Computers & Industrial Engineering 169 (2022) 108165

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

New complex proportional assessment approach using Einstein aggregation operators and improved score function for interval-valued Fermatean fuzzy sets

Pratibha Rani [a], Arunodaya Raj Mishra [b], Muhammet Deveci [c,d,*], Jurgita Antucheviciene [e]

[a] Department of Mathematics, Rajiv Gandhi National Institute of Youth Development, Sriperumbudur, Tamil Nadu 602105, India
[b] Department of Mathematics, Government College Raigaon, Satna, Madhya Pradesh-485441, India
[c] Royal School of Mines, Imperial College London, London SW7 2AZ, UK
[d] Department of Industrial Engineering, Turkish Naval Academy, National Defence University, 34940 Tuzla, Istanbul, Turkey
[e] Department of Construction Management and Real Estate, Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius, Lithuania

A R T I C L E I N F O

Keywords:
Interval-valued Fermatean fuzzy sets
Einstein operators
Community based tourism
Sustainable tourism development
Score function
MCDM
COPRAS

A B S T R A C T

As a generalization of the Fermatean fuzzy set, the theory of interval-valued Fermatean fuzzy set (IVFFS) is a more robust and reliable tool to address the imprecise and incomplete information in the process of multi-criteria decision making (MCDM), thus can be employed on wider range of applications. The aim of this study is to purpose a novel decision-making approach by combining two well-recognized methods, named as the criteria interaction through inter-criteria correlation (CRITIC) and the complex proportional assessment (COPRAS) with IVFFSs. In this line, to compare the interval-valued Fermatean fuzzy numbers (IVFFNs), a new score function is proposed and its feasibility in comparison with existing interval-valued Fermatean fuzzy score and accuracy functions is discussed. To combine the various IVFFNs, some interval-valued Fermatean fuzzy Einstein aggregation operators are introduced. Further, the CRITIC is utilized to derive the objective weights of attributes within IVFFS context. To prioritize the alternatives, the IVFF-COPRAS method is presented on IVFFSs settings. Later, to assess the performance quality of the developed methodology, an illustrative case study is discussed to evaluate and rank the sustainable community-based tourism (CBT) location candidates. Moreover, the comparative study and sensitivity investigation are conducted to prove that the developed framework efficiently handles the problem of sustainable CBT locations evaluation and selection problem under IVFFSs environment. The findings of this study conclude that the developed method is a systematic, more comprehensive, accurate, and structured approach in the assessment of sustainable CBT locations under uncertain environment.

1. Introduction

Due to wide-spread changes and intricacy of the decision-making problems, there are several alternatives and criteria to be evaluated in the context of *"community-based tourism (CBT)"* from sustainable perspective (He et al., 2021). In this process, the participation of numerous participants in the tourism sector has discussed the *"multi-criteria decision making (MCDM)"* procedures even more complicated (Amoako et al., 2021). MCDM, one of the renowned outlets of decision theory, has widely been studied and effectively employed to numerous disciplines (Božanić et al., 2020; Petrovic and Kankaras, 2020; Đalić et al., 2021). It is a process to find out the most suitable choice from a set of the available options under the numerous qualitative and quantitative criteria (Youssef and Webster, 2022). However, due to human errors, insufficient knowledge about the systems and complex environment, sometimes it is very tricky to make a suitable decision in a reasonable time (Karamaşa et al., 2021; Gorcun et al., 2021). To treat the uncertain information, Zadeh (1965) proposed the conception of *"fuzzy sets (FSs)"*, which has accomplished a great utilization in different fields. Inspired by the FSs doctrine, Atanassov (1986) proposed the theory of *"intuitionistic fuzzy sets (IFSs)"*, which is portrayed by the *"membership degree (MD)"* and *"non-membership degree (ND)"* along with the restriction that addition of these two degrees must not one. Since its advantage in modeling uncertain information systems, several scholars have paid much attention on the IFS theory (Kushwaha et al., 2020; Das

* Corresponding author.
E-mail addresses: pratibha138@gmail.com (P. Rani), arunodaya87@outlook.com (A.R. Mishra), muhammetdeveci@gmail.com (M. Deveci), jurgita.antucheviciene@vilniustech.lt (J. Antucheviciene).

https://doi.org/10.1016/j.cie.2022.108165
Received 9 February 2022; Received in revised form 24 March 2022; Accepted 6 April 2022
Available online 10 April 2022
0360-8352/

List of abbreviations

AHP	Analytic hierarchy process
ANP	Analytic network process
AOs	Aggregation operators
ARAS	Additive ratio assessment
BWM	Best-worst method
CBT	Community-based tourism
CRC	Correlation coefficient
CRITIC	Criteria interaction through inter-criteria correlation
COPRAS	Complex proportional assessment
CoCoSo	Combined compromise solution
DEs	Decision experts
DEMATEL	Decision-making trial and evaluation laboratory
EDAS	Evaluation based on distance from average solution
ELECTRE	Elimination et choix traduisant la realité
FMEA	Failure modes and effects analysis
FS	Fuzzy set
FFS	Fermatean fuzzy set
FWTT	Food waste treatment technology
GBES	Green building energy saving
HWRFE	Hazardous waste recycling facility evaluation
IFS	Intuitionistic fuzzy set
IT2FS	Interval type-2 fuzzy set
ITARA	Indifference threshold-based attribute ratio analysis
IVIFS	Interval-valued intuitionistic fuzzy set
IVFF	Interval-valued Fermatean fuzzy
IVFF-DM	Interval-valued Fermatean fuzzy-decision matrix
IVFFEOWA	Interval-valued Fermatean fuzzy Einstein ordered weighted averaging
IVFFEOWG	Interval-valued Fermatean fuzzy Einstein ordered weighted geometric
IVFFEWA	Interval-valued Fermatean fuzzy Einstein weighted averaging
IVFFEWA	Interval-valued Fermatean fuzzy Einstein weighted geometric
IVFFI	Interval-valued Fermatean fuzzy information
IVFFN	Interval-valued Fermatean fuzzy number
IVFFS	Interval-valued Fermatean fuzzy set
IVFFWA	Interval-valued Fermatean fuzzy weighted averaging
IVFFWG	Interval-valued Fermatean fuzzy weighted geometric
IVPFEWA	Interval-valued Pythagorean fuzzy Einstein weighted averaging
MCDM	Multi-criteria decision making
MEREC	Method based on the removal effects of criteria
MD	Membership degree
MMDE	Maximum mean de-entropy
MULTIMOORA	Multi-attribute multiple objective optimization on the basis of ratio analysis
ND	Non-membership degree
PCIM	Performance calculation technique of the integrated multiple
PFS	Pythagorean fuzzy set
RD	Relative degree
RSs	Rough sets
SAW	Simple additive weighting
SCNs	Supply chain networks
SD	Standard deviation
S3PRLP	Sustainable third party reverse logistic providers
SPSS	Smart product-service systems
SWARA	Stepwise weight assessment ratio analysis
ST	Sustainable tourism
UD	Utility degree
VIKOR	Visekriterijumska optimizacija I kompromisno resenje
WASPAS	Weighted aggregated sum product assessment

and Granados, 2022). However, the sum of MD and ND to which an alternative satisfying a criterion may be larger than unity in some of the real-life applications, but their squares sum is restricted to unity. To overcome these difficulties, Yager (2014) generalized the scope of IFSs to *"Pythagorean fuzzy sets (PFSs)"*, in which the squares sum of the MD and ND is ≤ 1. When considering some real-life problems, PFS can better cope with uncertain information and solve several complicated real-life MCDM problems that IFS cannot. To expand the spaces of IFSs and PFSs, Senapati and Yager (2020) pioneered the conception of *"Fermatean fuzzy sets (FFSs)"*. It is characterized by both MD and ND of an element and the sum of their 3rd powers must be at most equal to 1. Numerous authors have contributed to FFSs in different disciplines such as information measures, decision-making and aggregation information (Deng and Wang, 2021; Xu and Shen, 2021; Hadi et al., 2021).

As a matter of fact, the role of uncertain information becomes more and more diversified. In many realistic MADM processes, due to the fuzziness of information, and lack of knowledge and experience of the DEs, it is always tough for DEs to precisely illustrate their ideas with a crisp number; however, they can be articulated by an interval number within [0, 1]. In order to get the broader information, the notion of FFSs has been expanded to *"interval-valued Fermatean fuzzy sets (IVFFS)"* (Jeevaraj, 2021; Sergi et al., 2021; Rani and Mishra, 2022). The advantage of such extended theory is that it represents uncertain information more closely to the DE's expectation. It should be considered that when the lower and upper limits of the interval values are same, then the IVFFSs reduce to FFSs. As an expressing the information, the IVFFSs are more superior to extant tools in defining human's subjective intellects.

Since the MCDM procedure strives to utilize the consistent and rational models for optimal MCDM, the developed interval-valued Fermatean fuzzy-based MCDM method imposes itself as a logical option for treating MCDM problems. The *"complex proportional assessment (COPRAS)"* pioneered by Zavadskas et al. (1994), is established as one of the well-known MCDM approaches to rank the alternatives. This method has various merits, which as (i) it is valuable and way to handle the realistic MCDM patterns. (ii) It considers both the facets of the criteria (i.e., benefit and cost), and (iii) it illustrates the ratio to the worst and the ideal outcomes simultaneously (Roozbahini et al., 2020; Mishra et al., 2020). Recently, few studies (Jeevaraj, 2021; Sergi et al., 2021; Rani and Mishra, 2022) have been focused on some different types of MCDM techniques on "*interval-valued Fermatean fuzzy (IVFF)*" setting, but there is no work regarding the modified COPRAS method from IVFFS perspective. From the literature, it is also evident that there has been no work regarding a hybridized MCDM method based on *"criteria interaction through inter-criteria correlation (CRITIC)"* and COPRAS methods with *"interval-valued Fermatean fuzzy information (IVFFI)"*, whereas the CRITIC method provides an useful and easy way to obtain the criteria weights. Thus, to the best of authors' information, this is the first work which introduces an integrated IVFF-CRITIC-COPRAS method by combining the score function, Einstein aggregation operators (AOs), CRITIC model and COPRAS approach within IVFFSs context, which provides a simple calculation procedure with precise and consistent results for assessing sustainable *"community-based tourism (CBT)"* alternative selection.

The key challenges behind the current work are presented as follows:

- In the context of IVFFSs, existing score and accuracy functions (Jeevaraj, 2021; Rani and Mishra, 2022) have counter intuitive phenomena.
- As the Einstein t-norm and t-conorm operations have more flexible and valuable than the basic operations. However, there is no study about the Einstein AOs for IVFFSs in order to aggregate the IVFF information.
- In the literature, no research efforts have been made to obtain the objective weights of criteria on IVFF setting.
- The conventional COPRAS model has been generalized under different uncertain contexts but most of the existing COPRAS methods have limitations in dealing with interval-valued Fermatean fuzzy information.
- Previous studies have been introduced several methods for evaluating the sustainable CBT alternatives under diverse settings but these approaches have limitations in treating with the uncertain sustainable CBT alternative selection problem from IVFFSs.

The key contributions of the study are discussed as.

- To compare the IVFFNs, a new score function is presented with its pioneering axioms and also, presented its advantages in comparison with extant score and accuracy functions.
- To aggregate the IVFFI, some Einstein AOs are proposed for IVFFSs.
- To obtain the objective weights, the CRITIC method is utilized under IVFFSs environment.
- A hybrid IVFFI-based COPRAS model is introduced with the combination of the score function, Einstein aggregation operators and CRITIC method in order to solve the MCDM concerns with unknown DEs and criteria weights.
- To exhibit the stability and permanence of introduced model, a case study of sustainable CBT assessment is taken on IVFF setting.

The remaining of this study is prepared as: Section 2 presents the literatures related to the study. Section 3 discusses the basic ideas of IVFFSs. Section 4 firstly presents the novel score function for comparing IVFFNs and shows the comparison results with existing ones. Secondly, Einstein AOs are proposed to aggregate the IVFFIs. Section 5 proposes a novel IVFF-COPRAS framework to evaluate and rank the alternatives in MCDM context. Section 6 exhibits the applicability of the presented method in a case study of sustainable CBT alternatives selection. In addition, this section presents comparative and sensitivity studies. Lastly, conclusions and recommendations for further research are depicted in Section 7.

2. Literature review

In this section, we present the comprehensive review about the present study.

2.1. Interval-valued Fermatean fuzzy sets (IVFFSs)

Due to time pressure, human's cognitions and limited knowledge or data, it is often difficult to model the work situations using the primitive data processing methods by means of crisp numbers. To conquer these situations, the theory of FS has been pioneered by Zadeh (1965), wherein each object is measured using the degree of membership to lessen the ambiguity of information. Further, numerous generalizations of FS have been initiated, such as IFS (Atanassov, 1986), "*interval-valued intuitionistic fuzzy set (IVIFS)*" (Atanassov and Gargov, 1989), "*hesitant fuzzy set (HFS)*" (Torra, 2010), PFS (Yager, 2014), "*interval-valued Pythagorean fuzzy set (IVPFS)*" (Peng and Yang, 2016), FFS (Senapati and Yager, 2019) and so on.

To the useful information depiction capability, FFS tackles more uncertainty than the PFS and the IFS, and its application is more widespread than that of the PFS and the IFS. In the context of FFSs, several theories and applications have been presented recently. For instance, Aydın (2021) firstly proposed the Fermatean fuzzy cosine similarity and entropy measures with their enviable properties. Hadi et al. (2021) presented a series of Hamacher AOs for FFSs with their utilization in MCDM problems. Deng and Wang (2021) introduced two innovative distance measure models for addressing the medical diagnosis and pattern recognition problems within the context of FFSs. In accordance with the proposed Fermatean fuzzy similarity measures, Xu and Shen (2021) studied a novel decision-making methodology for solving medical diagnosis problems within FFSs context. Gül (2021) proposed three methods, named as "*simple additive weighting (SAW)*", "*additive ratio assessment (ARAS)*" and "*visekriterijumska optimizacija I kompromisno resenje (VIKOR)*" from Fermatean fuzzy perspective. Mishra and Rani (2021) developed new entropy and score function-based "*weighted aggregated sum product assessment (WASPAS)*" tool for selecting the healthcare waste disposal locations. Later, Zhou et al. (2021) put forward a combined methodology by combining Jensen–Shannon divergence based distance measure and "*elimination et choix traduisant la realité (ELECTRE)*" tool with FFSs and used to handle the decision-making problems from FFSs perspective. Further, Rani et al. (2022) presented a hybridized model by combining the heronian mean operators, "*method using the removal effects of criteria (MEREC)*" and the ARAS technique with FFSs and applied for treating the "*food waste treatment technology (FWTT)*" problem. Gonzales et al. (2022) integrated the "*maximum mean de-entropy (MMDE)*" and the "*decision-making trial and evaluation laboratory (DEMATEL)*" model with FFSs for evaluating the barriers to the implementation of Education 4.0.

Owing to the increasing intricacy of MCDM problems in engineering and the deficiency of exact information, the IVFFSs are more appropriate way to signify uncertain information. In the context of IVFFS, Sergi et al. (2021) studied a series of AOs for "*interval-valued Fermatean fuzzy numbers (IVFFNs)*". Further, Jeevaraj (2021) introduced several score functions for IVFFNs and studied their properties. Rani and Mishra (2022) proposed the score and accuracy functions for IVFFNs with their enviable properties. They suggested two AOs for aggregating the IVFFI and also, conferred some axioms. However, there has been little investigation on the theories and applications of IVFFSs; thus, it is necessary and significant to develop a new theory in the context of IVFFSs.

2.2. Einstein AOs

From the MCDM perspective, the concept of AOs has been studied by numerous researchers. Nowadays, the fuzzy information AOs are essentially interesting in significant research areas and are given insightful consideration among the scientific community. Einstein operations contain Einstein product and Einstein sum, which are worthy options to the algebraic product and algebraic sum, respectively. These operators eliminate the irrationality and inconsistency of the operational laws and offer us an extensive range for the MCDM applications. Rahman et al. (2020) developed some generalized Einstein AOs and their application to MCDM problems on IVPFSs. Ali et al. (2021) presented some complex Einstein weighted geometric AOs and their applicability in suppler chain management on IVPFSs. Rani and Mishra (2021) proposed a series of Einstein AOs for FFSs and presented their application in multi-criteria electric vehicle charging station problems. Based on Einstein operations, Kamacı et al. (2021) presented some dynamic interval-valued picture hesitant fuzzy AOs with their enviable properties. Iampan et al. (2021) gave linear diophantine fuzzy Einstein AOs to treat the numerous MCDM problems. Kumar and Chen (2022) studied some advanced linguistic intuitionistic fuzzy Einstein weighted averaging and geometric AOs of linguistic intuitionistic fuzzy numbers. The discussions so far revealed that the aforementioned studies on IVFFSs are based on the algebraic sum and product operations, which are not unique operations that can be preferred to model the union and intersection of FFSs.

2.3. CRITIC model

For the implementation of MCDM, the estimation of the attributes' weights is a significant concern for *"decision experts (DEs)"*. Several researchers have presented diverse tools for obtaining the attribute weights (Diakoulaki et al., 1995; Kersuliene et al., 2010). The criteria weighting procedures are characterized in two kinds as objective and subjective weights (Saraji et al., 2021). The CRITIC approach, initiated by Diakoulaki et al. (1995), is an objective weighting tool. The objective weight of criteria is obtained with the use of the contrast-intensity of each criterion, by *"standard deviation (SD)"*, while conflict among the criteria are obtained with the *"correlation coefficient (CRC)"*. It has effectively been applied in diverse realistic concerns to estimate the objective weight of each criterion (Hashemkhani Zolfani et al., 2020; Adali and Tus, 2021). For instance, Keshavarz Ghorabaee et al. (2018) presented the *"evaluation based on distance from average solution (EDAS)"* model with the CRITIC and *"stepwise weight assessment ratio analysis (SWARA)"* models for solving the construction management problem. Peng and Huang (2020) presented the CRITIC-CoCoSo tool for financial risk analysis. Liang (2020) gave the CRITIC-EDAS tool to prioritize the *"green building energy saving (GBES)"*alternatives on IFSs. Mishra et al. (2021b) discussed the hybrid CRITIC-EDAS model to treat the *"sustainable third party reverse logistic providers (S3PRLP)"* problem on Fermatean fuzzy information. Baidya et al. (2021) developed BCF-Archimedean power weighted operator-based CRITIC-MULTIMOORA method to solve *S3PRLP* assessment problem. Saraji et al. (2021) gave a Fermatean fuzzy CRITIC-COPRAS method to recognize the challenges to adopt the *"Industry 4.0"* in the firms and assess the performance of firms regarding the weighting procedure based on the DEs' opinions. Song et al. (2021) presented the BWM-CRITIC tool to evaluate the *"smart product-service systems (SPSS)"* with sustainability perspectives on *"rough sets (RSs)"*. Until now, there is no work to apply the CRITIC tool for obtaining the criteria weights through the assessing the sustainable CBT problem with IVFFSs setting.

