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A Fuzzy Extension of Simplified Best-Worst Method (F-SBWM) and Its Applications to Decision-Making Problems

Maghsoud Amiri [1], Mohammad Hashemi-Tabatabaei [1], Mehdi Keshavarz-Ghorabaee [2,*], Arturas Kaklauskas [3], Edmundas Kazimieras Zavadskas [4,*] and Jurgita Antucheviciene [3]

1 Department of Industrial Management, Faculty of Management and Accounting, Allameh Tabataba'i University, Tehran 14348-63111, Iran
2 Department of Management, Faculty of Humanities (Azadshahr Branch), Gonbad Kavous University, Gonbad Kavous 49717-99151, Iran
3 Department of Construction Management and Real Estate, Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius, Lithuania
4 Institute of Sustainable Construction, Civil Engineering Faculty, Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius, Lithuania
* Correspondence: m.keshavarz@gonbad.ac.ir (M.K.-G.); edmundas.zavadskas@vilniustech.lt (E.K.Z.)

Abstract: Today, most of the issues and challenges faced by managers and decision makers are complex and multifaceted. More clearly, due to the developments of technologies, emerging trends in various industries, competitive markets, and rapid and transformative changes in the business environment, managers and decision makers have faced an uncertain environments and issues that cannot be resolved definitively. The use of multi-criteria decision-making (MCDM) methods as a practical and decision-supporting tool allows managers to examine decision-making issues in various organizations and industries based on various criteria, alternatives, and objectives and make decisions with greater reliability. The use of fuzzy techniques and concepts in MCDM methods and their mathematical relationships makes it possible to consider complexities and uncertainties in decisions related to various issues and it can lead to better and more realistic decisions. In this paper, the simplified best-worst method (SBWM), which is one of the methods based on pairwise comparisons, has been developed using triangular fuzzy numbers (TFNs) to propose a fuzzy extension of SBWM (F-SBWM). Triangular fuzzy numbers in different symmetric and asymmetric forms have widely been used in MCDM approaches and pairwise comparisons. It is noteworthy that symmetric numbers are used when we are using equal division of the domain due to an increased ambiguity and lack of information. The proposed approach as a simplified fuzzy MCDM method helps managers and decision makers in various industries to solve decision-making problems under uncertainty without the need for complex calculations, specialized skills, and software packages. To check the feasibility and applicability of the proposed approach, two numerical examples and a computational experiment with real data are presented, and the results are analyzed and discussed. Furthermore, to check the robustness of the results obtained from the proposed approach, sensitivity analysis and comparison of methods have been performed.

Keywords: multi-criteria decision making (MCDM); simplified best-worst method (SBWM); fuzzy MCDM; triangular fuzzy numbers (TFNs); symmetric and asymmetric forms; pairwise comparisons; warehouse location problem; medical mask selection problem; sensitivity analysis

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1. Introduction

In the multi-criteria decision-making (MCDM) process, alternatives are examined based on various criteria. Decision makers tend to make logical and rational decisions using MCDM methods so that they can use them to formulate plans and implement various activities in the organization. In general, MCDM has three main stages: (1) identifying and defining decision criteria and alternatives; (2) calculating the importance of each criterion using linguistic terms and their corresponding numerical sets; (3) performing numerical calculations to prioritize and rank the available alternatives based on the importance of each criterion [1]. By reviewing the literature in the field of management issues and challenges, we find that researchers widely use MCDM in various fields for issues such as supplier selection [2,3], location selection [4], selection of urban sewage network design [5], equipment selection [6], and water resource management [7]. Today, due to the increasing complexity and range of decisions, the data necessary to make decisions in organizations are more insufficient and uncertain than in the past and decision makers prefer to consider many indicators in the decision-making process to deal with the uncertainties [8]. Solving MCDM problems is not easy because criteria and alternatives have different dimensions and are expressed with different scales. In many cases, it may not be possible to choose the best criterion or option accurately, and in this situation, decision makers rely on reasonable and logical solutions [9]. An important aspect in MCDM methods is determining the relative importance of each criterion. The relative importance of each criterion is determined by a set of priorities called weights, which are represented between 0 and 1. The weight of criteria affects the outcome of any decision-making process, so it is necessary to use appropriate methods according to the nature of the problem [10].

Access to sufficient and definitive information about criteria and alternatives is one of the important and influential factors in the decision-making process. It is clear that in most cases there is no access to sufficient and accurate information about the criteria for decision making, and in real conditions it is very difficult to obtain sufficient and complete information about the criteria. On the other hand, although the decision makers in the organization make their decisions by considering an acceptable error threshold, their logical and subjective preferences alone cannot fulfill the organization's goals. If experts solve problems using insufficient information and limited knowledge, the results of their decisions will lead to increased levels of uncertainty and ambiguity in the organization [11]. To deal with the uncertainties and lack of information in the decision-making environment, many methods and theories have been developed and combined with different MCDM approaches [12]. In addition, many studies have examined various MCDM techniques as effective multi-objective optimization tools to identify the most suitable solutions and have dealt with uncertainty in the decision-making environment by using various mathematical tools [13].

