce different values (Fig. 1). Therefore, the question arises *do normalization techniques affect the ranking of alternatives?*

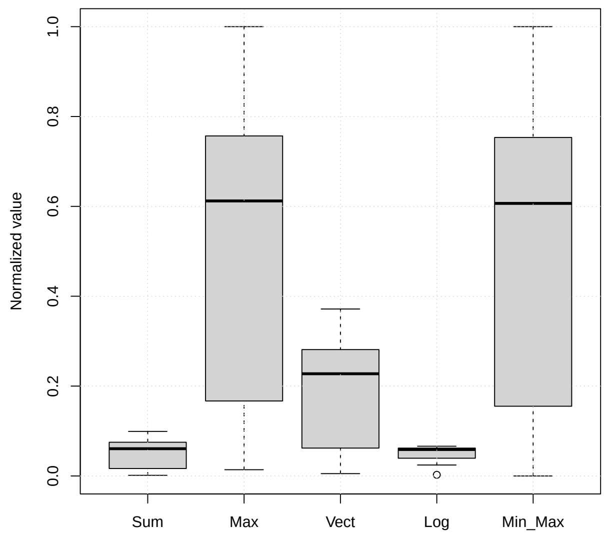


Fig. 1: Boxplots of normalized values of benefit attributes

Summing up, we need to investigate how normalization techiniques influence the ranking of alternatives applying the same MCDM method.

3 Algorithm of the study of the influence of normalization on the results of MADM methods

The study of the influence of normalization is carried out in the chosen MADM method by changing the normalization techniques (described above) and the results obtained by performing statistical analysis. The proposed algorithm for influencing normalization techniques is based on the sensitivity analysis algorithm described in the article by Simanavičienė (2016). The steps of the described algorithm are presented in the block diagram (Fig. 2).

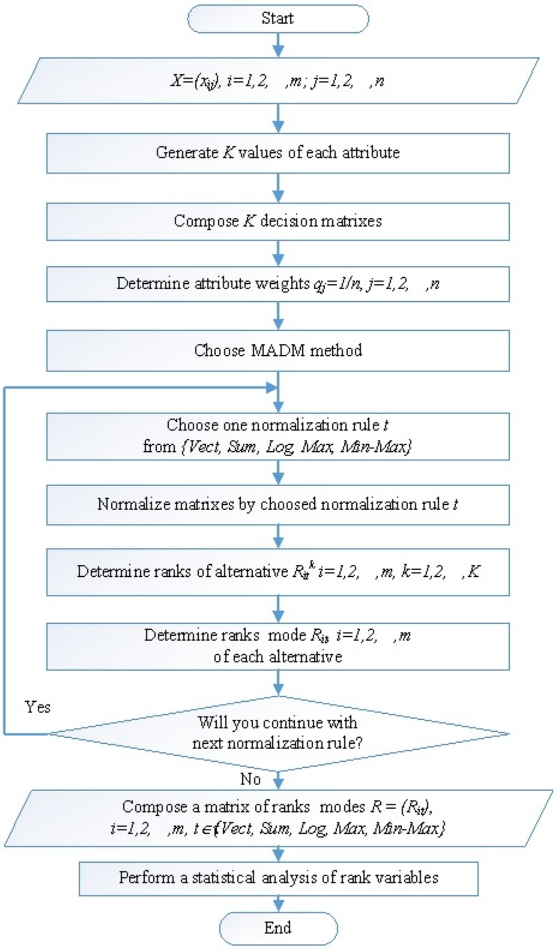


Fig. 2: Algorithm of sensitivity analysis

The detailed steps of the research algorithm are the following:

1. A multiple attribute decision-making problem is formulated, consisting of a set of alternatives under consideration $\{A_i\}, (i = \overline{1, m})$ and a set of attributes $\{X_j\}, (j = \overline{1, n})$.
2. A decision-making matrix $X = \left(x_{ij}\right), (i = \overline{1, m}), (j = \overline{1, n})$ is created, here x_{ij} is the value of the i-th alternative of j-th attribute.
3. Based on the values of the attributes x_{ij}, intervals are formed $\left[x_{ij} - \delta; x_{ij} + \delta\right]$, which will generate a sequence of equal pseudo-random values for each attribute x_{ij}^k, $(i = \overline{1, m}), (j = \overline{1, n}), (k = \overline{1, K})$ with K elements in each one, here $\delta > 0$ is possible error of attribute values.
4. The K matrixes are composing from the generated values: $X_k = \left(x_{ij}^k\right)$, $(i = \overline{1, m}), (j = \overline{1, n}), (k = \overline{1, K})$.