2.4. COPRAS method

The COPRAS approach (Zavadskas et al., 1994), is a well-known MCDM approach, which determines not only an outcome but also shows the discriminations between each alternative and an ideal or non-ideal alternative, and thus, can be considered as a compromising model. It has been utilized in diverse fields such as gully erosion modeling in prone areas (Arabameri et al., 2019), location selection for photovoltaic plants installation (Furtado and Sola, 2020), COVID-19 local safety assessment (Hezer et al., 2021), green supplier selection (Lu et al., 2021) etc. Wang et al. (2016) discussed the COPRAS, *"analytic network process (ANP)"* and *"failure modes and effects analysis (FMEA)"* models for ranking the failure modes on IVIFSs setting. Mishra et al. (2020) extended the COPRAS model using information measures to treat the *"hazardous waste recycling facility evaluation (HWRFE)"* problem on IVIFSs. Roozbahini et al. (2020) designed an incorporated model using fuzzy and grey COPRAS methods for inter-basin water transfer planning project. Song and Chen (2021) used a comprehensive model by integrating probabilistic hesitant fuzzy COPRAS technique with maximizing deviation method to tackle practical MCDM problems. Krishankumar et al. (2021) gave the COPRAS method based on the Hamy mean operator to enlighten the cloud vendor assessment problem from probabilistic hesitant fuzzy perspective. Narang et al. (2021) studied a hybridized triangular fuzzy COPRAS-base-criterion model with the application in stock selection. In a study, Chaurasiya and Jain (2022) designed a hybridized MCDM tool by combining the entropy and COPRAS model for solving healthcare waste treatment problem within PFS context. In order to sustainable assessment of metropolitan cities, Kusakci et al. (2022) discussed a collective method using the COPRAS and *"analytic hierarchy process (AHP)"* with *"interval type-2 fuzzy sets (IT2FSs)"*. As per our investigation, there is no work regarding the hybridized COPRAS method based on score function, Einstein AOs and CRITIC model under interval-valued Fermatean fuzzy environment.

3. Prerequisites

Since a crisp number cannot define the uncertainty and imprecision information appropriately, therefore, Zadeh (1965) initiated the idea of FSs. Subsequently, several extended theories have been proposed, such as IFS, IVIFS, PFS, IVPFS, FFS, IVFFS and others. The notion of IVFFS (Jeevaraj, 2021; Sergi et al. 2021) emerged from the FFS (Senapati and Yager, 2019, 2020), which generalizes the MD and ND from exact numbers to interval numbers. In the following, we present some fundamental ideas about the IVFFSs.

Definition 3.1. (*(Senapati and Yager, 2019, 2020).*) A FFS F on a finite universal set V is defined as $F = \{\langle x_i,\ \mu_F(x_i),\ \ \nu_F(x_i),\rangle : x_i \in V\}$, where $\mu_F,\ \nu_F : V \rightarrow [0,\ 1]$ present the MD and ND of the object $x_i \in V$ to F, respectively, with the condition $0 \le (\mu_F(x_i))^3 + (\nu_F(x_i))^3 \le 1$. The indeterminacy grade of element $x_i \in V$ is presented as $\pi_F(x_i) = \sqrt[3]{1 - (\mu_F(x_i))^3 - (\nu_F(x_i))^3}$. For the sake of simplicity, Senapati and Yager (2019, 2020) defined $(\mu_F(x_i),\ \nu_F(x_i))$ as a "Fermatean fuzzy number (FFN)", represented by $(\mu_\alpha,\ \nu_\alpha)$ where $\mu_\alpha,\ \nu_\alpha \in [0,1], \pi_\alpha = \sqrt[3]{1 - (\mu_\alpha)^3 - (\nu_\alpha)^3}$. and $0 \le (\mu_\alpha)^3 + (\nu_\alpha)^3 \le 1$.

Definition 3.2. (*(Rani and Mishra (2022).*) The mathematical definition of IVFFS T on V is given by.

$$T = \left\{ \left\langle x_i,\ \left[\mu_T^{lb}(x_i),\ \mu_T^{ub}(x_i)\right],\ \left[\nu_T^{lb}(x_i),\ \nu_T^{ub}(x_i)\right] \right\rangle : x_i \in V \right\},$$

where $0 \leqslant \mu_T^{lb}(x_i) \leqslant \mu_T^{ub}(x_i) \leqslant 1, 0 \leqslant \nu_T^{lb}(x_i) \leqslant \nu_T^{ub}(x_i) \leqslant 1$ and $(\mu_T^{ub}(x_i))^3 + (\nu_T^{ub}(x_i))^3 \leqslant 1$. Here, $\mu_T(x_i) = [\mu_T^{lb}(x_i),\ \mu_T^{ub}(x_i)]$ and $\nu_T(x_i) = [\nu_T^{lb}(x_i),\ \nu_T^{ub}(x_i)]$ represent the degrees of the interval-valued membership and non-membership degrees of $x_i \in V$, correspondingly. The function $\pi_T(x_i) = [\pi_T^{lb}(x_i),\ \pi_T^{ub}(x_i)]$ denotes the IVFF-hesitancy index of x_i to T, where $\pi_T^{lb}(x_i) = \sqrt[3]{1 - (\mu_T^{ub}(x_i))^3 - (\nu_T^{ub}(x_i))^3}$ and $\pi_T^{ub}(x_i) = \sqrt[3]{1 - (\mu_T^{lb}(x_i))^3 - (\nu_T^{lb}(x_i))^3}$. For simplicity, an IVFFN is denoted by $\varpi = ([\mu_\varpi^{lb}(x_i),\ \mu_\varpi^{ub}(x_i)],\ [\nu_\varpi^{lb}(x_i),\ \nu_\varpi^{ub}(x_i)])$ which fulfills $(\mu_\varpi^{ub})^3 + (\nu_\varpi^{ub})^3 \leqslant 1$. For convenience, the pair $([\mu_\varpi^{lb}(x_i),\ \mu_\varpi^{ub}(x_i)],\ [\nu_\varpi^{lb}(x_i),\ \nu_\varpi^{ub}(x_i)])$ is denoted by$([a,b],[c,d])$..

Some special cases of IVFFS are defined as.

(i) If $\mu_T^{lb}(x_i) = \mu_T^{ub}(x_i)$ and $\nu_T^{lb}(x_i) = \nu_T^{ub}(x_i)$, $x_i \in V$, then an IVFFS reduces to a FFS.
(ii) If $\mu_T^{ub}(x_i) + \nu_T^{ub}(x_i) \le 1$, then an IVFFS reduces to an IVIFS.
(iii) If $(\mu_T^{ub}(x_i))^2 + (\nu_T^{ub}(x_i))^2 \le 1$, then an IVFFS diminishes to an IVPFS.

Definition 3.3. Let $\varpi_1 = ([\mu_1^{lb}, \mu_1^{ub}],\ [\nu_1^{lb}, \nu_1^{ub}])$ and $\varpi_2 = ([\mu_2^{lb}, \mu_2^{ub}],\ [\nu_2^{lb}, \nu_2^{ub}])$, be two IVFFNs. Then, the relation between two IVFFNs is given as follows (Jeevaraj, 2021; Sergi et al., 2021; Rani and Mishra, 2022):

(i) $\varpi_1 = \varpi_2$ iff $\mu_1^{lb} = \mu_2^{lb}, \mu_1^{ub} = \mu_2^{ub}, \nu_1^{lb} = \nu_2^{lb}$, and $\nu_1^{ub} = \nu_2^{ub}$.
(ii) $\varpi_1 \prec \varpi_2$ iff $\mu_1^{lb} \le \mu_2^{lb}, \mu_1^{ub} \le \mu_2^{ub}, \nu_1^{lb} \ge \nu_2^{lb}$, and $\nu_1^{ub} \ge \nu_2^{ub}$.

Definition 3.4. Let $\varpi = ([\mu^{lb}, \mu^{ub}],\ [\nu^{lb}, \nu^{ub}])$, $\varpi_1 = ([\mu_1^{lb}, \mu_1^{ub}],\ [\nu_1^{lb}, \nu_1^{ub}])$ and $\varpi_2 = ([\mu_2^{lb}, \mu_2^{ub}],\ [\nu_2^{lb}, \nu_2^{ub}])$ be three IVFFNs and $\lambda > 0$. Then, the following operational laws are presented (Sergi et al., 2021; Rani and Mishra, 2022):

(i) $\varpi_1 \cup \varpi_2 = \left(\left[max\{\mu_1^{lb},\ \mu_2^{lb}\}, max\{\mu_1^{ub},\ \mu_2^{ub}\}\right],\ \left[min\ \{\nu_1^{lb},\ \nu_2^{lb}\},\ min\ \{\nu_1^{ub},\ \nu_2^{ub}\}\right]\right)$,

(ii) $\varpi_1 \cap \varpi_2 = \left(\left[min\{\mu_1^{lb},\ \mu_2^{lb}\}, min\{\mu_1^{ub},\ \mu_2^{ub}\}\right],\ \left[max\ \{\nu_1^{lb},\ \nu_2^{lb}\},\ max\ \{\nu_1^{ub},\ \nu_2^{ub}\}\right]\right)$,

(iii) $\varpi_1 \oplus \varpi_2 = \left(\left[\sqrt[3]{(\mu_1^{lb})^3 + (\mu_2^{lb})^3 - (\mu_1^{lb})^3(\mu_2^{lb})^3},\ \sqrt[3]{(\mu_1^{ub})^3 + (\mu_2^{ub})^3 - (\mu_1^{ub})^3(\mu_2^{ub})^3}\right],\ \left[\nu_1^{lb}\nu_2^{lb},\ \nu_1^{ub}\nu_2^{ub}\right]\right)$,

(iv) $\varpi_1 \otimes \varpi_2 = \left(\left[\mu_1^{lb}\mu_2^{lb},\ \mu_1^{ub}\mu_2^{ub}\right],\ \left[\sqrt[3]{(\nu_1^{lb})^3 + (\nu_2^{lb})^3 - (\nu_1^{lb})^3(\nu_2^{lb})^3},\ \sqrt[3]{(\nu_1^{ub})^3 + (\nu_2^{ub})^3 - (\nu_1^{ub})^3(\nu_2^{ub})^3}\right]\right)$,

(v) $\lambda\varpi = \left(\left[\sqrt[3]{1-\left(1-(\mu^{lb})^3\right)^{\lambda}},\ \sqrt[3]{1-\left(1-(\mu^{ub})^3\right)^{\lambda}}\right],\ \left[(\nu^{lb})^{\lambda},\ (\nu^{ub})^{\lambda}\right]\right)$,

(vi) $\varpi^{\lambda} = \left(\left[(\mu^{lb})^{\lambda},\ (\mu^{ub})^{\lambda}\right],\ \left[\sqrt[3]{1-\left(1-(\nu^{lb})^3\right)^{\lambda}},\ \sqrt[3]{1-\left(1-(\nu^{ub})^3\right)^{\lambda}}\right]\right)$.

Definition 3.5. Let $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ be a collection of IVFFNs, where $j = 1,\ 2,\ \ldots,\ r$.. Then, the “interval-valued Fermatean fuzzy weighted averaging (IVFFWA)” and “interval-valued Fermatean fuzzy weighted geometric (IVFFWG)” operators are presented as (Rani and Mishra, 2022).

$$IVFFWA\ (\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_r) = \bigoplus_{j=1}^{r} \wp_j\ \varpi_j$$

$$= \left(\left[\sqrt[3]{1-\prod_{j=1}^{r}\left(1-\left(\mu_j^{lb}\right)^3\right)^{\wp_j}}, \sqrt[3]{1-\prod_{j=1}^{r}\left(1-\left(\mu_j^{ub}\right)^3\right)^{\wp_j}}\right],\ \left[\prod_{j=1}^{r}\left(\nu_j^{lb}\right)^{\wp_j},\ \prod_{j=1}^{r}\left(\nu_j^{ub}\right)^{\wp_j}\right]\right), \tag{1}$$

$$IVFFWG\left(\varpi_1,\ \varpi_2, \ldots,\ \varpi_r\right) = \bigotimes_{j=1}^{r} \varpi_j^{\wp_j}$$

$$= \left(\left[\prod_{j=1}^{r}\left(\mu_j^{lb}\right)^{\wp_j},\ \prod_{j=1}^{r}\left(\mu_j^{ub}\right)^{\wp_j}\right],\ \left[\sqrt[3]{1-\prod_{j=1}^{r}\left(1-\left(\nu_j^{lb}\right)^3\right)^{\wp_j}}, \sqrt[3]{1-\prod_{j=1}^{r}\left(1-\left(\nu_j^{ub}\right)^3\right)^{\wp_j}}\right]\right), \tag{2}$$

wherein $\wp_j$ be the weights of ϖ_j satisfying $\wp_j \in [0,\ 1]$ and $\sum_{j=1}^{r}\wp_j = 1$..

4. New score function for IVFFNs

This section firstly reviews some extant score and accuracy functions for prioritizing arbitrary IVFFNs and then discusses their counter-intuitive cases. Further, to conquer the shortcomings of extant functions, a new score function for IVFFN is presented with its enviable properties.

Definition 4.1. ((*Rani and Mishra, 2022*).) For any IVFFN $\varpi = \left(\left[\mu_{\varpi}^{lb}, \mu_{\varpi}^{ub}\right],\ \left[\nu_{\varpi}^{lb}, \nu_{\varpi}^{ub}\right]\right)$, the score function of ϖ is given by.

$$\Im_1\left(\varpi\right) = \frac{1}{2}\left(\left(\mu_{\varpi}^{lb}\right)^3 + \left(\mu_{\varpi}^{ub}\right)^3 - \left(\nu_{\varpi}^{lb}\right)^3 - \left(\nu_{\varpi}^{ub}\right)^3\right),\quad \Im_1\left(\varpi\right) \in [-1, 1]. \tag{3}$$

The higher the score function $\Im_1(\varpi)$ the superior the ϖ Particularly, if $\Im_1(\varpi) = 1$, then ϖ is the largest IVFFN $([1,1],[0,0])$ and if $\Im_1(\varpi) = -1$, then ϖ is the least IVFFN $([0,0],[1,1])$.

Example 4.1. Let $\varpi_1 = ([0.45, 0.55], [0.30, 0.50])$ and $\varpi_2 = ([0.7, 0.9], [0.1, 0.3]) \in$ *IVFFNs*. If we utilize Eq. (3) to ϖ_1 and ϖ_2 then we get $\Im_1(\varpi_1) = 0.102 < \Im_1(\varpi_2) = 0.890$. Hence, we get $\varpi_1 < \varpi_2$.

In the subsequent example, we can see that $\Im_1(\varpi)$ is not sufficient for ordering the arbitrary IVFFNs.

Example 4.2. If we take $\varpi_1 = ([0.42, 0.75], [0.42, 0.75])$ and $\varpi_2 = ([0.25, 0.60], [0.25, 0.60])$, then $\Im_1(\varpi_1) = \Im_1(\varpi_2) = 0$. In this case, the function (3) cannot differentiate the IVFFNs ϖ_1 and ϖ_2.

Example 4.3. Let $\varpi_1 = \left(\left[\sqrt[3]{0.2}, \sqrt[3]{0.3}\right], \left[\sqrt[3]{0.35}, \sqrt[3]{0.5}\right]\right)$ and $\varpi_2 = \left(\left[\sqrt[3]{0.3}, \sqrt[3]{0.4}\right], \left[\sqrt[3]{0.45}, \sqrt[3]{0.6}\right]\right)$. If we utilize Eq. (3) to ϖ_1 and ϖ_2, then we obtain $\Im_1(\varpi_1) = \Im_1(\varpi_2) = -0.125$. Hence, we get $\varpi_1 = \varpi_2$, which implies incomparable using the score function.

To handle such types of issues, Rani and Mishra (2022) presented the following definition:

Definition 4.2. ((*Rani and Mishra, 2022*).) For any IVFFN $\varpi = \left(\left[\mu_{\varpi}^{lb}, \mu_{\varpi}^{ub}\right],\ \left[\nu_{\varpi}^{lb}, \nu_{\varpi}^{ub}\right]\right)$, the accuracy function of ϖ is presented as follows:

$$\rho\left(\varpi\right) = \frac{1}{2}\left(\left(\mu_{\varpi}^{lb}\right)^3 + \left(\mu_{\varpi}^{ub}\right)^3 + \left(\nu_{\varpi}^{lb}\right)^3 + \left(\nu_{\varpi}^{ub}\right)^3\right),\quad \rho\left(\varpi\right) \in [0, 1]. \tag{4}$$

Example 4.4. Let $\varpi_1 = ([\sqrt[3]{0.2}, \sqrt[3]{0.3}], [\sqrt[3]{0.35}, \sqrt[3]{0.5}])$ and $\varpi_2 = ([\sqrt[3]{0.3}, \sqrt[3]{0.4}], [\sqrt[3]{0.45}, \sqrt[3]{0.6}])$. If we utilize the score function $\Im_1$ to rank the IVFFNs ϖ_1 and ϖ_2, then we obtain $\Im_1(\varpi_1) = \Im_1(\varpi_2) = -0.125$. This implies $\varpi_1 = \varpi_2$ but $\varpi_1 \neq \varpi_2$, thus, the function (3) fails to rank the IVFFNs ϖ_1 and ϖ_2. Next, we apply formula (4) to the IVFFNs ϖ_1 and ϖ_2, then we get$\rho(\varpi_1) = 0.675 < \rho(\varpi_2) = 0.875 \Rightarrow \varpi_1 < \varpi_2$.

In the following example, Eq. (3) and Eq. (4) are failed to rank the IVFFNs ϖ_1 andϖ_2 :.

Example 4.5. For any two IVFFNs $\varpi_1 = ([\sqrt[3]{0.30}, \sqrt[3]{0.35}], [\sqrt[3]{0.45}, \sqrt[3]{0.60}])$ and $\varpi_2 = ([\sqrt[3]{0.25}, \sqrt[3]{0.40}], [\sqrt[3]{0.50}, \sqrt[3]{0.55}])$, we have $\Im_1(\varpi_1) = \Im_1(\varpi_2) = -0.100 \Rightarrow \varpi_1 = \varpi_2$. But $\varpi_1 \neq \varpi_2$, then, we utilize accuracy function to these IVFFNs, then we obtain $\rho(\varpi_1) = \rho(\varpi_2) = 0.850 \Rightarrow \varpi_1 = \varpi_2$. However,$\varpi_1 \neq \varpi_2$.