5. In order to check the effect of the normalization techniques on the results of the MADM method, the the attribute significances are selected $q_j = \frac{1}{n}$, $(j = \overline{1, n})$.
6. When analyzing the sensitivity of the selected MADM method to the normalization techniques under consideration, $t \in \{Vect, Sum, Log, Max, Min - Max\}$, calculations are performed with all K decision matrices $X_k = (x_{ij}^k)$, $(i = \overline{1, m})$, $(j = \overline{1, n})$, $(k = \overline{1, K})$.
7. The results of the calculation are presented as samples $A_t^k = (a_{1t}^k, a_{2t}^k, \ldots, a_{mt}^k)$, $(k = \overline{1, K})$, the elements of which are MADM method criterion values for each alternative.
8. Depending on the values of MADM criterion a_{it}^k, $(i = \overline{1, m})$, $(k = \overline{1, K})$ for the i-th alternative, all alternatives are ranked by assigning the highest value to grade 1, the least to rank *m*. This way the rankings of data samples are obtained in $R_t^k = (R_{1t}^k, R_{2t}^k, \ldots, R_{mt}^k)$, $(k = \overline{1, K})$, according to the t-th normalization technique.
9. Statistical analysis of results is performed.

If the aim is to compare the results of several MADM methods against the techniques of normalization, then the described algorithm is applied to several MADM methods. The rank correlation coefficients of Spearman or Kendall's are counted for samples of the ranked data. Based on the fact that the effect of normalization was observed using the same decision matrixes and the same algorithm of chosen MADM method, it is assumed that the samples of corresponding alternatives A_i, $(i = \overline{1, m})$ ranks $R_t = (R_{1t}, R_{2t}, \ldots, R_{mt})$, here $t \in \{Vect, Sum, Log, Max, Min - Max\}$, are dependent. Depending on the assumption given, in order to determine whether in the chosen MADM method and changing the normalization techniques the obtained rank lines R_t and R_l, $t, l \in \{Vect, Sum, Log, Max, Min - Max\}$ correlate, Kendall's rank correlation coefficient is calculated this way:

$$\tau = 1 - \frac{4c}{m^2 - m}, \tag{2}$$

where *c* is the difference between the concordant and the discordant pairs, *m* is the sample size. The hypothesis described in equation (3) is used to determine the statistical significance of Kendall's rank correlation (Cleff, 2014). The null hypothesis indicates that the correlation coefficient equals zero, i.e., there is no correlation between the ranks obtained by different normalization techniques. The alternative hypothesis indicates that the correlation is statistically significant.

$$\begin{cases} H_0 : \tau(R_t, R_l) = 0, \\ H_1 : \tau(R_t, R_l) \neq 0, \end{cases} \tag{3}$$

where τ is a correlation coefficient between rank lines R_t and R_l. Those ranks are obtained applying the MADM method with different normalization techniques $(t, l \in \{Vect, Sum, Log, Max, Min - Max\})$.

Variables correlate if $p < \alpha$, and the variables do not correlate if $p \geq \alpha$, where the significance level is $\alpha = 0.05$.

4 Multiple attribute decision-making methods

4.1 TOPSIS method

TOPSIS method was developed by Hwang and Yoon (1981). This technique is based on the idea that the optimal alternative is most similar to an ideal solution (being closest to it and at the longest distance from the negatively ideal solution). This method is known as TOPSIS – *Technique for Order Preference by Similarity to Ideal Solution*. Suppose we have a decision matrix X where rows mark the alternatives (m – number of alternatives), columns – attributes (n – number of attributes).