In order to overcome the limitations of Eqs. (3)-(4), Jeevaraj (2021) introduced a new score function for comparing such types of IVFFNs as follows:

Definition 4.3. ((*Jeevaraj, 2021*).) For any IVFFN $\varpi = ([\mu_\varpi^{lb}, \mu_\varpi^{ub}], [\nu_\varpi^{lb}, \nu_\varpi^{ub}])$, the score function of ϖ is given by.

$$\Im_2\left(\varpi\right) = \frac{1}{2}\left(-\left(\mu_\varpi^{lb}\right)^3 + \left(\mu_\varpi^{ub}\right)^3 + \left(\nu_\varpi^{lb}\right)^3 - \left(\nu_\varpi^{ub}\right)^3\right),\ \Im_2\left(\varpi\right) \in [-0.5, 0.5], \tag{5a}$$

$$\Im_3\left(\varpi\right) = \frac{1}{2}\left(-\left(\mu_\varpi^{lb}\right)^3 + \left(\mu_\varpi^{ub}\right)^3 - \left(\nu_\varpi^{lb}\right)^3 + \left(\nu_\varpi^{ub}\right)^3\right),\ \Im_3\left(\varpi\right) \in [-0.5, 0.5]. \tag{5b}$$

The following examples demonstrate the advantage and drawback of the score functions (5a) and (5b):

Example 4.6. Let $\varpi_1 = ([\sqrt[3]{0.30}, \sqrt[3]{0.35}], [\sqrt[3]{0.45}, \sqrt[3]{0.60}])$ and $\varpi_2 = ([\sqrt[3]{0.25}, \sqrt[3]{0.40}], [\sqrt[3]{0.50}, \sqrt[3]{0.55}])$. If utilize the score function $\Im_1$ to find the best alternative out of these two alternatives ϖ_1 and ϖ_2, then we obtain $\Im_1(\varpi_1) = \Im_1(\varpi_2) = -0.100 \Rightarrow \varpi_1 = \varpi_2$, but $\varpi_1 \neq \varpi_2$. Next, we use accuracy function to the IVFFNs, then we get $\rho(\varpi_1) = \rho(\varpi_2) = 0.850 \Rightarrow \varpi_1 = \varpi_2$. However, ϖ_1 and ϖ_2 are two different IVFFNs. Next, if we use score function $\Im_2$ to the IVFFNs ϖ_1 and ϖ_2 then we get $\Im_2(\varpi_1) = -0.050 < \Im_2(\varpi_2) = 0.050$, which implies that $\varpi_1 < \varpi_2$..

Example 4.7. Let $\varpi_1 = ([\sqrt[3]{0.25}, \sqrt[3]{0.3}], [\sqrt[3]{0.35}, \sqrt[3]{0.5}])$ and $\varpi_2 = ([\sqrt[3]{0.15}, \sqrt[3]{0.4}], [\sqrt[3]{0.25}, \sqrt[3]{0.6}])$. If we apply score function $\Im_2$ to ϖ_1 and ϖ_2, then we get $\Im_2(\varpi_1) = \Im_2(\varpi_2) = -0.050$ and $\Im_3(\varpi_1) = \Im_3(\varpi_2) = 0.150$ which implies that $\varpi_1 = \varpi_2$, i.e., alternatives ϖ_1 and ϖ_2 are not discriminable using score function $\Im_2$ and $\Im_3$. Consequently, it has been obtained that the extant score and accuracy values are failed to rank the IVFFNs in several situations. This is the inspiration for developing a novel score function.

In the following, we propose a novel modified score function by considering MD, ND and indeterminacy degree:

Definition 4.4. The score function of $\varpi = ([\mu_\varpi^{lb}, \mu_\varpi^{ub}], [\nu_\varpi^{lb}, \nu_\varpi^{ub}])$ is defined as follows:

$$\Im\left(\varpi\right) = \frac{1}{2}\left(\left(\left(\mu_\varpi^{lb}\right)^3 - \left(\nu_\varpi^{lb}\right)^3\right)\left(1 + \sqrt[3]{1 - \left(\mu_\varpi^{lb}\right)^3 - \left(\nu_\varpi^{lb}\right)^3}\right) + \left(\left(\mu_\varpi^{ub}\right)^3 - \left(\nu_\varpi^{ub}\right)^3\right)\left(1 + \sqrt[3]{1 - \left(\mu_\varpi^{ub}\right)^3 - \left(\nu_\varpi^{ub}\right)^3}\right)\right), \tag{6}$$

where $\Im(\varpi) \in [-1, 1]$. The higher value of $\Im(\varpi)$ determines the best alternative.

The developed score function (6) is applied on Example 3.7 to rank the IVFFNs $\varpi_1 = ([\sqrt[3]{0.25}, \sqrt[3]{0.3}], [\sqrt[3]{0.35}, \sqrt[3]{0.5}])$ and $\varpi_2 = ([\sqrt[3]{0.15}, \sqrt[3]{0.4}], [\sqrt[3]{0.25}, \sqrt[3]{0.6}])$, we get $\Im(\varpi_1) = -0.245$, $\Im(\varpi_2) = -0.192$, this implies $\varpi_1 < \varpi_2$. Thus, it will be more preferable and reliable than the existing score and accuracy functions.

Proposition 4.1. *If $\varpi = (\mu_\varpi^{lb},\ 1 - \mu_\varpi^{lb})$ is a fuzzy number, then* $\Im(\varpi) = \left(\left(\left(\mu_\varpi^{lb}\right)^3 - \left(1 - \mu_\varpi^{lb}\right)^3\right)\left(1 + \sqrt[3]{3\ \mu_\varpi^{lb}\left(1 - \mu_\varpi^{lb}\right)}\right)\right)$.

Proposition 4.2. *If $\varpi = (\mu_\varpi^{lb}, \nu_\varpi^{lb})$ is an intuitionistic fuzzy number, then*$\Im(\varpi) = \left(\left(\left(\mu_\varpi^{lb}\right)^3 - \left(\nu_\varpi^{lb}\right)^3\right)\left(1 + \sqrt[3]{1 - \left(\mu_\varpi^{lb}\right)^3 - \left(\nu_\varpi^{lb}\right)^3}\right)\right)$.

Proposition 4.3. ((*Maximum value*).) *If $\varpi = ([1.0, 1.0], [0.0, 0.0]) \in$ IVFFN, then $\Im(\varpi) = 1$.*

Proposition 4.4. ((*Minimum value*).) *If $\varpi = ([0.0, 0.0], [1.0, 1.0]) \in$ IVFFN, then $\Im(\varpi) = -1$.*

Theorem 4.1. *Let $\varpi_1, \varpi_2 \in$ IVFFNs. If $\varpi_1 \subseteq \varpi_2$, then$\Im(\varpi_1) \leq \Im(\varpi_2)$.*

Proof. It is equivalent to the result of *Theorem 4.10* in Jeevaraj (2021).

Theorem 4.2. *Let $\varpi_1, \varpi_2, \varpi_3 \in$ IVFFNs and $\varpi_1 \subseteq \varpi_2$. If $\Im(\varpi_1) \leq \Im(\varpi_2)$, then$\Im(\varpi_1 + \varpi_3) \leq \Im(\varpi_2 + \varpi_3)$.*

Proof. Let $\varpi_1 = \left(\left[\mu_{\varpi_1}^{lb}, \mu_{\varpi_1}^{ub}\right], \left[\nu_{\varpi_1}^{lb}, \nu_{\varpi_1}^{ub}\right]\right)$,$\varpi_2 = \left(\left[\mu_{\varpi_2}^{lb}, \mu_{\varpi_2}^{ub}\right], \left[\nu_{\varpi_2}^{lb}, \nu_{\varpi_2}^{ub}\right]\right)$ and $\varpi_3 = \left(\left[\mu_{\varpi_3}^{lb}, \mu_{\varpi_3}^{ub}\right], \left[\nu_{\varpi_3}^{lb}, \nu_{\varpi_3}^{ub}\right]\right)$ be three IVFFNs and$\varpi_1 \subseteq \varpi_2$.

Since $\varpi_1 \subseteq \varpi_2$, it implies.

$$\mu_{\varpi_1}^{lb} \leq \mu_{\varpi_2}^{lb},\ \mu_{\varpi_1}^{ub} \leq \mu_{\varpi_2}^{ub},\ \nu_{\varpi_1}^{lb} \geq \nu_{\varpi_2}^{lb},\ \nu_{\varpi_1}^{ub} \geq \nu_{\varpi_2}^{ub}. \tag{7}$$

From *Definition 3.4*, we obtain.

$$\varpi_1 + \varpi_3 = \left(\left[\sqrt[3]{\left(\mu_{\varpi_1}^{lb}\right)^3 + \left(\mu_{\varpi_3}^{lb}\right)^3 - \left(\mu_{\varpi_1}^{lb}\right)^3\left(\mu_{\varpi_3}^{lb}\right)^3},\ \sqrt[3]{\left(\mu_{\varpi_1}^{ub}\right)^3 + \left(\mu_{\varpi_3}^{ub}\right)^3 - \left(\mu_{\varpi_1}^{ub}\right)^3\left(\mu_{\varpi_3}^{ub}\right)^3}\right],\ \left[\nu_{\varpi_1}^{lb}\nu_{\varpi_3}^{lb},\ \nu_{\varpi_1}^{ub}\nu_{\varpi_3}^{ub}\right]\right)$$

and .

$$\varpi_2 + \varpi_3 = \left(\left[\sqrt[3]{\left(\mu_{\varpi_2}^{lb}\right)^3 + \left(\mu_{\varpi_3}^{lb}\right)^3 - \left(\mu_{\varpi_2}^{lb}\right)^3\left(\mu_{\varpi_3}^{lb}\right)^3},\ \sqrt[3]{\left(\mu_{\varpi_2}^{ub}\right)^3 + \left(\mu_{\varpi_3}^{ub}\right)^3 - \left(\mu_{\varpi_2}^{ub}\right)^3\left(\mu_{\varpi_3}^{ub}\right)^3}\right],\ \left[\nu_{\varpi_2}^{lb}\nu_{\varpi_3}^{lb},\ \nu_{\varpi_2}^{ub}\nu_{\varpi_3}^{ub}\right]\right).$$

From Eq. (7), it is very clear that.

$$\sqrt[3]{\left(\mu^{lb}_{\varpi_1}\right)^3+\left(\mu^{lb}_{\varpi_3}\right)^3-\left(\mu^{lb}_{\varpi_1}\right)^3\left(\mu^{lb}_{\varpi_3}\right)^3} \leq \sqrt[3]{\left(\mu^{lb}_{\varpi_2}\right)^3+\left(\mu^{lb}_{\varpi_3}\right)^3-\left(\mu^{lb}_{\varpi_2}\right)^3\left(\mu^{lb}_{\varpi_3}\right)^3},$$

$$\sqrt[3]{\left(\mu^{ub}_{\varpi_1}\right)^3+\left(\mu^{ub}_{\varpi_3}\right)^3-\left(\mu^{ub}_{\varpi_1}\right)^3\left(\mu^{ub}_{\varpi_3}\right)^3} \leq \sqrt[3]{\left(\mu^{ub}_{\varpi_2}\right)^3+\left(\mu^{ub}_{\varpi_3}\right)^3-\left(\mu^{ub}_{\varpi_2}\right)^3\left(\mu^{ub}_{\varpi_3}\right)^3},$$

$$\nu^{lb}_{\varpi_1}\ \nu^{lb}_{\varpi_3}\ \geq \nu^{lb}_{\varpi_2}\ \nu^{lb}_{\varpi_3}\ \text{and}\ \nu^{ub}_{\varpi_1}\ \nu^{ub}_{\varpi_3}\ \geq \nu^{ub}_{\varpi_2}\ \nu^{ub}_{\varpi_3}..$$

$$\Rightarrow \varpi_1+\varpi_3\ \subseteq\ \varpi_2+\varpi_3. \tag{8}$$

From Eq. (8) and *Theorem 4.1*, we get.

$$\Im(\varpi_1+\varpi_3)\ \leq \Im(\varpi_2+\varpi_3)$$

Theorem 4.3. *Let* $\varpi_1,\ \varpi_2 \in$ *IVFFNs*,$\gamma\ >0$ *and* $\varpi_1\subseteq\varpi_2$. *If* $\Im(\varpi_1)\subseteq\Im(\varpi_2)$, *then*$\Im(\gamma\ \varpi_1)\subseteq\Im(\gamma\ \varpi_2)$.

Proof. It is similar to the proof of *Theorem* 4.2.

Definition 4.6. Let $\varpi_1,\ \varpi_2 \in$ *IVFFNs*. Then corresponding to the extant score and accuracy values and proposed score function, a comparative scheme is presented to compare any two IVFFNs ϖ_1 andϖ_2 :

- If $\Im_1(\varpi_1)\rangle\Im_1(\varpi_2)$, then $\varpi_1\ \succ\ \varpi_2$;
- If $\Im_1(\varpi_1)\langle\Im_1(\varpi_2)$, then $\varpi_1\ \prec\ \varpi_2$;
- If $\Im_1(\varpi_1)=\Im_1(\varpi_2)$, then
  - If $\rho(\varpi_1)>\rho(\varpi_2)$, then $\varpi_1\ \succ\ \varpi_2$;
  - If $\rho(\varpi_1)\ <\ \rho(\varpi_2)$, then $\varpi_1\ \prec\ \varpi_2$;
  - If $\rho(\varpi_1)=\rho(\varpi_2)$, then $\varpi_1\ =\ \varpi_2$.
    - o If $\Im_2(\varpi_1)\rangle\Im_2(\varpi_2)$, then $\varpi_1\ \succ\ \varpi_2$;
    - o If $\Im_2(\varpi_1)\langle\Im_2(\varpi_2)$, then $\varpi_1\ \prec\ \varpi_2$;
    - o If $\Im_2(\varpi_1)=\Im_2(\varpi_2)$, then
      - If $\Im(\varpi_1)\ >\ \Im(\varpi_2)$, then $\varpi_1\ \succ\ \varpi_2$;
      - If $\Im(\varpi_1)\ <\ \Im(\varpi_2)$, then $\varpi_1\ \prec\ \varpi_2$;
      - If $\Im(\varpi_1)\ =\ \Im(\varpi_2)$, then $\varpi_1\ =\ \varpi_2$.

5. Interval-valued Fermatean fuzzy Einstein aggregation operators

The current section introduces the concept of *"interval-valued Fermatean fuzzy Einstein averaging (IVFFEA)"* and *"interval-valued Fermatean fuzzy Einstein geometric (IVFFEG)"* operators with some desirable properties. For this, firstly we discuss Einstein operations of IVFFSs.

5.1. Einstein operations for IVFFSs

Definition 5.1. Let $\varpi=([a,b],\ [c,d]),\varpi_1=([a_1,b_1],\ [c_1,d_1])$ and $\varpi_2=([a_2,b_2],\ [c_2,d_2])$ be three IVFFNs and $\lambda>0$ be any real number. Then, some fundamental Einstein operations for IVFFNs are presented as.

(i)$\varpi_1\oplus_\varepsilon\varpi_2$

$$=\left(\begin{array}{c}\left[\dfrac{\sqrt[3]{a_1^3+a_2^3}}{\sqrt[3]{1+a_1^3a_2^3}},\dfrac{\sqrt[3]{b_1^3+b_2^3}}{\sqrt[3]{1+b_1^3b_2^3}}\right],\\ \left[\dfrac{c_1\,c_2}{\sqrt[3]{1+(1-c_1^3)(1-c_2^3)}},\dfrac{d_1\,d_2}{\sqrt[3]{1+\left(1-d_1^3\right)\left(1-d_2^3\right)}}\right]\end{array}\right);$$

(ii)$\varpi_1\otimes_\varepsilon\varpi_2$

$$=\left(\begin{array}{c}\left[\dfrac{a_1\,a_2}{\sqrt[3]{1+(1-a_1^3)(1-a_2^3)}},\dfrac{b_1\,b_2}{\sqrt[3]{1+\left(1-b_1^3\right)\left(1-b_2^3\right)}}\right],\\ \left[\dfrac{\sqrt[3]{c_1^3+c_2^3}}{\sqrt[3]{1+c_1^3c_2^3}},\dfrac{\sqrt[3]{d_1^3+d_2^3}}{\sqrt[3]{1+d_1^3d_2^3}}\right]\end{array}\right);$$

(iii)$\lambda\cdot_\varepsilon\varpi$

$$=\left(\begin{array}{c}\left[\sqrt[3]{\dfrac{(1+a^3)^\lambda-(1-a^3)^\lambda}{(1+a^3)^\lambda+(1-a^3)^\lambda}},\sqrt[3]{\dfrac{\left(1+b^3\right)^\lambda-\left(1-b^3\right)^\lambda}{\left(1+b^3\right)^\lambda+\left(1-b^3\right)^\lambda}}\right],\\ \left[\dfrac{\sqrt[3]{2\left(c^3\right)^\lambda}}{\sqrt[3]{(2-c^3)^\lambda+(c^3)^\lambda}},\dfrac{\sqrt[3]{2\left(d^3\right)^\lambda}}{\sqrt[3]{\left(2-d^3\right)^\lambda+\left(d^3\right)^\lambda}}\right]\end{array}\right);$$

(iv)ϖ^λ

$$=\left(\begin{array}{c}\left[\dfrac{\sqrt[3]{2\left(a^3\right)^\lambda}}{\sqrt[3]{(2-a^3)^\lambda+(a^3)^\lambda}},\dfrac{\sqrt[3]{2\left(b^3\right)^\lambda}}{\sqrt[3]{\left(2-b^3\right)^\lambda+\left(b^3\right)^\lambda}}\right],\\ \left[\sqrt[3]{\dfrac{(1+c^3)^\lambda-(1-c^3)^\lambda}{(1+c^3)^\lambda+(1-c^3)^\lambda}},\sqrt[3]{\dfrac{\left(1+d^3\right)^\lambda-\left(1-d^3\right)^\lambda}{\left(1+d^3\right)^\lambda+\left(1-d^3\right)^\lambda}}\right]\end{array}\right).$$

Corresponding to the *Definition 5.1*, we further discuss the following relations:

Proposition 5.1. *Let* $\varpi=([a,b],\ [c,d]),\varpi_1=([a_1,b_1],\ [c_1,d_1])$ *and* $\varpi_2=([a_2,b_2],\ [c_2,d_2])$ *be three IVFFNs and*$\lambda,\ \lambda_1,\ \lambda_2\ >0$. *Then, we have.*

(i) $\varpi_1\oplus_\varepsilon\varpi_2\ =\ \varpi_2\oplus_\varepsilon\ \varpi_1$;
(ii) $\varpi_1\otimes_\varepsilon\varpi_2\ =\ \varpi_2\otimes_\varepsilon\varpi_1$;
(iii) $\lambda\,(\varpi_1\oplus_\varepsilon\varpi_2)\ =\ \lambda\,\varpi_1\oplus_\varepsilon\lambda\,\varpi_2$;
(iv) $\lambda_1\,\varpi\oplus_\varepsilon\lambda_2\,\varpi\ =\ (\lambda_1\ +\ \lambda_2)\cdot_\varepsilon\ \varpi$;
(v) $(\varpi_1\otimes_\varepsilon\varpi_2)^\lambda\ =\ \varpi_1^\lambda\otimes_\varepsilon\varpi_2^\lambda$;
(vi) $\varpi^{\lambda_1}\otimes_\varepsilon\ \varpi^{\lambda_2}\ =\ \varpi^{\lambda_1+\lambda_2}$.

Proof. It is trivial by *Definition 5.1*. Hence, we omitted the proof.

5.2. Interval-valued Fermatean fuzzy Einstein weighted averaging (IVFFEWA) operator

Definition 5.2. Assume that $\varpi_j=\left(\left[\mu_j^{lb},\mu_j^{ub}\right],\left[\nu_j^{lb},\nu_j^{ub}\right]\right)$ $(j=1,\ 2,...,\ r)$ be a set of IVFFNs and $\psi=(\psi_1,\psi_2,\ ...,\ \psi_r)^T$ is the weight value of ϖ_j satisfying $\psi_j\in[0,1]$ with $\sum_{j=1}^r\psi_j=1$. Then, the IVFFEWA operator of dimension r is a function $IVFFEWA:\Lambda^r\to\Lambda$, and defined as.

$$IVFFEWA(\varpi_1,\ \varpi_2,\ ...,\ \varpi_r)=\psi_1\cdot_\varepsilon\varpi_1\oplus_\varepsilon\ \psi_2\cdot_\varepsilon\varpi_2\oplus_\varepsilon...\oplus_\varepsilon\psi_r\cdot_\varepsilon\varpi_r,$$

wherein Λ is the set of all IVFFNs. In particular, if $\psi=\left(\frac{1}{r},\frac{1}{r},\ ...,\ \frac{1}{r}\right)^T$, then the IVFFEWA operator reduces to *"interval-valued Fermatean fuzzy weighted averaging (IVFFWA)"* operator of dimension r, defined as.

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = \frac{1}{r}\cdot_\varepsilon(\varpi_1 \oplus_\varepsilon \varpi_2 \oplus_\varepsilon ... \oplus_\varepsilon \varpi_r)$$
$$= IVFFWA(\varpi_1, \varpi_2, ..., \varpi_r).$$

By means of Definition 5.2, we find the subsequent outcome:

Theorem 5.1. *Let $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)(j = 1, 2, ..., r)$ be a set of IVFFNs and $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ be the weight vector of ϖ_j with $\psi_j \in [0, 1]$ and $\sum_{j=1}^r \psi_j = 1$. Then, the aggregation by an IVFFEWA operator is again an IVFFN and.*

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = \left(\left[\frac{\sqrt[3]{\prod_{j=1}^r\left(1+(\mu_j^{lb})^3\right)^{\psi_j} - \prod_{j=1}^r\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+(\mu_j^{lb})^3\right)^{\psi_j} + \prod_{j=1}^r\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^r\left(1+(\mu_j^{ub})^3\right)^{\psi_j} - \prod_{j=1}^r\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+(\mu_j^{ub})^3\right)^{\psi_j} + \prod_{j=1}^r\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}\right], \left[\frac{\sqrt[3]{2\prod_{j=1}^r\left((\nu_j^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-(\nu_j^{lb})^3\right)^{\psi_j} + \prod_{j=1}^r\left((\nu_j^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^r\left((\nu_j^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-(\nu_j^{ub})^3\right)^{\psi_j} + \prod_{j=1}^r\left((\nu_j^{ub})^3\right)^{\psi_j}}}\right]\right). \tag{9}$$

Proof. By mathematical induction on positive integer r, we will prove this theorem.

When $r = 1$, Eq. (9) becomes.

$$IVFFEWA(\varpi_1) = \left(\left[\frac{\sqrt[3]{\left(1+(\mu^{lb})^3\right)^{\psi_1} - \left(1-(\mu^{lb})^3\right)^{\psi_1}}}{\sqrt[3]{\left(1+(\mu^{lb})^3\right)^{\psi_1} + \left(1-(\mu^{lb})^3\right)^{\psi_1}}}, \frac{\sqrt[3]{\left(1+(\mu^{ub})^3\right)^{\psi_1} - \left(1-(\mu^{ub})^3\right)^{\psi_1}}}{\sqrt[3]{\left(1+(\mu^{ub})^3\right)^{\psi_1} + \left(1-(\mu^{ub})^3\right)^{\psi_1}}}\right],\right.$$
$$\left.\left[\frac{\sqrt[3]{2\left((\nu^{lb})^3\right)^{\psi_1}}}{\sqrt[3]{\left(2-(\nu^{lb})^3\right)^{\psi_1} + \left((\nu^{lb})^3\right)^{\psi_1}}}, \frac{\sqrt[3]{2\left((\nu^{ub})^3\right)^{\psi_1}}}{\sqrt[3]{\left(2-(\nu^{ub})^3\right)^{\psi_1} + \left((\nu^{ub})^3\right)^{\psi_1}}}\right]\right).$$

Thus, Eq. (9) holds for $r = 1$..

Assume that Eq. (9) satisfies for $r = k$, i.e.,

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_k) = \left(\left[\frac{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{lb})^3\right)^{\psi_j} - \prod_{j=1}^k\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{lb})^3\right)^{\psi_j} + \prod_{j=1}^k\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{ub})^3\right)^{\psi_j} - \prod_{j=1}^k\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{ub})^3\right)^{\psi_j} + \prod_{j=1}^k\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}\right], \left[\frac{\sqrt[3]{2\prod_{j=1}^k\left((\nu_j^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(2-(\nu_j^{lb})^3\right)^{\psi_j} + \prod_{j=1}^k\left((\nu_j^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^k\left((\nu_j^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(2-(\nu_j^{ub})^3\right)^{\psi_j} + \prod_{j=1}^k\left((\nu_j^{ub})^3\right)^{\psi_j}}}\right]\right). \tag{10}$$

If $r = k + 1$, then by Definition 5.2, we get

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_k, \varpi_{k+1}) =$$

$$\left(\left[\frac{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{lb})^3\right)^{\psi_j} - \prod_{j=1}^k\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{lb})^3\right)^{\psi_j} + \prod_{j=1}^k\left(1-(\mu_j^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{ub})^3\right)^{\psi_j} - \prod_{j=1}^k\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^k\left(1+(\mu_j^{ub})^3\right)^{\psi_j} + \prod_{j=1}^k\left(1-(\mu_j^{ub})^3\right)^{\psi_j}}}\right],\right.$$

$$\left[\frac{\sqrt[3]{2\prod_{j=1}^{k}\left(\left(\nu_j^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k}\left(2-\left(\nu_j^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k}\left(\left(\nu_j^{lb}\right)^3\right)^{\psi_j}}},\frac{\sqrt[3]{2\prod_{j=1}^{k}\left(\left(\nu_j^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k}\left(2-\left(\nu_j^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k}\left(\left(\nu_j^{ub}\right)^3\right)^{\psi_j}}}\right]\Bigg)$$

$$\oplus_\varepsilon\left(\left[\frac{\sqrt[3]{\left(1+\left(\mu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}-\left(1-\left(\mu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}}}{\sqrt[3]{\left(1+\left(\mu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}+\left(1-\left(\mu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}}},\frac{\sqrt[3]{\left(1+\left(\mu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}-\left(1-\left(\mu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}}}{\sqrt[3]{\left(1+\left(\mu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}+\left(1-\left(\mu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}}}\right],\right.$$

$$\left.\left[\frac{\sqrt[3]{2\left(\left(\nu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}}}{\sqrt[3]{\left(2-\left(\nu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}+\left(\left(\nu_{k+1}^{lb}\right)^3\right)^{\psi_{k+1}}}},\frac{\sqrt[3]{2\left(\left(\nu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}}}{\sqrt[3]{\left(2-\left(\nu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}+\left(\left(\nu_{k+1}^{ub}\right)^3\right)^{\psi_{k+1}}}}\right]\right).$$

$$=\left(\left[\frac{\sqrt[3]{\prod_{j=1}^{k+1}\left(1+\left(\mu_j^{lb}\right)^3\right)^{\psi_j}-\prod_{j=1}^{k+1}\left(1-\left(\mu_j^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k+1}\left(1+\left(\mu_j^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k+1}\left(1-\left(\mu_j^{lb}\right)^3\right)^{\psi_j}}},\frac{\sqrt[3]{\prod_{j=1}^{k+1}\left(1+\left(\mu_j^{ub}\right)^3\right)^{\psi_j}-\prod_{j=1}^{k+1}\left(1-\left(\mu_j^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k+1}\left(1+\left(\mu_j^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k+1}\left(1-\left(\mu_j^{ub}\right)^3\right)^{\psi_j}}}\right],\right.$$

$$\left.\left[\frac{\sqrt[3]{2\prod_{j=1}^{k+1}\left(\left(\nu_j^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k+1}\left(2-\left(\nu_j^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k+1}\left(\left(\nu_j^{lb}\right)^3\right)^{\psi_j}}},\frac{\sqrt[3]{2\prod_{j=1}^{k+1}\left(\left(\nu_j^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{k+1}\left(2-\left(\nu_j^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{k+1}\left(\left(\nu_j^{ub}\right)^3\right)^{\psi_j}}}\right]\right).$$

Therefore, by mathematical induction, Eq. (9) holds for all r.

Corresponding to the Theorem 5.1, we figure out the following axioms:

Property 5.1. ((*Idempotency*).) *Let* $\varpi_j = \left(\left[\mu_j^{lb},\mu_j^{ub}\right],\left[\nu_j^{lb},\nu_j^{ub}\right]\right)$ *($j=1,2,\ldots,r$) be the set of IVFFNs. If* $\varpi_j = \varpi = \left(\left[\mu^{lb},\mu^{ub}\right],\left[\nu^{lb},\nu^{ub}\right]\right)$, *for all* $j=1,2,\ldots,r$, *then* $IVFFEWA(\varpi_1,\varpi_2,\ldots,\varpi_r) = \varpi$.

Proof. As $\varpi_j = \varpi = \left(\left[\mu^{lb},\mu^{ub}\right],\left[\nu^{lb},\nu^{ub}\right]\right)$, then by *Definition 5.2*, we have.

$IVFFEWA(\varpi_1,\varpi_2,\ldots,\varpi_r) =$

$$=\left(\left[\frac{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\mu^{lb}\right)^3\right)^{\psi_j}-\prod_{j=1}^{r}\left(1-\left(\mu^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\mu^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(1-\left(\mu^{lb}\right)^3\right)^{\psi_j}}},\frac{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\mu^{ub}\right)^3\right)^{\psi_j}-\prod_{j=1}^{r}\left(1-\left(\mu^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\mu^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(1-\left(\mu^{ub}\right)^3\right)^{\psi_j}}}\right],\right.$$

$$\left.\left[\frac{\sqrt[3]{2\prod_{j=1}^{r}\left(\left(\nu^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(2-\left(\nu^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(\left(\nu^{lb}\right)^3\right)^{\psi_j}}},\frac{\sqrt[3]{2\prod_{j=1}^{r}\left(\left(\nu^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(2-\left(\nu^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(\left(\nu^{ub}\right)^3\right)^{\psi_j}}}\right]\right)$$

$$=\left(\left[\frac{\sqrt[3]{\left(1+\left(\mu^{lb}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}-\left(1-\left(\mu^{lb}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}}}{\sqrt[3]{\left(1+\left(\mu^{lb}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}+\left(1-\left(\mu^{lb}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}}},\frac{\sqrt[3]{\left(1+\left(\mu^{ub}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}-\left(1-\left(\mu^{ub}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}}}{\sqrt[3]{\left(1+\left(\mu^{ub}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}+\left(1-\left(\mu^{ub}\right)^3\right)^{\sum_{j=1}^{r}\psi_j}}}\right],\right.$$

$$\left[\frac{\sqrt[3]{2\left((\nu^{lb})^3\right)^{\sum_{j=1}^{r}\psi_j}}}{\sqrt[3]{\left(2-(\nu^{lb})^3\right)^{\sum_{j=1}^{r}\psi_j}+\left((\nu^{lb})^3\right)^{\sum_{j=1}^{r}\psi_j}}}, \frac{\sqrt[3]{2\left((\nu^{ub})^3\right)^{\sum_{j=1}^{r}\psi_j}}}{\sqrt[3]{\left(2-(\nu^{ub})^3\right)^{\sum_{j=1}^{r}\psi_j}+\left((\nu^{ub})^3\right)^{\sum_{j=1}^{r}\psi_j}}}\right]\Bigg)$$

$$= \left(\left[\mu^{lb}, \mu^{ub}\right], \left[\nu^{lb}, \nu^{ub}\right]\right) = \varpi.$$

Property 5.2. ((*Commutativity*).) *Let* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *and* $\overline{\varpi}_j = \left(\left[\overline{\mu}_j^{lb}, \overline{\mu}_j^{ub}\right], \left[\overline{\nu}_j^{lb}, \overline{\nu}_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) be the set of IVFFNs. Then.*

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = IVFFEWA(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r),$$

wherein $(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r)$ is any permutation of$(\varpi_1, \varpi_2, ..., \varpi_r)$.

Proof. As we know that.

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = \psi_1 \cdot_\varepsilon \varpi_1 \oplus_\varepsilon \psi_2 \cdot_\varepsilon \varpi_2 \oplus_\varepsilon ... \oplus_\varepsilon \psi_r \cdot_\varepsilon \varpi_r,$$

$$IVFFEWA(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r) = \psi_1 \cdot_\varepsilon \overline{\varpi}_1 \oplus_\varepsilon \psi_2 \cdot_\varepsilon \overline{\varpi}_2 \oplus_\varepsilon ... \oplus_\varepsilon \psi_r \cdot_\varepsilon \overline{\varpi}_r.$$

Since $(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r)$ is any permutation of $(\varpi_1, \varpi_2, ..., \varpi_r)$, thus the property $IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = IVFFEWA(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r)$ always holds.

Property 5.3. ((*Monotonicity*).) *Consider two collections* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *and* $\widetilde{\varpi}_j = \left(\left[\widetilde{\mu}_j^{lb}, \widetilde{\mu}_j^{ub}\right], \left[\widetilde{\nu}_j^{lb}, \widetilde{\nu}_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) such that* $\mu_j^{lb} \geq \widetilde{\mu}_j^{lb}, \mu_j^{ub} \geq \widetilde{\mu}_j^{ub}, \nu_j^{lb} \leq \widetilde{\nu}_j^{lb}$ *and* $\nu_j^{ub} \leq \widetilde{\nu}_j^{ub}$, *then*IVFFEWA $(\varpi_1, \varpi_2, ..., \varpi_r) \geq$ IVFFEWA$(\widetilde{\varpi}_1, \widetilde{\varpi}_2, ..., \widetilde{\varpi}_r)$..

Proof. As we know that.

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = \psi_1 \cdot_\varepsilon \varpi_1 \oplus_\varepsilon \psi_2 \cdot_\varepsilon \varpi_2 \oplus_\varepsilon ... \oplus_\varepsilon \psi_r \cdot_\varepsilon \varpi_r,$$

and

$$IVFFEWA(\widetilde{\varpi}_1, \widetilde{\varpi}_2, ..., \widetilde{\varpi}_r) = \psi_1 \cdot_\varepsilon \widetilde{\varpi}_1 \oplus_\varepsilon \psi_2 \cdot_\varepsilon \widetilde{\varpi}_2 \oplus_\varepsilon ... \oplus_\varepsilon \psi_r \cdot_\varepsilon \widetilde{\varpi}_r.$$

As $\mu_j^{lb} \geq \widetilde{\mu}_j^{lb}, \mu_j^{ub} \geq \widetilde{\mu}_j^{ub}, \nu_j^{lb} \leq \widetilde{\nu}_j^{lb}$ and $\nu_j^{ub} \leq \widetilde{\nu}_j^{ub}$, then $IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) \geq IVFFEWA(\widetilde{\varpi}_1, \widetilde{\varpi}_2, ..., \widetilde{\varpi}_r)$ always holds.

Property 5.4. ((*Boundedness*).) *Let* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) be a collection of IVFFNs and let* $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ *be the weight vector of* ϖ_j *such that* $\sum_{j=1}^{r}\psi_j = 1$. *Then,*

$$\varpi_{min} \leq IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) \leq \varpi_{max}, \text{ where } \varpi_{min} = \min_j(\varpi_j)$$

and$\varpi_{max} = \max_j(\varpi_j)$.

Proof. Let $\mu_{min}^{lb} = \min_j\left(\mu_j^{lb}\right), \mu_{min}^{ub} = \min_j\left(\mu_j^{ub}\right), \nu_{min}^{lb} = \min_j\left(\nu_j^{lb}\right), \nu_{min}^{ub} = \min_j\left(\nu_j^{ub}\right),$

$\mu_{max}^{lb} = \max_j\left(\mu_j^{lb}\right), \mu_{max}^{ub} = \max_j\left(\mu_j^{ub}\right), \nu_{max}^{lb} = \max_j\left(\nu_j^{lb}\right)$ and $\nu_{max}^{ub} = \max_j\left(\nu_j^{ub}\right)$.