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix}. \tag{4}$$

where x_{ij}– i-th alternatives, value of the j-th attribute. By applying the TOPSIS method, a decision matrix X is normalized by making a vector normalization:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \tag{5}$$

Suppose known values of attribute significance $q_j, (j = \overline{1,n})$, then a weighted normalized matrix is formed $\overline{X}^* = (v_{ij}), (i = \overline{1,m}, j = \overline{1,n})$, whose elements are calculated according to the formula:

$$v_{ij} = \bar{x}_{ij} \cdot q_j. \tag{6}$$

The “ideal” alternative, denoted as A^+, is determined according to the following formula:

$$A^+ = \{(\max_i v_{ij} | j \in J), (\min_i v_{ij} | j \in J') | i = \overline{1,m}\} = \{a_1^+, a_2^+, ..., a_n^+\}, \tag{7}$$

where J – a set of attribute indexes whose higher values are better; J' – a set of attribute indexes whose lower values are better. The “negative-idea” alternative, denoted as A^-, is determined according to the following formula:

$$A^- = \{(\min_i v_{ij} | j \in J), (\max_i v_{ij} | j \in J') | i = \overline{1,m}\} = \{a_1^-, a_2^-, ..., a_n^-\}. \tag{8}$$

A distance between a comparative i-th and A^+ is determined by calculating a distance in the n-dimensional Euclidean space upon the following formula:

$$L_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - a_j^+)^2}, (i = \overline{1,m}) \tag{9}$$

and between the i-th and A^-, upon following the formula:

$$L_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - a_j^-)^2}, (i = \overline{1,m}). \tag{10}$$

The criterion of the TOPSIS method is a relative distance of the i-th alternative to the A^- alternative:

$$K_i = \frac{L_i^-}{L_i^+ + L_i^-}, (i = \overline{1,m}). \tag{11}$$

4.2 COPRAS method

The COPRAS (*COmplex PRoportional ASsessment*) method was created by Lithuanian researchers Zavadskas and Kaklauskas (1996). COPRAS method consists of several phases of calculation:

Phase 1. Normalization of the elements of the decision matrix is conducted by using the formula:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, (i = \overline{1,m}; j = \overline{1,n}). \tag{12}$$

Suppose known values of attribute significance $q_j, (j = \overline{1,n})$, then a weighted normalized matrix is formed $\overline{X}^* = (v_{ij}), (i = \overline{1,m}, j = \overline{1,n})$, whose elements are calculated according to the formula, like in the TOPSIS method $v_{ij} = \bar{x}_{ij} \cdot q_j$, where x_{ij} is the j-th attribute value of the i-th alternative; q_j is the significance value of the j-th attribute.

Phase 2. The sums of minimizing S_{-i} and maximizing S_{+i} weighted normalized values of each alternative are calculated. The following formulas are used:

$$S_{+i} = \sum_{j=1}^{n} v_{ij}^+, (i = \overline{1,m}, j = \overline{1,n}), \tag{13}$$

$$S_{-i} = \sum_{j=1}^{n} v_{ij}^-, (i = \overline{1,m}, j = \overline{1,n}), \tag{14}$$

where v_{ij}^+ is the weighted normalized values of j-th benefit attribute of the i-th alternative; v_{ij}^- is the weighted normalized values of j-th cost attribute of the i-th alternative; q_j is the significance value of the j-th attribute.

Phase 3. The relative significance of comparable alternatives is identified on the basis of the positive S_{+i} and negative S_{+i} characteristics that describe the i-th alternative. The relative significance (rationality) Q_i of i-th alternative is identified using the formula:

$$Q_i = S_{+i} + \frac{S_{-min} \sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{S_{-\min}}{S_{-i}}}, \qquad i = \overline{1,m}. \tag{15}$$

The higher the Q_i value, the more i-th alternative complies with the needs (preferences) of a decision-making person (Zavadskas and Kaklauskas, 1996).

5 Case study

The proposed sensitivity algorithm has been applied for 12 most advanced countries in the world, according to the World Economic Forum (WEF, 2018) presented the inclusive development index 2018. The countries concerned were assessed on the basis of 12 attributes: 6 cost and 6 benefit attributes. Benefit (max) attributes are X_1 - GDP per capita, (\$), X_2 - Labor productivity, (\$), X_3 - Healty life expectancy, (yrs), X_4 - Employment, (%), X_8 - Median income, (\$), X_9 - Adjusted net saving, (%). Cost (min) attributes are X_5 - Net income gini, X_6 - Poverty rate, (%), X_7 - Wealth gini, X_{10} - Carbon intensity, (kg per \$ of GDP), X_{11} - Public debt, (%), X_{12} - Dependency ratio, (%). The initial decision-making matrix is presented in Table 2.