Consider that

$$IVFFEWA(\varpi_1, \varpi_2, ..., \varpi_r) = \varpi = \left(\left[\mu^{lb}, \mu^{ub}\right], \left[\nu^{lb}, \nu^{ub}\right]\right).$$

Then obviously

$$\left(\left[\mu_{min}^{lb}, \mu_{min}^{ub}\right], \left[\nu_{max}^{lb}, \nu_{max}^{ub}\right]\right) \leq \left(\left[\mu^{lb}, \mu^{ub}\right], \left[\nu^{lb}, \nu^{ub}\right]\right), \tag{11}$$

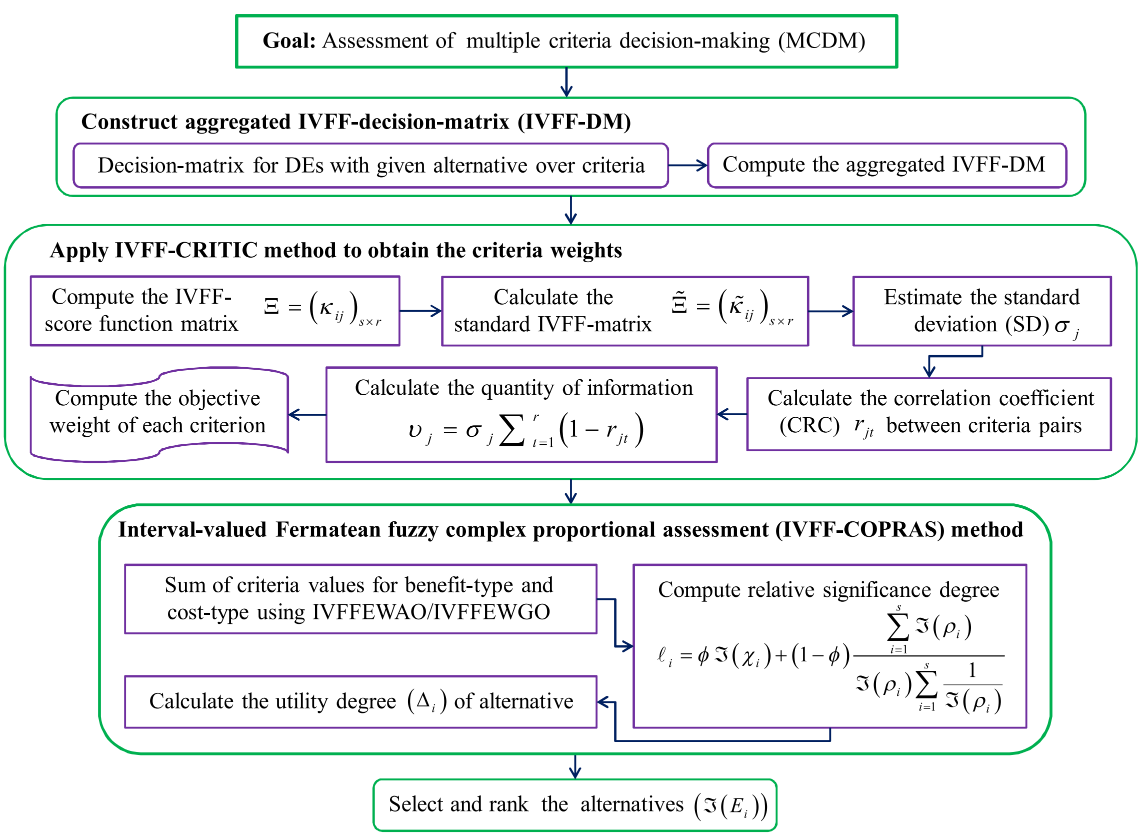

Fig. 1. Representation of proposed IVFF-COPRAS methodology.

$$\left(\left[\mu^{lb}_{max},\ \mu^{ub}_{max}\right],\ \left[\nu^{lb}_{min},\ \nu^{ub}_{min}\right]\right) \geq \left(\left[\mu^{lb},\ \mu^{ub}\right],\ \left[\nu^{lb},\ \nu^{ub}\right]\right). \tag{12}$$

Thus, from Eq. (11) and Eq. (12), we get.

$$\varpi_{min} \leq IVFFEWA(\varpi_1,\ \varpi_2,\ \ldots,\ \varpi_r) \leq \varpi_{max}.$$

Definition 5.3. Let $\varpi_j (j = 1, 2, ..., r)$ be a set of IVFFNs. Then "interval-valued Fermatean fuzzy Einstein ordered weighted averaging (IVFFEOWA)" operator is a function $IVFFEOWA : \Lambda^r \rightarrow \Lambda$ and defined as.

$$IVFFEOWA(\varpi_1, \varpi_2, ..., \varpi_r) = \psi_1 \cdot_\varepsilon \varpi_{\sigma(1)} \oplus_\varepsilon \psi_2 \cdot_\varepsilon \varpi_{\sigma(2)} \oplus_\varepsilon ... \oplus_\varepsilon \psi_r \cdot_\varepsilon \varpi_{\sigma(r)},$$

wherein Λ is the set of all IVFFNs, $(\sigma(1), \sigma(2), ..., \sigma(r))^T$ is a permutation of $(1, 2, ..., r)$ such that $\sigma(j) \leqslant \sigma(j-1),\ \forall\ j$, and $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ is weight value of $\varpi_{\sigma(j)}\ (j = 1, 2, ..., r)$..

Theorem 5.2. *Let* $\varpi_j = \left(\left[\mu^{lb}_j, \mu^{ub}_j\right], \left[\nu^{lb}_j, \nu^{ub}_j\right]\right)$ *(j = 1, 2, ..., r) be a set of IVFFNs and* $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ *be the weight value of* ϖ_j *(j = 1, 2, ..., r), satisfying* $\psi_j \in [0, 1]$ *and* $\sum_{j=1}^r \psi_j = 1$*. Then, the aggregation by IVFFEOWA operator is again an IVFFN and.*

$$IVFFEOWA(\varpi_1, \varpi_2, ..., \varpi_r) =$$

$$\left(\left[\frac{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\mu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}-\prod_{j=1}^r\left(1-\left(\mu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\mu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(1-\left(\mu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\mu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}-\prod_{j=1}^r\left(1-\left(\mu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\mu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(1-\left(\mu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}}}\right],\right.$$
$$\left.\left[\frac{\sqrt[3]{2\prod_{j=1}^r\left(\left(\nu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-\left(\nu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(\left(\nu^{lb}_{\sigma(j)}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^r\left(\left(\nu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-\left(\nu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(\left(\nu^{ub}_{\sigma(j)}\right)^3\right)^{\psi_j}}}\right]\right). \tag{13}$$

Here, $(\sigma(1), \sigma(2), ..., \sigma(r))^T$ is a permutation of $(1, 2, ..., r)$ with $\sigma(j) \leqslant \sigma(j-1),\ \forall\ j$..

Proof. Same as *Theorem 5.1*.

5.3. Interval-valued Fermatean fuzzy Einstein weighted geometric (IVFFEWG) operator

Definition 5.4. Let $\varpi_j = \left(\left[\mu^{lb}_j, \mu^{ub}_j\right], \left[\nu^{lb}_j, \nu^{ub}_j\right]\right)$ $(j = 1, 2, ..., r)$ be a set of IVFFNs and $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ be the weight value of ϖ_j $(j = 1, 2, ..., r)$, satisfying $\psi_j \in [0, 1]$ and $\sum_{j=1}^r \psi_j = 1$. Then, the IVFFEWG operator of dimension r is a function $IVFFEWG : \Lambda^r \rightarrow \Lambda$ and defined as.

$$IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_r) = \varpi_1^{\psi_1} \oplus_\varepsilon \varpi_2^{\psi_2} \oplus_\varepsilon ... \oplus_\varepsilon \varpi_r^{\psi_r},$$

where Λ is the set of all IVFFNs. In particular, if $\psi = \left(\frac{1}{r}, \frac{1}{r}, ..., \frac{1}{r}\right)^T$, then the IVFFEWG operator reduces to an *"interval-valued Fermatean fuzzy weighted Geometric (IVFFWG)"* operator and is defined as.

$$IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_r) = (\varpi_1 \otimes_\varepsilon \varpi_2 \otimes_\varepsilon ... \otimes_\varepsilon \varpi_r)^{1/r}.$$

Corresponding to the operations on IVFFNs in *Definition 5.4*, we show the following outcome:

Theorem 5.3. *Let* $\varpi_j = \left(\left[\mu^{lb}_j, \mu^{ub}_j\right], \left[\nu^{lb}_j, \nu^{ub}_j\right]\right)$ *(j = 1, 2, ..., r) be a set of IVFFNs and* $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ *be the weight value of* ϖ_j *(j = 1, 2, ..., r), with* $\psi_j \in [0, 1]$ *and* $\sum_{j=1}^n \psi_j = 1$*. Then, the aggregation by IVFFEWG operator is again an IVFFN and.*

$$IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_r) =$$

$$\left(\left[\frac{\sqrt[3]{2\prod_{j=1}^r\left(\left(\mu^{lb}_j\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-\left(\mu^{lb}_j\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(\left(\mu^{lb}_j\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^r\left(\left(\mu^{ub}_j\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(2-\left(\mu^{ub}_j\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(\left(\mu^{ub}_j\right)^3\right)^{\psi_j}}}\right],\right.$$
$$\left.\left[\frac{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\nu^{lb}_j\right)^3\right)^{\psi_j}-\prod_{j=1}^r\left(1-\left(\nu^{lb}_j\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\nu^{lb}_j\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(1-\left(\nu^{lb}_j\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\nu^{ub}_j\right)^3\right)^{\psi_j}-\prod_{j=1}^r\left(1-\left(\nu^{ub}_j\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^r\left(1+\left(\nu^{ub}_j\right)^3\right)^{\psi_j}+\prod_{j=1}^r\left(1-\left(\nu^{ub}_j\right)^3\right)^{\psi_j}}}\right]\right). \tag{14}$$

Proof. Similar as *Theorem 5.1*. Hence, we omitted the proof.

Corresponding to the *Theorem 5.3*, we obtain the following properties:

Property 5.5. (*(Idempotency).*) *If all* $\varpi_j = \left(\left[\mu^{lb}_j, \mu^{ub}_j\right], \left[\nu^{lb}_j, \nu^{ub}_j\right]\right)$ *(j = 1, 2, ..., r) be the set of IVFFNs and* $\varpi_j = \varpi = \left(\left[\mu^{lb}, \mu^{ub}\right], \left[\nu^{lb}, \nu^{ub}\right]\right)$*, then* $IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_n) = \varpi$..

Property 5.6. *((Monotonicity).) Consider two collections* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *and* $\widetilde{\varpi}_j = \left(\left[\widetilde{\mu}_j^{lb}, \widetilde{\mu}_j^{ub}\right], \left[\widetilde{\nu}_j^{lb}, \widetilde{\nu}_j^{ub}\right]\right)$ *such that* $\mu_j^{lb} \geqslant \widetilde{\mu}_j^{lb}, \mu_j^{ub} \geqslant \widetilde{\mu}_j^{ub}, \nu_j^{lb} \leqslant \widetilde{\nu}_j^{lb}$ *and* $\nu_j^{ub} \leqslant \widetilde{\nu}_j^{ub}$, *for all* $j = 1, 2, ..., r$, *then* $IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_r) \geqslant IVFFEWG\left(\widetilde{\varpi}_1, \widetilde{\varpi}_2, ..., \widetilde{\varpi}_r\right)$..

Property 5.7. *((Commutativity).) Let* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *and* $\overline{\varpi}_j = \left(\left[\overline{\mu}_j^{lb}, \overline{\mu}_j^{ub}\right], \left[\overline{\nu}_j^{lb}, \overline{\nu}_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) be the set of IVFFNs. Then.*

$$IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_r) = IVFFEWG(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r),$$

where $(\overline{\varpi}_1, \overline{\varpi}_2, ..., \overline{\varpi}_r)$ is any permutation of $(\varpi_1, \varpi_2, ..., \varpi_r)$..

Property 5.8. *((Boundedness).) Let* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) be a set of IVFFNs and let* $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ *be the weight vector of* ϖ_j *such that* $\sum_{j=1}^{r} \psi_j = 1$. *Then,*

$\varpi_{\min} \leqslant IVFFEWG(\varpi_1, \varpi_2, ..., \varpi_n) \leqslant \varpi_{\max}$, where $\varpi_{\min} = \min_j (\varpi_j)$ and $\varpi_{\max} = \max_j (\varpi_j)$..

Definition 5.5. Let ϖ_j $(j = 1, 2, ..., r)$ be a set of IVFFNs. An "interval-valued Fermatean fuzzy Einstein ordered weighted geometric (IVFFEOWG)" operator r is a function $IVFFEOWG : \Lambda^n \rightarrow \Lambda$ and defined as.

$$IVFFEOWG(\varpi_1, \varpi_2, ..., \varpi_n) = \varpi_{\sigma(1)}^{\psi_1} \otimes_\varepsilon \varpi_{\sigma(2)}^{\psi_2} \otimes_\varepsilon ... \otimes_\varepsilon \varpi_{\sigma(r)}^{\psi_r},$$

where Λ is the set of all IVFFNs, $(\sigma(1), \sigma(2), ..., \sigma(r))^T$ is a permutation of $(1, 2, ..., r)$ such that $\sigma(j) \leqslant \sigma(j-1)$, $\forall j$, and $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ is weight value of $\varpi_{\sigma(j)}$ $(j = 1, 2, ..., r)$..

By means of Einstein operational laws of IVFFNs, we show the subsequent theorem:

Theorem 5.4. *Let* $\varpi_j = \left(\left[\mu_j^{lb}, \mu_j^{ub}\right], \left[\nu_j^{lb}, \nu_j^{ub}\right]\right)$ *(j = 1, 2, ..., r) be a set of IVFFNs and* $\psi = (\psi_1, \psi_2, ..., \psi_r)^T$ *be the weight value of* ψ_j $(j = 1, 2, ..., r)$, *satisfying* $\psi_j \in [0, 1]$ *and* $\sum_{j=1}^{r} \psi_j = 1$. *Then, the aggregation by IVFFEOWA operator is again an IVFFN and.*

$$IVFFEOWG(\varpi_1, \varpi_2, ..., \varpi_r) =$$

$$\left(\left[\frac{\sqrt[3]{2\prod_{j=1}^{r}\left(\left(\mu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(2-\left(\mu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(\left(\mu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^{r}\left(\left(\mu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(2-\left(\mu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(\left(\mu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}}}\right],\right.$$
$$\left.\left[\frac{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\nu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}-\prod_{j=1}^{r}\left(1-\left(\nu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\nu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(1-\left(\nu_{\sigma(j)}^{lb}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\nu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}-\prod_{j=1}^{r}\left(1-\left(\nu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{r}\left(1+\left(\nu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}+\prod_{j=1}^{r}\left(1-\left(\nu_{\sigma(j)}^{ub}\right)^3\right)^{\psi_j}}}\right]\right), \quad (15)$$

Table 1
List of considered criteria for sustainable CBT location assessment.

Main criteria		Sub-criteria	References
Economic	It requires to produce and manage the prosperity at diverse levels with focus on the cost-efficiency of all economic activities.	Employment Quality	Thongdejsri and Nitivattananon (2019); Pfueller et al (2011); Tseng et al. (2018); Urtasun and Gutiérrez (2006); Gössling (2017); de Grosbois (2016)
		Economical Capacity	Tseng et al. (2018); Janusz and Bajdor (2013); Pomering et al. (2011); Torres-Delgado and Palomeque (2014); Sgroi (2020); Fletcher et al (2016); He et al. (2021)
Social	It refers the human rights, education and equal opportunities for all the community.	Quality of Life	Dangi and Jamal (2016); Lee & Jan (2019); Dodds et al. (2018); Burgos and Mertens (2017); Álvarez-García et al. (2018)
		Community Participation	Mayaka et al. (2018); Jaafar et al (2020); Dangi and Jamal (2016); Sebele (2010); Kayat (2002)
		Socio-Cultural Policy	Dangi and Jamal (2016); Bhalla and Bhattacharya (2019); Basak et al. (2021); He et al., (2021)
Environmental	This considers the management and conservation of natural resources. It requires to reduce the carbon emission, air pollution, water pollution, and protect natural heritage and biological diversity.	Conservation of natural resources	Stănciulescu and Țîrca (2010); McLoughlin and Hanrahan (2016); Basak et al. (2021); He et al., (2021)
		Integration of homestay and nature	Bhalla and Bhattacharya (2019); Jaafar et al (2020); He et al. (2021); Basak et al. (2021)
		Local community concern regarding environmental sustainability	Tseng et al (2018); He et al. (2021); Basak et al. (2021)

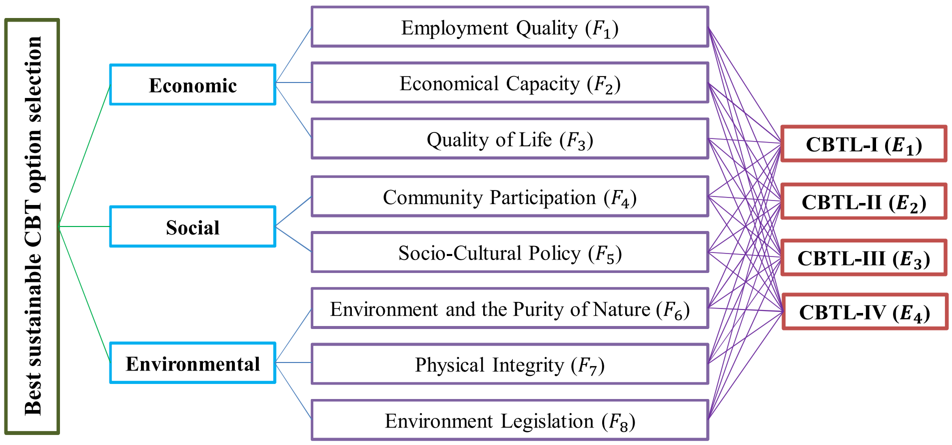

Fig. 2. Hierarchical frame of sustainable CBTL selection.

Table 2
IVFF decision-matrix.

	E_1	E_2	E_3	E_4
F_1	([0.5,0.65], [0.60,0.80])	([0.45,0.55], [0.60,0.75])	([0.40,0.50], [0.55,0.75])	([0.35,0.45], [0.65,0.75])
F_2	([0.30,0.40], [0.50,0.70])	([0.35,0.40], [0.60,0.75])	([0.40,0.45], [0.65,0.70])	([0.30,0.45], [0.55, 0.70])
F_3	([0.45,0.50], [0.60,0.65])	([0.40,0.50], [0.70,0.75])	([0.45,0.55], [0.65,0.75])	([0.30,0.35], [0.70,0.80])
F_4	([0.60,0.70], [0.45,0.50])	([0.65,0.75], [0.42,0.60])	([0.70,0.75], [0.50,0.55])	([0.64,0.70], [0.35,0.45])
F_5	([0.68,0.77], [0.37,0.48])	([0.70,0.78], [0.45,0.55])	([0.63,0.75], [0.44,0.50])	([0.60,0.65], [0.30,0.40])
F_6	([0.70,0.75], [0.50,0.55])	([0.60,0.68], [0.50,0.60])	([0.64,0.70], [0.30,0.35])	([0.65,0.75], [0.45,0.55])
F_7	([0.64,0.72], [0.38,0.50])	([0.70,0.75], [0.45,0.57])	([0.66,0.74], [0.46,0.54])	([0.60,0.70], [0.48,0.58])
F_8	([0.63, 0.70], [0.35,0.50])	([0.68,0.76], [0.40,0.52])	([0.55,0.60], [0.42,0.54])	([0.69, 0.78], [0.30,0.35])

Table 3
The standard IVFF-DM $\widetilde{\Xi} = \left(\widetilde{\kappa}_{ji}\right)_{t\times s}$, and weight for each criterion.