Table 2: Dashboard of National Key Performance Indicators (WEF, 2018)

Alternative	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
A_1-NO	89818	126236	72.0	61.7	24.9	8.1	80.5	63.8	20.6	16.3	33.2	52.5
A_2-IS	48614	78278	72.7	71.1	24.4	6.5	46.7	43.4	14.7	21.2	53.2	52.1
A_3-LU	111001	206734	71.8	55.4	28.4	8.1	68.1	61.8	20.9	32.5	22.6	44.0
A_4-CH	75726	98724	73.1	65.4	29.3	7.8	69.4	55.6	17.9	11.8	45.4	49.4
A_5-DK	60268	89010	71.2	58.3	25.3	5.5	80.9	44.7	18.5	18.2	39.9	56.3
A_6-SE	56319	94533	72.0	59.9	25.7	8.0	83.4	48.3	19.0	14.2	41.7	59.3
A_7-NL	52111	94244	72.2	59.7	26.6	7.9	73.0	43.3	15.2	38.9	62.6	53.8
A_8-IE	66787	146230	71.5	54.9	30.3	9.2	81.3	38.0	26.2	19.5	76.4	54.5
A_9-AU	55671	88981	71.9	60.9	33.2	12.8	65.2	44.4	8.1	57.1	41.1	51.9
A_{10}-AT	47704	92169	72.0	56.5	27.8	9.0	78.8	49.2	12.7	22.6	83.9	49.5
A_{11}-FI	45709	86923	71.0	53.0	25.6	6.3	76.7	43.5	7.4	27.6	63.6	59.1
A_{12}-DE	45552	89805	71.3	57.7	29.0	9.5	79.1	45.3	13.8	58.9	67.6	52.3
	max	*max*	*max*	*max*	*min*	*min*	*min*	*max*	*max*	*min*	*min*	*min*

Table 3: p-values of attribute Pearson correlation coefficients

Attributes	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
X_1	0.00	**0.00**	0.60	0.85	0.83	0.95	0.96	**0.00**	**0.04**	0.42	**0.01**	**0.04**
X_2	**0.00**	0.00	0.78	0.24	0.55	0.82	0.78	0.08	**0.03**	0.79	0.21	0.05
X_3	0.60	0.78	0.00	**0.00**	0.98	0.89	0.06	0.33	0.72	0.32	0.58	0.18
X_4	0.85	0.24	**0.00**	0.00	0.52	0.78	**0.01**	0.75	0.97	0.55	0.42	0.66
X_5	0.83	0.55	0.98	0.52	0.00	**0.00**	0.98	0.65	0.77	0.07	0.73	0.24
X_6	0.95	0.82	0.89	0.78	**0.00**	0.00	1.00	0.87	0.54	**0.03**	0.78	0.36
X_7	0.96	0.78	0.06	**0.01**	0.98	1.00	0.00	0.90	0.39	0.68	0.55	0.21
X_8	**0.00**	0.08	0.33	0.75	0.65	0.87	0.90	0.00	0.41	0.43	**0.02**	0.06
X_9	**0.04**	**0.03**	0.72	0.97	0.77	0.54	0.39	0.41	0.00	0.08	0.53	0.62
X_{10}	0.42	0.79	0.32	0.55	0.07	**0.03**	0.68	0.43	0.08	0.00	0.74	0.59
X_{11}	**0.01**	0.21	0.58	0.42	0.73	0.78	0.55	**0.02**	0.53	0.74	0.00	0.47
X_{12}	**0.04**	0.05	0.18	0.66	0.24	0.36	0.21	0.06	0.62	0.59	0.47	0.00

For further analysis, we have calculated the Pearson correlation coefficients between attributes to check their correlation Table 3. Since attributes X_1, X_4, X_6, X_8 and X_9 have strong correlation $\left(p < 0.05\right)$, they there excluded from analysis.

For the sensitivity analysis, based on the initial decision-making matrix, we have generated 100 pseudo-random values ($K = 100$) using uniform distribution of value ranges $x_{ij} \pm 10\%$. Those obtained 100 decision-making matrices are used to analyze the sensitivity of TOPSIS and COPRAS by changing normalization techniques.

The Table 4 shows the rank mode of the most common rank of the TOPSIS and COPRAS methods, using the default normalization technique, and the percentage of all rank values of the TOPSIS and the COPRAS methods with different normalizations.