Criteria	E_1	E_2	E_3	E_4	σ_j	υ_j	ψ_j
F_1	0.000	0.276	0.360	1.000	0.366	2.386	0.1010
F_2	0.000	0.926	1.000	0.168	0.444	3.247	0.1375
F_3	0.000	0.584	0.305	1.000	0.368	2.370	0.1004
F_4	0.000	0.398	0.863	1.000	0.395	2.544	0.1077
F_5	1.000	0.677	0.228	0.000	0.388	3.705	0.1569
F_6	0.756	0.000	1.000	0.674	0.371	3.064	0.1297
F_7	0.915	1.000	0.799	0.000	0.398	3.618	0.1532
F_8	0.479	0.669	0.000	1.000	0.362	2.685	0.1137

Table 4
The computational outcome of IVFF-COPRAS method.

RETs	χ_i	$\Im(\chi_i)$	δ_i	$\Im(\delta_i)$	ℓ_i	Δ_i
E_1	([0.574, 0.645], [0.579, 0.663])	0.4931	([0.295, 0.368], [0.853, 0.909])	0.1758	0.3176	94.58
E_2	([0.588, 0.659], [0.614, 0.711])	0.4748	([0.278, 0.337], [0.880, 0.914])	0.1539	0.3186	94.87
E_3	([0.560, 0.631], [0.591, 0.650])	0.4863	([0.290, 0.348], [0.876, 0.914])	0.1576	0.3225	96.01
E_4	([0.555, 0.629], [0.550, 0.629])	0.5011	([0.221, 0.297], [0.877, 0.919])	0.1465	0.3359	100.00

wherein $(\sigma(1), \sigma(2), ..., \sigma(r))^T$ is a permutation of $(1, 2, ..., r)$ such that $\sigma(j) \leqslant \sigma(j-1),\ \forall j$..

6. Proposed model

In this part of the study, an extended COPRAS model is introduced to treat the MCDM problems with IVFF-information. The calculation steps of the present approach are as below (see Fig. 1):

Algorithm 1. (*Pseudo code representation of IVFF-COPRAS for MCDA problem assessment*)

Input: s – number of alternatives, t – number of criteria
Output: Rank the alternatives over considered criteria
Begin
Step 1: Input the IVFF-DM $\mathbb{Z} = (z_{ji})_{t\times s}$
Step 2: **for** $j = 1$ to t
  for $i = 1$ to s
  Calculate score matrix $\Xi = (\kappa_{ij})_{s\times t}$, using Eq. (17) # κ_{ij} is the IVFFN.
  Compute the standard IVFF-DM $\widetilde{\Xi} = \left(\widetilde{\kappa}_{ji}\right)_{t\times s}$ using Eq. (18)
  Estimate the criteria standard deviations (σ_j) using Eq. (19)
  Calculate the correlation coefficient (r_{jm}) between criteria using Eq. (20)
  end for
  Assess the quantity of information (υ_j) of each criterion using Eq. (21)
  Compute the weight (ϖ_j) of each criterion using Eq. (22)
  end for
Step 3: **for** $i = 1$ to s
  Use the IVFFEWA operator to output χ_i for beneficial criteria using Eq. (23) and output δ_i for non- beneficial criteria using Eq. (24)
  end for
Step 4: **for** $i = 1$ to s Compute the relative degree (ℓ_i) for option w. r. t. strategy coefficient $\sigma \in [0, 1]$ using Eq. (26)
  end for
Step 5:for $i = 1$ to s Compute the utility degree (Δ_i) for alternative using Eq. (28)
  end for
Step 6: Rank the alternatives in decreasing score values of Δ_i
End

Step 1: Construct the "IVFF-decision matrix (IVFF-DM)".

In the MCDM practice, the aim is to choose the most suitable option among a set of choices $\{E_1, E_2, ..., E_s\}$ concerning a criterion set $\{F_1, F_2, ..., F_t\}$ such that the characteristics of each alternative is given in the form of IVFFNs $z_{ij} = \left(\left[\mu_{ij}^{lb}, \mu_{ij}^{ub}\right], \left[\nu_{ij}^{lb}, \nu_{ij}^{ub}\right]\right)$, where $\left[\mu_{ij}^{lb}, \mu_{ij}^{ub}\right]$ offers the degree of alternative in terms of favors whilst $\left[\nu_{ij}^{lb}, \nu_{ij}^{ub}\right]$ provides in terms of against for i^{th} alternative by means of j^{th} criterion. Thus, an IVFF-DM $\mathbb{Z} = (z_{ji})_{t\times s}\ \left(or\ \mathbb{Z} = (z_{ij})_{s\times t}\right)$ can be created as.

Table 5
Utility degree of option with diverse parameter values.

ϕ	E_1	E_2	E_3	E_4
$\phi = 0.0$	0.1422	0.1624	0.1586	0.1706
$\phi = 0.1$	0.1773	0.1937	0.1914	0.2037
$\phi = 0.2$	0.2124	0.2249	0.2242	0.2367
$\phi = 0.3$	0.2475	0.2561	0.2569	0.2698
$\phi = 0.4$	0.2826	0.2874	0.2897	0.3028
$\phi = 0.5$	0.3176	0.3186	0.3225	0.3359
$\phi = 0.6$	0.3527	0.3499	0.3552	0.3689
$\phi = 0.7$	0.3878	0.3811	0.3880	0.4019
$\phi = 0.8$	0.4229	0.4124	0.4207	0.4350
$\phi = 0.9$	0.4580	0.4436	0.4535	0.4680
$\phi = 1.0$	0.4931	0.4748	0.4863	0.5011

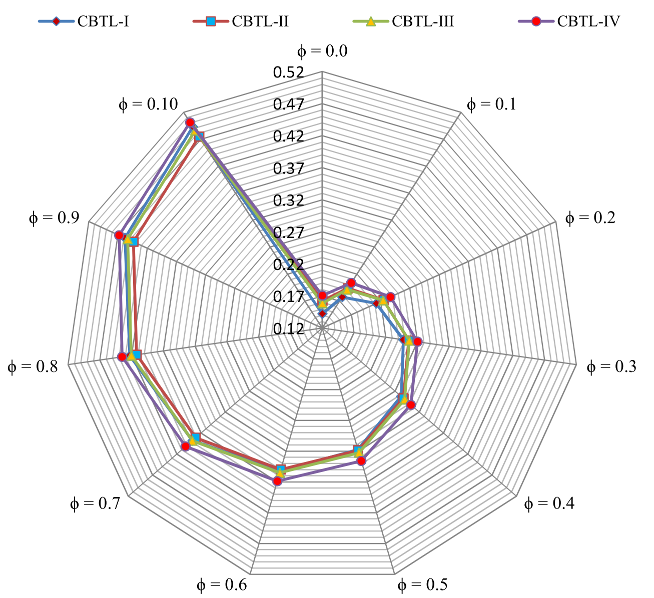

Fig. 3. Sensitivity assessments of utility degree over decision coefficient parameter (ϕ).

$$\mathbb{Z} = \begin{array}{c} \\ E_1 \\ E_2 \\ \vdots \\ E_s \end{array} \begin{array}{c} \begin{array}{cccc} F_1 & F_2 & \cdots & F_t \end{array} \\ \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1t} \\ z_{21} & z_{22} & \cdots & z_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ z_{s1} & z_{s2} & \cdots & z_{st} \end{pmatrix} \end{array} \tag{16}$$

Step 2: Evaluation of criteria weights based on CRITIC approach.

First of all, suppose $\psi = (\psi_1, \psi_2, ..., \psi_t)^T$ be the set of criteria weights with $\psi_j \in [0,\ 1]$ and $\sum_{j=1}^{t}\psi_j = 1$. In the following, the computational steps are presented for the determination of criteria weights using CRITIC method:

Step 2.1: Determine the score matrix $\Xi = (\kappa_{ji})_{t\times s}\left(or\, \Xi = (\kappa_{ij})_{s\times t}\right)$, where.

$$\kappa_{ij} = \frac{1}{2}\left(\left(\left(\mu_{ij}^{lb}\right)^3 - \left(\nu_{ij}^{lb}\right)^3\right)\left(1 + \sqrt[3]{1 - \left(\mu_{ij}^{lb}\right)^3 - \left(\nu_{ij}^{lb}\right)^3}\right) + \left(\left(\mu_{ij}^{ub}\right)^3 - \left(\nu_{ij}^{ub}\right)^3\right)\left(1 + \sqrt[3]{1 - \left(\mu_{ij}^{ub}\right)^3 - \left(\nu_{ij}^{ub}\right)^3}\right)\right), \tag{17}$$

Step 2.2: Change the score-IVFF-DM Ξ into a standard IVFF-DM$\widetilde{\Xi} = \left(\widetilde{\kappa}_{ij}\right)_{s\times t}$.

$$\widetilde{\kappa}_{ij} = \begin{cases} \dfrac{\kappa_{ij} - \kappa_j^-}{\kappa_j^+ - \kappa_j^-}, & j \in F_b, \\ \dfrac{\kappa_j^+ - \kappa_{ij}}{\kappa_j^+ - \kappa_j^-}, & j \in F_n, \end{cases} \tag{18}$$

where $\kappa_j^- = \min_i \kappa_{ij}$ and $\kappa_j^+ = \max_i \kappa_{ij}$..

Step 2.3: Compute the criteria' standard deviations.

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{s}\left(\widetilde{\kappa}_{ij} - \overline{\kappa}_j\right)^2}{s}}, \tag{19}$$

where $\overline{\kappa}_j = \sum_{i=1}^{s}\widetilde{\kappa}_{ij}/s$.

Step 2.4: Derive the correlation between criteria.

$$r_{jm} = \frac{\sum_{i=1}^{s}\left(\widetilde{\kappa}_{ij} - \overline{\kappa}_j\right)\left(\widetilde{\kappa}_{im} - \overline{\kappa}_m\right)}{\sqrt{\sum_{i=1}^{s}\left(\widetilde{\kappa}_{ij} - \overline{\kappa}_j\right)^2 \sum_{i=1}^{s}\left(\widetilde{\kappa}_{im} - \overline{\kappa}_m\right)^2}}. \tag{20}$$

Step 2.5: Determine the quantity of information of each attribute.

$$\upsilon_j = \sigma_j \sum_{m=1}^{t}\left(1 - r_{jm}\right). \tag{21}$$

Step 2.6: Compute the weight of each criterion.

$$\psi_j = \frac{\upsilon_j}{\sum_{j=1}^{t}\upsilon_j}. \tag{22}$$

Step 3: Add the criteria values for benefit and cost.

In IVFF-COPRAS procedure, each option is evaluated with its sums of maximizing criteria,χ_i, as considered to benefit-type and minimizing criteria, δ_i, as considered to cost-type and computed by the following way:

$$\chi_i = IVFFEWA(z_{i1}, z_{i2}, ..., z_{il})$$

Table 6
Comparison of preference order of presented methodology with extant models.

CBTLs	IVFFWAO-COPRAS	IVFF-WSM	IVFF-WPM	IVFF-WASPAS	IVFF-TOPSIS	IVPFEWA-based method	Introduced model
E_1	0.3393	0.6122	0.6016	0.6069	0.3677	0.2574	0.3176
E_2	0.3432	0.6173	0.6102	0.6138	0.6330	0.2603	0.3186
E_3	0.3460	0.6113	0.6026	0.6070	0.6178	0.2555	0.3225
E_4	0.3592	0.6275	0.6166	0.6220	0.5121	0.3039	0.3359
Ranking	$E_4 \succ E_3 \succ E_2 \succ E_1$	$E_4 \succ E_2 \succ E_1 \succ E_3$	$E_4 \succ E_2 \succ E_3 \succ E_1$	$E_4 \succ E_2 \succ E_3 \succ E_1$	$E_2 \succ E_3 \succ E_4 \succ E_1$	$E_4 \succ E_2 \succ E_1 \succ E_3$	$E_4 \succ E_3 \succ E_2 \succ E_1$
Best RET	E_4	E_4	E_4	E_4	E_2	E_4	E_4
SRCC	1.00	0.40	0.80	0.80	0.20	0.40	–
WS Coefficient	1.000	0.667	0.813	0.813	0.542	0.667	–

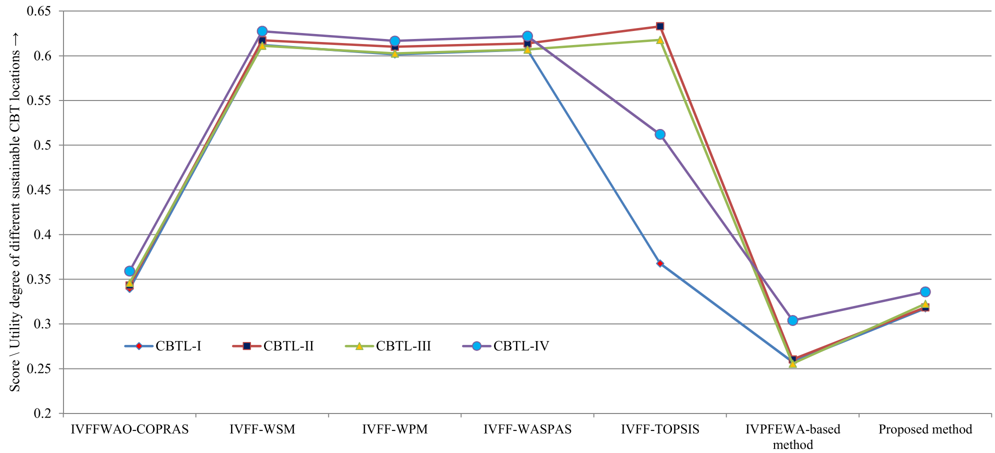

Fig. 4. Comparison of *UDs* of each CBTL with different extant approaches.

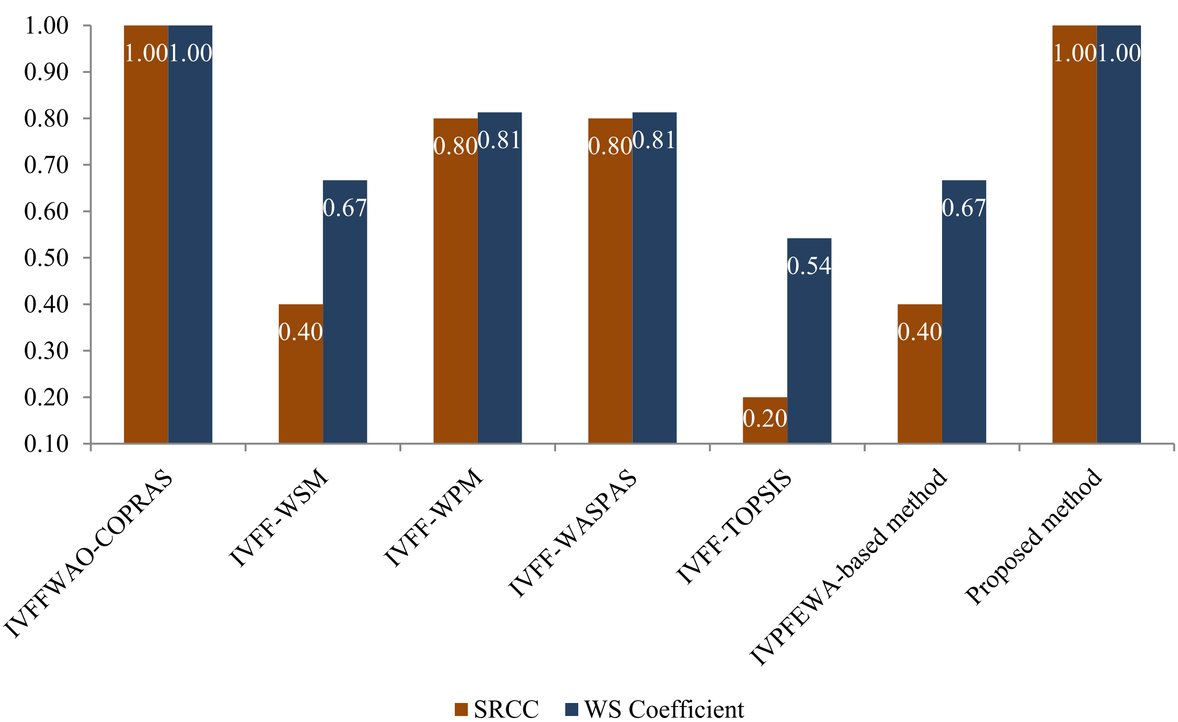

Fig. 5. Correlation and similarity design of ranking orders with different methods.

$$= \left(\left[\frac{\sqrt[3]{\prod_{j=1}^{l}\left(1+\left(\mu_{ij}^{lb}\right)^3\right)^{\psi_j} - \prod_{j=1}^{l}\left(1-\left(\mu_{ij}^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{l}\left(1+\left(\mu_{ij}^{lb}\right)^3\right)^{\psi_j} + \prod_{j=1}^{l}\left(1-\left(\mu_{ij}^{lb}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=1}^{l}\left(1+\left(\mu_{ij}^{ub}\right)^3\right)^{\psi_j} - \prod_{j=1}^{l}\left(1-\left(\mu_{ij}^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{l}\left(1+\left(\mu_{ij}^{ub}\right)^3\right)^{w_j} + \prod_{j=1}^{l}\left(1-\left(\mu_{ij}^{ub}\right)^3\right)^{\psi_j}}} \right], \right.$$

$$\left. \left[\frac{\sqrt[3]{2\prod_{j=1}^{l}\left(\left(\nu_{ij}^{lb}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{l}\left(2-\left(\nu_{ij}^{lb}\right)^3\right)^{\psi_j} + \prod_{j=1}^{l}\left(\left(\nu_{ij}^{lb}\right)^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=1}^{l}\left(\left(\nu_{ij}^{ub}\right)^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=1}^{l}\left(2-\left(\nu_{ij}^{ub}\right)^3\right)^{\psi_j} + \prod_{j=1}^{l}\left(\left(\nu_{ij}^{ub}\right)^3\right)^{\psi_j}}} \right] \right). \tag{23}$$

$$\delta_i = IVFFEWA\left(z_{i(l+1)}, z_{i(l+2)}, ..., z_{it}\right)$$

$$= \left(\left[\frac{\sqrt[3]{\prod_{j=l+1}^{t}\left(1+(\mu_{ij}^{lb})^3\right)^{\psi_j} - \prod_{j=l+1}^{t}\left(1-(\mu_{ij}^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=l+1}^{t}\left(1+(\mu_{ij}^{lb})^3\right)^{\psi_j} + \prod_{j=l+1}^{t}\left(1-(\mu_{ij}^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{\prod_{j=l+1}^{t}\left(1+(\mu_{ij}^{ub})^3\right)^{\psi_j} - \prod_{j=l+1}^{t}\left(1-(\mu_{ij}^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=l+1}^{t}\left(1+(\mu_{ij}^{ub})^3\right)^{\psi_j} + \prod_{j=l+1}^{t}\left(1-(\mu_{ij}^{ub})^3\right)^{\psi_j}}}\right],\right.$$

$$\left.\left[\frac{\sqrt[3]{2\prod_{j=l+1}^{t}\left((\nu_{ij}^{lb})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=l+1}^{t}\left(2-(\nu_{ij}^{lb})^3\right)^{\psi_j} + \prod_{j=l+1}^{t}\left((\nu_{ij}^{lb})^3\right)^{\psi_j}}}, \frac{\sqrt[3]{2\prod_{j=l+1}^{t}\left((\nu_{ij}^{ub})^3\right)^{\psi_j}}}{\sqrt[3]{\prod_{j=l+1}^{t}\left(2-(\nu_{ij}^{ub})^3\right)^{\psi_j} + \prod_{j=l+1}^{t}\left((\nu_{ij}^{ub})^3\right)^{\psi_j}}}\right]\right). \quad (24)$$

Here, r is number of benefit criteria.

Step 4: Define the "relative degree (RD)" of each option

$$\ell_i = \phi\,\Im(\chi_i) + (1-\phi)\frac{\min_i \Im(\delta_i)\sum_{i=1}^{r}\Im(\delta_i)}{\Im(\delta_i)\sum_{i=1}^{r}\frac{\min_i \Im(\delta_i)}{\Im(\delta_i)}};\ i = 1, 2, ..., r. \quad (25)$$

Here, $\Im(\chi_i)$ and $\Im(\delta_i)$ denote the score degrees of χ_i and δ_i, respectively.

Also, Eq. (25) can be given by.

$$\ell_i = \phi\,\Im(\chi_i) + (1-\phi)\frac{\sum_{i=1}^{r}\Im(\delta_i)}{\Im(\delta_i)\sum_{i=1}^{r}\frac{1}{\Im(\delta_i)}},\ i = 1, 2, ..., r. \quad (26)$$

Here, the parameter $\phi \in [0, 1]$ denotes the strategic coefficient of DE. The assessment of strategic parameter is as follows: if $\phi < 0.5$, then DE shows pessimistic behavior, if $\phi > 0.5$, then DE presents optimistic behavior, which means, the higher degree is related to the weights of benefit-type criteria and if $\phi = 0.5$, then the DE presents neutral performance, which means the equal weight is related to the both benefit-type and cost-type attributes.

Step 5: Find the prioritization of each alternative.

As per the relative weight, the order of priority of each alternative is estimated. The maximum relative weight of option has been given as higher preference degree, and it is the optimal option.

$$E^* = \left\{E_i \middle| \max_i \ell_i\right\},\ i = 1, 2, ..., m. \quad (27)$$

Step 6: Obtain the "utility degree (UD)" of each candidate.

$$\Delta_i = \frac{\ell_i}{\ell_{\max}} \times 100\,\%, \quad (28)$$

wherein ℓ_i and $\ell_{\max}$ are the *RDs* is given by Eq. (26).

The present IVFF-COPRAS methodology permits to determine the straight and comparative confidence of the *RDs* and *UDs* of option(s) related to the criteria.

7. Case study

In developing countries, tourism has emerged as a significant activity to boost the nation's economy, conserve and restore the biological diversity, protect the environment and generate the economic and social benefits for local people (Pasanchay and Schott, 2021). It is defined as one of the most important community development tools, particularly in peripheral or marginal communities such as remote, indigenous and rural communities. The circumstances that gave eminence to tourism may differ from place to place but the common point is that it could improve the standard of living through economic diversity, educational and employment opportunities, natural and cultural attractions, boosting food and hospitality sectors without harming the local community and nature (Quevedo et al., 2021). Although tourism is a key driving economic booster to a community, but improper tourism development and practice can deplete natural resources, degrade habitats and landscapes and cause waste and pollution.

The notion of "*community-based tourism (CBT)*" has been at the forefront of the encouragement of rural development, both in developing and developed nations. As a community improvement initiative, CBT assists to reinforce and empower indigenous, remote and rural communities (Schott and Nhem, 2018). In view of tourism as a special means to strengthen local economies, it becomes a key to reduce poverty and assist the local community in a variety of ways, such as by providing employment opportunities, environment protection, outdoor recreation and get-together opportunities and revenue-producing activities (Lo and Janta, 2020; Quevedo et al., 2021). It can not only give you a real insight into local lives, but also ensure your travel experience makes a true difference to local people. Lo et al. (2021) proposed a two-way decision-making model for sustainable supplier assessment and transportation design in multi-phase *"supply chain networks (SCNs)"*. They used *"indifference threshold-based attribute ratio analysis (ITARA)"* and the *"performance calculation technique of the integrated multiple (PCIM)"*-based MCDM to obtain the supplier's assessment index.

The key principle of CBT is to incorporate local people in tourism by preserving tourism resources and by offering basic structure namely food, lodging, and further services to host travelers (Schott and Nhem, 2018). It grew as an off shoot of responsible tourism and can be described as 'tourism that considers social, economic, cultural and environmental aspects of sustainable perspectives into consideration. It is accomplished and possessed by the community, and for the community, with the aim of facilitating travelers to raise their knowledge and discover related to the local customs of life of the community'. In a more general way, CBT can be defined as an alternate procedure of tourism development that focuses on involvement of local people in all procedures from idea creation to scheduling, execution, management, observation, assessment, and benefit sharing (He et al.,2021; Basak et al. 2021). It is similar to *"sustainable tourism (ST)"* by including environmental, economic and socio-cultural aspects (Dangi and Jamal, 2016).

CBT is now as an option to being able to offer community empowerment and community welfare towards sustainable tourism. In CBT, local communities work as a team to manage tourist resources and tourism services in an appropriate manner. Their welcoming demeanor and traditional ways add value to the experiences of tourists. Travel and tourism are one of the largest sectors in India. Due to its rich culture of varied traditions, customs and festivals, India has been attracting a huge number of visitors from all over the world. Government of India has made several strategies to turn India become a global tourist hotspot, recognizing the industry's potential. They set up January 25th as the

"*National Tourism Day*" to raise awareness about the significance of tourism for the country's economic prosperity. Indian government is also planning to improve the tourism by leveraging on the lighthouses in the country. 71 lighthouses have been recognized in India which will be grown as traveler spots. The lighthouses will feature *amphi*-theatres, open air theatres, museums, restaurants, children's parks, green cottages and landscaping in accordance with its capacity.

The region selected for this study has been delineated as cultural region. '*Darjeeling*' is the northernmost district in India's West Bengal state, situated in the Himalayan foothills at a height of 6700 ft (2042.2 m). The district comprises four subdivisions: Darjeeling Sadar, Kurseong, Mirik and Siliguri. By means of its temperate climate and picturesque settings in the hills, Darjeeling came to be called as '*Queen of the Hills*.' Darjeeling is also known for its top-class aromatic tea that is exported around the world. In this region, tourism is one of the crucial driving services in the economic growth of the whole district. The wonderful mountain landscape related with visualizations of snowy mountain peaks, charming 'European' hill station ambiance, and world-distinguished tea estates of this area portraying individuals of all ages, interests, experience levels and as a consequence, a large number of local and foreign tourists visit this region annually (Basak et al., 2021). Taking into consideration the characteristics and components of rural tourism and sustainable development planning such as the community participation, employment opportunity, local benefits, tourists' satisfaction, sustainable tourism management, environment security, and infrastructure, this study plans a sustainable tourism development in the rural areas of Darjeeling district.

According to the literature survey, several criteria are identified for evaluation of sustainable CBT, see Table 1 and Fig. 2.

Furthermore, four sustainable CBT locations (CBTL-I, CBTL-II, CBTL-III and CBTL-IV) are identified as options by a panel of experts. Subsequently, experts are requested to present their assessments and experiences to weigh the considered criteria and to assess the CBT locations over considered criteria. Experts state their preferences by opinions, as per their domain knowledge in Table 2.

Step 1: Suppose that the given alternatives are assessed by means of each criterion and their corresponding judgments are provided by the panel of experts, which is shown by $\mathbb{Z} = (z_{ji})_{t\times s}\left(or\ \mathbb{Z} = (z_{ij})_{s\times t}\right)$ and depicted in Table 2.

Step 2: In the following, the procedure of the CRITIC approach is given to estimate the criteria weights:

Step 2.1: Firstly, by means of Eq. (17) and Table 2, the score matrix $\Xi = (\kappa_{ji})_{t\times s}\left(or\ \Xi = (\kappa_{ij})_{s\times t}\right)$ is determined.

Step 2.2: With the use of Eq. (18), the standard IVFF-DM $\widetilde{\Xi} = \left(\widetilde{\kappa}_{ji}\right)_{t\times s}$ is created.

Steps 2.3–2.6: In accordance with Eq. (19)-Eq. (21), the SD, CRC and quantity of information of criterion are calculated and shown in Table 3. Further, by means of Eq. (22), the weight of each criterion is determined and then specified in Table 4.

Steps 3–6: Using Eq. (23)-Eq. (28), the computational outcomes of χ_i, δ_i, $\Im(\chi_i)$, $\Im(\delta_i)$, ℓ_i and Δ_i of E_i are determined, see Table 4. Consequently, the prioritization order of the candidate locations is obtained as $E_4 \succ E_3 \succ E_2 \succ E_1$ and thus, CBTL-IV (E_4) is the most desirable alternative.

7.1. Sensitivity analysis

We execute a sensitivity analysis with respect to the different values of strategy coefficient (ϕ). In what follows, we successively observe the influences of the coefficient to the sustainable CBT location selection in details. Various values of $\phi \in [0, 1]$ are considered for investigation. This investigation is conferred to investigate the variation of the presented IVFF-COPRAS model. The variation in ϕ can help us to appraise the sensitivity of the presented approach. The results of the sensitivity investigation (Table 5 and Fig. 3) show that the most optimal choice E_4 is same in coefficient value, whereas the prioritization of the candidate locations is varying over diverse set of parameter values. In accordance with Table 5 and Fig. 3, the ranking order of candidates locations is $E_4 \succ E_2 \succ E_3 \succ E_1$ when $\phi = 0.0$ to 0.2, while ranking order is $E_4 \succ E_3 \succ E_2 \succ E_1$ when $\phi = 0.3$ to 0.5, ranking order is $E_4 \succ E_3 \succ E_1 \succ E_2$ when $\phi = 0.6$ to 0.7 and ranking order is $E_4 \succ E_1 \succ E_3 \succ E_2$ when $\phi = 0.8$ to 1.0. Thus, it is concluded that the assessment of CBTL options is depend on and sensitive to the coefficient ϕ. Hence, the present model has an acceptable steadiness over diverse parameters values. Finally, we can say that the use of diverse parameter values will improve the stability of the presented IVFF-COPRAS methodology. Table 6.

7.2. Comparison with existing methods

The aforesaid application can only illustrate the applicability of the developed model. To show the efficiency of the found results, we execute some specific comparative analysis with extant methods which have high stability and robustness, namely, IVFF-TOPSIS (Jeevaraj, 2021) and IVFF-WASPAS (Rani and Mishra, 2022) and IVPFEWA-based method (Rahman et al., 2020). These classic techniques generally consisting of the *IVFFWA* operator-based COPRAS method, the *"weighted sum model (WSM)"* based on IVFFWA operator, the *"weighted product model (WPM)"* based on IVFFWG operator, IVPFEWA-based method and the WASPAS method. First of all, the discussed approaches are implemented on the above-mentioned application.

The comparative results confirmed that the proposed IVFF-COPRAS method has higher robustness in comparison with the above-discussed methods. Therefore, it is applicable to a wider range of problems. Here, the most important benefits of the introduced model are presented (See Fig. 4):

- The CRITIC method used in this study can be discussed to find the objective weights of criteria, by ignoring the biasness of the decision experts, while IVFF-WASPAS uses the score function-based model to acquire the criteria weights with partially known information. In IVFF-TOPSIS and IVPFEWA-based model, the attribute weights are assumed randomly by the DEs to evaluate the subjective weights of criteria.
- The IVFF-COPRAS method can appropriately process the available information from various points of view, e.g., the cost-type and benefit-type criteria. It is also used the improved score function and proposed IVFFEWA operators to select the alternatives. In IVFF-TOPSIS model, it is essential to estimate the distances among each option with IVFF-ideal and anti-ideal solutions, which is time-consuming and diminishes the accurateness of the results, while, in IVPFEWA-based method, the IVPFEWA operator to rank the alternative, which diminishes the exactness of the results and leaving no room to treat ambiguity.
- The IVFF-COPRAS and IVFF- TOPSIS approaches are basically extensions of COPRAS and TOPSIS methods respectively under IVFFSs environment. For measuring the distance between each alternative and corresponding reference points, the TOPSIS method plays a key role by providing the optimal solution. But, the guarantee of non-exactness of the solution achieved through TOPSIS with ideal solution has been put forth by Opricovic and Tzeng (2004). As pointed out by Rani and Mishra (2022), the IVFF-WASPAS and IVPFEWA-based methods have complex process of aggregating the preferences and beyond that WASPAS and IVPFEWA-based methods may express some degree of inconsistency. As the IVFF-COPRAS method is free from these difficulties, our developed method is more robust.

In Fig. 5, it is noticed that the presented method is extremely consistent with extant tools. To maintain uniformity in the technique-related comparison, various appraisal measures and existing methods viz., IVFF-TOPSIS (Jeevaraj, 2021) and IVFF-WASPAS (Rani and Mishra,

2022) and IVPFEWA-based method (Rahman et al., 2020) are considered. The *"spearman rank correlation coefficients (SRCCs)"* of different extant methods with compromise measure are presented by (1.00, 0.40, 0.80, 0.80, 0.2, 0.40, 1.00). From Fig. 5, the *SRCC* are higher than 0.4 except IVFF-TOPSIS method. In addition, the *"WS-coefficients (WSCs)"* (Sałabun and Urbaniak, 2020; Mishra et al., 2021) of different extant methods with compromise measure are presented by (1.000, 0.667, 0.813, 0.813, 0.542, 0.667, 1.000), which are higher than 0.65 except IVFF-TOPSIS model. The outcomes of the *WSC* state that it is an appropriate way for associating the similarity of prioritizations, which signifies the homogeneity of prioritization of CBTL is high. Thus, it is concluded that the developed methodology has resilient association between preference outcomes. Hence, the presented methodology is more reliable and has better stability with the formerly introduced models.

8. Conclusions

The goal of this study is to recommend a new MCDM methodology for the evaluation of sustainable CBT options from IVFFSs perspective. This method is based on a new score function, the CRITIC and the COPRAS approaches with IVFFSs. Firstly, to compare the IVFFNs, an innovative score function has proposed with its enviable properties. Secondly, new IVFF-Einstein operators are proposed to obtain the aggregated IVFFNs. Thirdly, to estimate the criteria's weights, the classical CRITIC method has been extended on IVFFNs settings. Fourthly, an integrated IVFF-COPRAS methodology with the use of score function, IVFF-Einstein operators and CRITIC method is developed to prioritize the alternatives. An illustrative example of sustainable CBT location options has been presented to reveal the feasibility of the presented IVFF-COPRAS model. Comparison with earlier methods has been presented to certify the results obtained by the proposed method. Thus, to manage the MCDM problems under IVFFSs context, the IVFF-COPRAS methodology gives an easy process of calculation with well-organized and accurate outcomes. The outcomes of the study indicated that the CBTL-IV (0.3359) has the best sustainable CBT locations over different significant criteria namely Socio-Cultural Policy (0.1569), Integration of homestay and nature (0.1532), Economical Capacity (0.1375), Conservation of natural resources (0.1297), and Local community concern regarding environmental sustainability (0.1137). One of the key advantages of the proposed method is that the IVFF-COPRAS model is not only proficient to solve the MCDM problem with IVFFNs information but also capable to deal with FFNs, PFNs, IFNs, IVIFNs and IVPFNs information.

In the following, we present the limitations of the introduced MCDM method: 1) This study is failed to derive the significance of DEs, which means that we have ignored the influences of the relative weights of DEs on decision results, 2) The Einstein AOs proposed in this study cannot take the interrelationship between input data, and 3) In this study, the evaluation index system should include more sustainability criteria.

Future research studies will try to handle the limitations of this work. Further, the proposed MCDM approach will be extended to covering based *"q-rung orthopair fuzzy rough sets (q-ROFRSs)"*, *"Plithogenic sets"*, *"soft rough q-rung orthopair m-polar fuzzy sets (SR-q-Rom-PFSs)"* and *"semiring-valued fuzzy sets (SVFSs)"*. Henceforth, we will introduce some new IVFF-information based AOs with their enviable characteristics and applications. In addition, we will expand this work to the interval-valued hesitant Fermatean fuzzy setting.

CRediT authorship contribution statement

Pratibha Rani: Conceptualization, Methodology, Validation, Investigation, Visualization, Writing – original draft, Writing – review & editing. **Arunodaya Raj Mishra:** Conceptualization, Methodology, Validation, Investigation, Visualization, Writing – original draft, Writing – review & editing. **Muhammet Deveci:** Investigation, Validation, Visualization, Writing – original draft, Writing – review & editing. **Jurgita Antucheviciene:** Writing – original draft, Writing – review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Adali, E. A., & Tus, A. (2021). Hospital site selection with distance-based multi-criteria decision-making methods. *International Journal of Healthcare Management, 14*(2), 534–544. https://doi.org/10.1080/20479700.2019.1674005

Ali, Z., Mahmood, T., Ullah, K., & Khan, Q. (2021). Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management. *Reports in Mechanical Engineering, 2*, 105–134.

Álvarez-García, J., Durán-Sánchez, A., & Del Río-Rama, M. D. I. C. (2018). Scientific coverage in community-based tourism: Sustainable tourism and strategy for social development. *Sustainability, 10*, 1–18. https://doi.org/10.3390/su10041158

Amoako, G. K., Obuobisa-Darko, T., & Marfo, S. O. (2021). Stakeholder role in tourism sustainability: The case of Kwame Nkrumah Mausoleum and centre for art and culture in Ghana. *International Hospitality Review*. https://doi.org/10.1108/IHR-09-2020-0057

Arabameri, A., Yamani, M., Pradhan, B., Melesse, A., Shirani, K., & Bui, D. T. (2019). Novel ensembles of COPRAS multi-criteria decision-making with logistic regression, boosted regression tree, and random forest for spatial prediction of gully erosion susceptibility. *Science of The Total Environment, 688*, 903–916.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and Systems, 20*, 87–96.

Atanassov, K. T., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31*, 343–349.