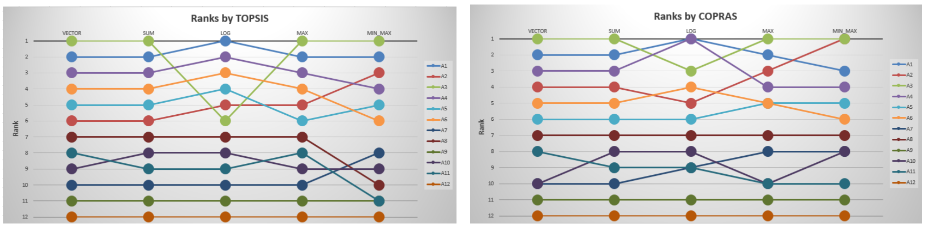
Based on the data in the Table 4, it can be observed that if alternatives was evaluated by TOPSIS and COPRAS, with no change in the normalization techniques, the modes of ranks differed slightly. Observing the effect of normalization techniques on rankings of alternatives, we found out that the most unmatched results by the TOPSIS and COPRAS methods (using default normalization techniques and using logarithmic or the Min-Max normalization techniques) are achieved. What was described above is best seen in the dominant alternative A_3. Using vector (Vect), linear scale (Sum) and (Max) normalization techniques results are similar.

Table 4: The most common rank value and its percentage of alternatives in the TOPSIS (T) and COPRAS (C) using different normalization techniques

Alternative	Mode of rank		Frequency of rank (%)									
	Default		Vect		Sum		Log		Max		Min-Max	
	T	C	T	C	T	C	T	C	T	C	T	C
A_1	2	2	67	60	63	55	32	24	71	46	29	18
A_2	6	4	39	29	39	32	33	20	21	25	4	11
A_3	1	1	77	88	71	84	0	15	90	87	79	56
A_4	3	3	58	37	52	41	19	22	44	27	17	18
A_5	5	6	41	36	40	39	20	46	27	28	21	10
A_6	4	5	42	30	42	27	18	25	31	33	13	17
A_7	10	10	68	34	72	41	62	52	36	16	13	10
A_8	7	7	53	59	48	60	61	49	64	39	13	21
A_9	11	11	80	70	81	74	64	68	56	44	26	28
A_{10}	9	8	42	29	42	32	23	33	34	19	18	17
A_{11}	8	9	46	29	44	30	19	40	35	21	14	15
A_{12}	12	12	98	94	99	94	98	96	96	91	65	51

Observing the most common values of alternative rankings, it was observed that alternatives ranked by TOPSIS and COPRAS correlated strongly enough, simultaneously applying the same normalization techniques. This conclusion can be drawn by observing the values of Kendall's correlation coefficient of the ranks (Table 5).

Another question that was to be answered in this study, is to determine which method of TOPSIS or COPRAS is more sensitive to normalization techniques. Using the data in Table 5, charts of the most common alternatives obtained by TOPSIS and COPRAS were drawn. The diagrams show that the TOPSIS method is more sensitive to the normalization techniques than the COPRAS method (Fig. 3).



(a) By TOPSIS **(b)** By COPRAS

Fig. 3: The most common rank value for TOPSIS and COPRAS results

Table 5: The most common rank of alternatives using these normalization techniques

Alternative	Vect		Sum		Log		Max		Min-Max	
	T	C	T	C	T	C	T	C	T	C
A_1	2	2	2	2	1	1	2	2	2	3
A_2	6	4	6	4	5	5	5	3	3	1
A_3	1	1	1	1	6	3	1	1	1	1
A_4	3	3	3	3	2	1	3	4	4	4
A_5	5	6	5	6	4	6	6	5	5	5
A_6	4	5	4	5	3	4	4	5	6	6
A_7	10	10	10	10	10	9	10	8	8	8
A_8	6	7	6	7	7	7	7	7	10	7
A_9	11	11	11	11	11	11	11	11	11	11
A_{10}	9	10	8	8	8	8	9	10	9	8
A_{11}	8	8	9	9	9	9	8	10	11	10
A_{12}	12	12	12	12	12	12	12	12	12	12

For the assumption of greater sensitivity to normalization techniques of the TOPSIS method, the Kendall's rank data correlation coefficient was calculated. Using the most common rank values for each study, we have calculated the Kendall's correlation coefficient for the TOPSIS method rank series pairs to obtain test results using different normalization techniques for the TOPSIS method (Table 6, Table 7).