Aydın, S. (2021). A fuzzy MCDM method based on new Fermatean fuzzy theories. *International Journal of Information Technology & Decision-making, 20*, 881–902.

Baidya, J., Garg, H., Saha, A., Mishra, A. R., Rani, P., & Dutta, D. (2021). Selection of third party reverses logistic providers: An approach of BCF-CRITIC-MULTIMOORA using Archimedean power aggregation operators. *Complex & Intelligent Systems*. https://doi.org/10.1007/s40747-021-00413-x

Basak, D., Bose, A., Roy, S., Chowdhury, I. R., Sarkar, B. C. (2021). Understanding sustainable homestay tourism as a driving factor of tourist's satisfaction through structural equation modelling: A case of Darjeeling Himalayan region, India. Current Research in Environmental Sustainability, Doi: 10.1016/j.crsust.2021.100098.

Bhalla, P., & Bhattacharya, P. (2019). Visitors' satisfaction from ecotourism in the protected area of the Indian Himalayan Region using importance–performance analysis. *Journal of Global Scholars of Marketing Science, 29*, 162–179.

Božanić, D., Ranđelović, A., Radovanović, M., & Tešić, D. (2020). A hybrid LBWA-IR-MAIRCA multi-criteria decision-making model for determination of constructive elements of weapons. *FACTA UNIVERSITATIS Series: Mechanical Engineering, 18*, 399–418.

Burgos, A., & Mertens, F. (2017). Participatory management of community-based tourism: A network perspective. *Community Development, 48*, 546–565.

Chaurasiya, R., & Jain, D. (2022). Pythagorean fuzzy entropy measure-based complex proportional assessment technique for solving multi-criteria healthcare waste treatment problem. *Granular Computing*. https://doi.org/10.1007/s41066-021-00304-z

Đalić, I., Stević, Ž., Ateljević, J., Turskis, Z., Zavadskas, E. K., & Mardani, A. (2021). A novel integrated MCDM-SWOT-TOWS model for the strategic decision analysis in transportation company. *FACTA UNIVERSITATIS Series: Mechanical Engineering, 19*, 401–422.

Dangi, T. B., & Jamal, T. (2016). An integrated approach to "sustainable community-based tourism". *Sustainability, 8*, 1–32. https://doi.org/10.3390/su8050475

Das, A. K., & Granados, C. (2022). FP-intuitionistic multi fuzzy N-soft set and its induced FP-Hesitant N soft set in decision-making. *Decision Making: Applications in Management and Engineering, 5*, 67–89.

de Grosbois, D. (2016). Corporate social responsibility reporting in the cruise tourism industry: A performance evaluation using a new institutional theory based model. *Journal of Sustainable Tourism, 24*, 245–269.

Deng, Z., & Wang, J. (2021). New distance measure for Fermatean fuzzy sets and its application. *International Journal of Intelligent Systems*. https://doi.org/10.1002/int.22760

Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The CRITIC method. *Computers & Operations Research, 22*, 763–770.

Dodds, R., Ali, A., & Galaski, K. (2018). Mobilizing knowledge: Determining key elements for success and pitfalls in developing community-based tourism. *Current Issues in Tourism, 21*, 1547–1568.

Fletcher, C., Pforr, C., & Brueckner, M. (2016). Factors influencing Indigenous engagement in tourism development: An international perspective. *Journal of Sustainable Tourism, 24*(8–9), 1100–1120.

Furtado, P. A. X., & Sola, A. V. H. (2020). Fuzzy complex proportional assessment applied in location selection for installation of photovoltaic plants. *Energies, 13*, 1–20. https://doi.org/10.3390/en13236260

Gonzales, G., Costan, F., Suladay, D., Gonzales, R., Enriquez, L., Costan, E., … Ocampo, L. (2022). Fermatean Fuzzy DEMATEL and MMDE Algorithm for Modelling the Barriers of Implementing Education 4.0: Insights from the Philippines. *Applied Sciences, 12*, 1–19. https://doi.org/10.3390/app12020689

Gorcun, O. F., Senthil, S., & Küçükönder, H. (2021). Evaluation of tanker vehicle selection using a novel hybrid fuzzy MCDM technique. *Decision Making: Applications in Management and Engineering, 4*, 140–162.

Gössling, S. (2017). Tourism, information technologies and sustainability: An exploratory review. *Journal of Sustainable Tourism, 25*, 1024–1041.

Gül, S. (2021). Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. *Expert Systems*. https://doi.org/10.1111/exsy.12769

Hadi, A., Khan, W., & Khan, A. (2021). A novel approach to MADM problems using Fermatean fuzzy Hamacher aggregation operators. *International Journal of Fuzzy Systems, 36*, 3464–3499.

Hashemkhani Zolfani, S., Yazdani, M., Torkayesh, A. E., & Derakhti, A. (2020). Application of a gray-based decision support framework for location selection of a temporary hospital during COVID-19 pandemic. *Symmetry, 12*(2020), 1–15.

He, J., Huang, Z., Mishra, A. R., & Alrasheedi, M. (2021). Developing a new framework for conceptualizing the emerging sustainable community-based tourism using an extended interval-valued Pythagorean fuzzy SWARA-MULTIMOORA. *Technological Forecasting and Social Change*. https://doi.org/10.1016/j.techfore.2021.120955

Hezer, S., Gelmez, E., & Özceylan, E. (2021). Comparative analysis of TOPSIS, VIKOR and COPRAS methods for the COVID-19 regional safety assessment. *Journal of Infection and Public Health, 14*, 775–786.

Iampan, A., Garcıa, G. S., Riaz, M., Farid, H. M. A., Chinram, R. (2021). Linear diophantine fuzzy einstein aggregation operators for multi-criteria decision-making problems. Journal of Mathematics 2021(Article ID 5548033), 1-31. Doi: 10.1155/2021/5548033.

Jaafar, M., Md Noor, S., Mohamad, D., Jalali, A., & Hashim, J. B. (2020). Motivational factors impacting rural community participation in community based tourism enterprise in Lenggong Valley, Malaysia. *Asia Pacific Journal of Tourism Research, 25*, 799–812.

Janusz, G. K., & Bajdor, P. (2013). Towards to sustainable tourism – framework, activities and dimensions. *Procedia Economics and Finance, 6*, 523–529.

Jeevaraj, S. (2021). Ordering of interval-valued Fermatean fuzzy sets and its applications. *Expert Systems with Applications, 185*, Article 115613. https://doi.org/10.1016/j.eswa.2021.115613

Kamacı, H., Petchimuthu, S., & Akçetin, E. (2021). Dynamic aggregation operators and Einstein operations based on interval-valued picture hesitant fuzzy information and their applications in multi-period decision making. *Computational and Applied Mathematics*. https://doi.org/10.1007/s40314-021-01510-w

Karamaşa, Ç., Karabasevic, D., Stanujkic, D., Kookhdan, A. R., Mishra, A. R., & Ertürk, M. (2021). An extended single-valued neutrosophic AHP and MULTIMOORA method to evaluate the optimal training aircraft for flight training organizations. *FACTA UNIVERSITATIS Series: Mechanical Engineering, 19*, 555–578.

Kayat, K. (2002). Exploring factors influencing individual participation in community based tourism: The case of Kampung Relau homestay program. *Malaysia. Asia Pacific J. Tourism Res., 7*, 19–27.

Kersuliene, V., Zavadskas, E. K., & Turskis, Z. (2010). Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA). *Journal of Business Economics and Management, 11*(2), 243–258.

Keshavarz Ghorabaee, M., Amiri, M., Zavadskas, E. K., & Antuchevicience, J. (2018). A new hybrid fuzzy MCDM approach for evaluation of construction equipment with sustainability considerations. *Archives of Civil and Mechanical Engineering, 18*(1), 32–49.

Krishankumar, R., Garg, H., Arun, K., Saha, A., Ravichandran, K. S., & Kar, S. (2021). An integrated decision-making COPRAS approach to probabilistic hesitant fuzzy set information. *Complex & Intelligent Systems, 7*, 2281–2298.

Kumar, K., & Chen, S. M. (2022). Multiple attribute group decision making based on advanced linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers. *Information Sciences, 587*, 813–824.

Kusakci, S., Yilmaz, M. K., Kusakci, A. O., Sowe, S., & Nantembelele, F. A. (2022). Towards sustainable cities: A sustainability assessment study for metropolitan cities in Turkey via a hybridized IT2F-AHP and COPRAS approach. *Sustainable Cities and Society*. https://doi.org/10.1016/j.scs.2021.103655

Kushwaha, D. K., Panchal, D., & Sachdeva, A. (2021). Risk analysis of cutting system under intuitionistic fuzzy environment. *Reports in Mechanical Engineering, 1*, 162–173.

Lee, T. H., & Jan, F.-H. (2019). Can community-based tourism contribute to sustainable development? Evidence from residents' perceptions of the sustainability. *Tourism Management, 70*, 368–380.

Liang, Y. (2020). An EDAS method for multiple attribute group decision-making under intuitionistic fuzzy environment and its application for evaluating green building energy-saving design projects. *Symmetry, 12*, 1–12. https://doi.org/10.3390/sym12030484

Lo, H.-W., Liaw, C.-F., Gul, M., & Lin, K.-Y. (2021). Sustainable supplier evaluation and transportation planning in multi-level supply chain networks using multi-attribute- and multi-objective decision making. *Computers & Industrial Engineering, 162*, Article 107756. https://doi.org/10.1016/j.cie.2021.107756

Lo, Y.-C., Janta, P. (2020). Resident's perspective on developing community-based tourism – a qualitative study of Muen Ngoen Kong community, Chiang Mai, Thailand. Frontiers in Psychology, doi: 10.3389/fpsyg.2020.01493.

Lu, J., Zhang, S., Wu, J., Wei, Y. (2021).COPRAS method for multiple attribute group decision-making under picture fuzzy environment and their application to green supplier selection. Technological and Economic Development of Economy 27, 369-385.

Mayaka, M., Croy, W. G., & Cox, J. W. (2018). Participation as motif in community-based tourism: A practice perspective. *Journal of Sustainable Tourism, 26*(3), 416–432.

McLoughlin, E., & Hanrahan, J. (2016). Local authority tourism planning in Ireland: An environmental perspective. *Journal of Policy Research in Tourism, Leisure and Events, 8*, 33–52.

Mishra, A. R., Chandel, A., & Saeidi, P. (2021). Low-carbon tourism strategy evaluation and selection using interval-valued intuitionistic fuzzy additive ratio assessment approach based on similarity measures. *Environment, Development and Sustainability,*. https://doi.org/10.1007/s10668-021-01746-w

Mishra, A. R., & Rani, P. (2021). Multi-criteria healthcare waste disposal location selection based on Fermatean fuzzy WASPAS method. *Complex & Intelligent Systems, 7*, 2469–2484.

Mishra, A. R., Rani, P., & Pandey, K. (2021). Fermatean fuzzy CRITIC-EDAS approach for the selection of sustainable third-party reverse logistics providers using improved generalized score function. *Journal of Ambient Intelligence and Humanized Computing*. https://doi.org/10.1007/s12652-021-02902-w

Mishra, A. R., Rani, P., Mardani, A., Pardasani, K. R., Govindan, K., & Alrasheedi, M. (2020). Healthcare evaluation in hazardous waste recycling using novel interval-valued intuitionistic fuzzy information based on complex proportional assessment method. *Computers & Industrial Engineering, 139*, Article 106140.

Narang, M., Joshi, M. C., & Pal, A. K. (2021). A hybrid fuzzy COPRAS-base-criterion method for multi-criteria decision-making. *Soft Computing, 25*, 8391–8399.

Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDA methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operations Research, 156*, 445–455.

Pasanchay, K., & Schott, C. (2021). Community-based tourism homestays' capacity to advance the Sustainable Development Goals: A holistic sustainable livelihood perspective. *Tourism Management Perspectives*. https://doi.org/10.1016/j.tmp.2020.100784

Peng, X., & Huang, H. (2020). Fuzzy decision making method based on CoCoSo with CRITIC for financial risk evaluation. *Technological and Economic Development of Economy, 26*(4), 695–724. https://doi.org/10.3846/tede.2020.11920

Peng, X., & Yang, Y. (2016). Fundamental properties of interval-valued pythagorean fuzzy aggregation operators. *International Journal of Intelligent Systems, 31*, 444–487.

Petrovic, I., & Kankaras, M. (2020). A hybridized IT2FS-DEMATEL-AHP-TOPSIS multicriteria decision making approach: Case study of selection and evaluation of criteria for determination of air traffic control radar position. *Decision Making: Applications in Management and Engineering, 3*, 146–164.

Pfueller, S. L., Lee, D., & Laing, J. (2011). Tourism partnerships in protected areas: Exploring contributions to sustainability. *Environmental Management, 48*(4), 734–749.

Pomering, A., Noble, G., & Johnson, L. W. (2011). Conceptualising a contemporary marketing mix for sustainable tourism. *Journal of Sustainable Tourism, 19*, 953–969.

Quevedo, J. M. D., Uchiyama, Y., & Kohsaka, R. (2021). Linking blue carbon ecosystems with sustainable tourism: Dichotomy of urban–rural local perspectives from the Philippines. *Regional Studies in Marine Science*. https://doi.org/10.1016/j.rsma.2021.101820

Rahman, K., Abdullah, S., & Khan, M. (2020). Some interval-valued Pythagorean fuzzy Einstein weighted averaging aggregation operators and their application to group decision-making. *J. Intell. Syst., 29*, 393–408. https://doi.org/10.1515/jisys-2017-0212

Rani, P., & Mishra, A. R. (2021). Fermatean fuzzy Einstein aggregation operators-based MULTIMOORA method for electric vehicle charging station selection. *Expert Systems with Applications*. https://doi.org/10.1016/j.eswa.2021.115267

Rani, P., & Mishra, A. R. (2022). Interval-valued fermatean fuzzy sets with multi-criteria weighted aggregated sum product assessment-based decision analysis framework. *Neural Computing and Applications*. https://doi.org/10.1007/s00521-021-06782-1.0123456789

Rani, P., Mishra, A. R., Saha, A., Hazem, I. M., & Pamucar, D. (2022). Fermatean fuzzy Heronian mean operators and MEREC-based additive ratio assessment method: An application to food waste treatment technology selection. *International Journal of Fuzzy Systems, 37*, 2612–2647.

Roozbahini, A., Ghased, H., & Shahedany, M. H. (2020). Inter-basin water transfer planning with grey COPRAS and fuzzy COPRAS techniques: A case study in Iranian Central Plateau. *Science of The Total Environment*. https://doi.org/10.1016/j.scitotenv.2020.138499

Sałabun, W., &Urbaniak, K. (2020). A new coefficient of rankings similarity in decision-making problems.In International Conference on Computational Science, 632-645. Springer, Cham, Doi: 10.1007/978-3-030-50417-5_47.

Saraji, M. K., Streimikiene, D., & Kyriakopoulos, G. L. (2021). Fermatean Fuzzy CRITIC-COPRAS Method for Evaluating the Challenges to Industry 4.0 Adoption for a Sustainable Digital Transformation. *Sustainability., 13*(17), 9577. https://doi.org/10.3390/su13179577

Schott, C., & Nhem, S. (2018). Paths to the market: Analysing tourism distribution channels for community-based tourism. *Tourism Recreat. Res., 43*(3), 356–371.

Sebele, L. S. (2010). Community-based tourism ventures, benefits and challenges: Khama Rhino Sanctuary Trust, Central District, Botswana. *Tourism Management, 31*, 136–146.

Senapati, T., & Yager, R. R. (2019). Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. *Engineering Applications of Artificial Intelligence, 85*, 112–121.

Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing, 11*, 663–674.
Sergi, D., Sari, I. U., & Senapati, T. (2021). Extension of capital budgeting techniques using interval-valued Fermatean fuzzy sets. *Journal of Intelligent & Fuzzy Systems, 1–12*. https://doi.org/10.3233/jifs-219196
Sgroi, F. (2020). Forest resources and sustainable tourism, a combination for the resilience of the landscape and development of mountain areas. *Science of The Total Environment*. https://doi.org/10.1016/j.scitotenv.2020.139539
Song, H., & Chen, Z.-C. (2021). Multi-attribute decision-making method based distance and COPRAS method with probabilistic hesitant fuzzy environment. *International Journal of Computational Intelligence Systems, 14*, 1229–1241.
Song, W., Niu, Z., & Zheng, P. (2021). Design concept evaluation of smart product-service systems considering sustainability: An integrated method. *Computers & Industrial Engineering, 159*, Article 107485. https://doi.org/10.1016/j.cie.2021.107485
Stănciulescu, G. C., & Ţîrca, A.-M. (2010). Implications of commercial activity within monastic settlements as a way to maintain the sustainable development of religious tourism in Romania. *The Amfiteatru Economic journal, 12*(27), 129–144.
Thongdejsri, M., & Nitivattananon, V. (2019). Assessing impacts of implementing low-carbon tourism program for sustainable tourism in a world heritage city. *Tourism Review, 74*(2), 216–234.
Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems, 25*, 529–539.
Torres-Delgado, A., & Palomeque, F. L. (2014). Measuring sustainable tourism at the municipal level. *Annals of Tourism Research, 49*, 122–137.
Tseng, M.-L., Wu, K.-J., Lee, C.-H., Lim, M. K., Bui, T.-D., & Chen, C.-C. (2018). Assessing sustainable tourism in Vietnam: A hierarchical structure approach. *Journal of Cleaner Production, 195*, 406–417.
Urtasun, A., & Gutiérrez, I. (2006). Tourism agglomeration and its impact on social welfare: An empirical approach to the Spanish case. *Tourism Management, 27*, 901–912.
Wang, L. E., Liu, H. C., & Quan, M. Y. (2016). Evaluating the risk of failure modes with a hybrid MCDM model under interval-valued intuitionistic fuzzy environments. *Computers & Industrial Engineering, 102*, 175–185.
Xu, C., & Shen, J. (2021). Multi-criteria decision-making and pattern recognition based on similarity measures for Fermatean fuzzy sets. *Journal of Intelligent & Fuzzy Systems, 41*, 5847–5863.
Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision-making. *IEEE Transactions on Fuzzy Systems, 22*, 958–965.
Youssef, M. I., & Webster, B. (2022). A multi-criteria decision making approach to the new product development process in industry. *Reports in Mechanical Engineering, 3*, 83–93.
Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.
Zavadskas, E. K., Kaklauskas, A., & Sarka, V. (1994). The new method of multicriteria complex proportional assessment of projects. *Technological and Economic Development of Economy, 1*, 131–139.
Zhou, L.-P., Wan, S.-P., & Dong, J.-Y. (2021). A Fermatean fuzzy ELECTRE method for multi-criteria group decision-making. *Informatica*.