Table 6: Kendall's correlation coefficients for the TOPSIS results and p-values

τ (p)	Vector	Sum	Log	Max	Min-Max
Vector	1	0.97 (0.00)	0.82 (0.00)	0.97 (0.00)	0.72 (0.01)
Sum	0.97 (0.00)	1	0.85 (0.00)	0.94 (0.00)	0.75 (0.01)
Log	0.82 (0.00)	0.85 (0.00)	1	0.79 (0.00)	0.60 (0.04)
Max	0.97 (0.00)	94 (0.00)	0.79 (0.00)	1	0.75 (0.01)
Min-Max	0.72 (0.01)	0.75 (0.01)	0.60 (0.04)	0.75 (0.01)	1

Table 7: Kendall's correlation coefficients for the COPRAS results and p-values

τ (p)	Vector	Sum	Log	Max	Min-Max
Vector	1	0.96 (0.00)	0.85 (0.00)	0.91 (0.00)	0.84 (0.00)
Sum	0.96 (0.00)	1	0.89 (0.00)	0.89 (0.00)	0.86 (0.00)
Log	0.85 (0.00)	0.89 (0.00)	1	0.81 (0.00)	0.78 (0.00)
Max	0.91 (0.00)	0.89 (0.00)	0.81 (0.00)	1	0.94 (0.00)
Min-Max	0.84 (0.00)	0.86 (0.00)	0.78 (0.00)	94 (0.00)	1

The Kendall's correlation coefficients and p-values have shown that the TOPSIS method is more sensitive to the normalization techniques, than the COPRAS method. This assumption is based on p-values. In the case of TOPSIS method, the p-values of the Kendall's correlation coefficients using linear Min-Max were greater than or equal to 0.01. In the case of the Copras method, all p-values were zero. It can be observed that, for both TOPSIS and COPRAS vector and linear Sum normalization, the ranking of alternatives will be quite similar, as evidenced by the corresponding correlation coefficients of 0.97 and 0.96 respectivelly.

6 Summary and conclusions of the study

Five normalization techniques were chosen in MADM methods: vector (Vect), linear scale (Max), (Sum) and (Min-Max) and non-linear (Log) normalizations. After performing a statistical analysis of these normalization techniques with the generated data, it was found out that using linear scale (Sum) and non-linear (Log) normalization techniques for the same data, the variance of the normalized values were significantly lower than applying other normalization techniques. By applying the techniques of linear scale (Max) and linear scale (Min-Max) normalization, the variance of normalized values were significantly greater than applying other normalization techniques (Fig. 1).

An algorithm was developed to investigate the sensitivity of MADM methods and normalization techniques; it was based on the work published by Simanavičienė (2016), supplemented by statistical analysis of the results, which allows to compare the sensitivity of several MADM methods. The described algorithm was applied to the study of the sensitivity, in relation to normalization techniques of the TOPSIS and COPRAS.

For evaluating the sensitivity of the TOPSIS and COPRAS alternative rank lines derived from the described algorithm were analyzed. After performing a statistical analysis of alternative rank lines, it was observed that either using TOPSIS method (applying vector normalization) or using COPRAS method (applying linear scale (Sum) normalization), the rank value 1 was most often found in alternative A_3. As an alternative to A_3, in the TOPSIS method the ranked value 1 made 77 % of all ranked values, and 84 % of all ranked values by the COPRAS method. Meanwhile, for TOPSIS and COPRAS methods with non-linear (Log) normalization for alternative A_3, the TOPSIS method ranked 0 % of all rank values, while COPRAS method ranked 15 % of all rank values. The results show that non-linear normalization (Log) strongly influences alternative rankings.

By observing the most common values of alternative rankings, it was observed that the first three dominant alternatives and the last rankings obtained by the TOPSIS and COPRAS coincide when these methods use vector (Vect) and linear scale (Sum) and (Max) normalization techniques. Meanwhile, using the non-linear (Log) technique of normalization, the most common rank values are radically different from the above mentioned results.

The values of the Kendall's correlation coefficient showed that the TOPSIS method is more sensitive to the normalization techniques than the COPRAS method, because the p-values of the Kendall's correlation coefficient of the COPRAS method are closer to 0.00 comparing it to the TOPSIS method. It can be observed that, for both TOPSIS and COPRAS vector (Vect) and linear scale (Sum) normalization, the ranking of alternatives will be sufficiently similar, this is indicated by the corresponding correlation coefficients of 0.97 and 0.96.

Summarizing the study, it can be concluded that the normalization techniques affect the ranking of alternatives. TOPSIS and COPRAS methods are more sensitive to normalization techniques for linear scale (Min-Max) and non-linear scale (Log).

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Received February 1, 2022 , revised April 2, 2022, accepted April 8, 2022