The theory of fuzzy sets was introduced by Zadeh [14] in 1965 to involve uncertainties in real-world problems. The theory of fuzzy sets was developed due to the existence of uncertainties in the problems faced by organizations. In fuzzy logic, a range of concepts and applications are defined so that experts and decision makers can choose their preferences according to real-world conditions. This theory is able to mathematically formulate many problems that have uncertain and ambiguous variables and provide a basis for reasoning, inference, control, and decision making in uncertain conditions. Fuzzy decision-making methods, unlike deterministic methods, are able to involve uncertainties in the decision-making process [15].

Decision making in organizations is one of the main duties of managers and is an inevitable process [16]. Fuzzy decision-making methods are widely used to solve managerial problems and challenges [17]. Despite the many advantages of MCDM methods, the implementation of these methods in a crisp deterministic manner may face significant limitations when making decisions in real-world situations. This is because these methods fail when there is a lack of information and uncertain conditions. Such decision-making situations can be investigated with the help of fuzzy set theory, which is an effective way to deal with the lack of information and imprecise data [18]. On the other hand, due to the existence of uncertainties and lack of information in real conditions, the need to use an appropriate decision-making approach that is both practical and easy to use and involves uncertainties in decision making is felt more than ever. Fuzzy MCDM methods by considering uncertainty in scenarios are suitable for decision making in ambiguous situations. To achieve the best solution and overcome various management issues, fuzzy MCDM models can be used with multiple criteria and multiple scenarios [19]. Some previous studies have pointed out the necessity of the fuzzification of MCDM methods [20,21]. In real MCDM problems, decision makers often make their preferences using incomplete information. However, decision makers may prefer to use a simple and easy technique instead of complex algorithms to obtain acceptable results [22]. In MCDM problems, it is recommended to simplify the processes and systems in such a way that they are understandable and can be easily implemented [23].

The best-worst method (BWM) is one of the efficient and popular MCDM methods, which was introduced in 2015 by Rezaei [24]. The BWM method requires less information for decision making and is able to investigate various multi-criteria problems with fewer pairwise comparisons than existing methods. In BWM, first the most important and least important criteria are selected by the decision maker, and then the rest of the criteria are compared with them. Due to the many advantages of this method compared to the previous methods, today many researchers and decision makers use this method to solve multi-criteria problems. The data required in BWM are completely subjective and depend on the preferences of decision makers. Compared to analytic hierarchy process (AHP) and analytic network process (ANP) methods, the BWM requires less comparative data, which makes the BWM simpler. The results of recent review studies show that the BWM has been used in more studies compared to the AHP and ANP methods [10].

Amiri et al. [25] proposed a simplified BWM to solve MCDM problems. The simplified BWM was able to solve multi-criteria problems using simple numerical calculations and without the need to solve the mathematical programming model and provide acceptable results compared to the original model of BWM. The main purpose of this study is to extend a fuzzy version of simplified BWM for dealing with uncertainty in decision-making problems. Therefore, the main research question is how to develop the fuzzy simplified BWM to face uncertainty in decision-making problems?

In this research, a new fuzzy MCDM approach called the fuzzy extension of simplified best-worst method (F-SBWM) is proposed. By maintaining the advantages of BWM and using triangular fuzzy numbers (TFNs), the proposed approach helps decision makers to make appropriate decisions in uncertain conditions only by using simple fuzzy relations without the need for software packages and complex calculations to solve the mathematical programming model in BWM. Using TFNs in MCDM problems has many advantages. One of the advantages of using TFNs is the simple implementation capability using linear functions and only three parameters. Moreover, the number of parameters can be reduced to two parameters if symmetric TFNs are used. Therefore, we can make efficient pairwise comparisons based on symmetric TFNs. In the proposed approach, first a set of necessary criteria are selected to carry out the decision-making process. Then the decision maker selects the most important criterion as the best criterion and the least important criterion as the worst criterion. Fuzzy reference comparisons (comparing the best criterion with the other criteria and comparing the other criteria with the worst criterion) are performed using linguistic terms and their corresponding TFNs. In this way, the best-to-others and others-to-worst fuzzy preferences vectors are obtained. Then, using fuzzy operators and simple mathematical relationships, the weights of the decision criteria are calculated, and by combining them with the weights resulted from the reference comparisons, the final fuzzy weights of the criteria are calculated. In addition to simplifying the decision-making process and saving time by reducing complex calculations, the proposed approach has high flexibility in solving management problems and challenges in various fields. The contributions of this research can be summarized as follows:

- Developing a simplified multi-criteria decision-making method in a fuzzy environment to take into account ambiguity in information, insufficient knowledge, and ambiguity in decision makers' preferences using TFNs.
- Introducing a simplified fuzzy approach as a decision support tool using fuzzy operators and simple calculations without the need to formulate mathematical programming models and software packages.

- Examining the accuracy of the decision makers' opinions and the reliability of the results by calculating the proposed consistency ratio in the pairwise comparisons made by the decision makers.
- The flexibility of the proposed approach in the face of various linguistic terms and the scales of their corresponding fuzzy numbers.

To check the applicability and effectiveness of the proposed approach, numerical examples and a computational experiment are provided, and the results are analyzed and discussed.

The rest of this paper is organized as follows. Section 2 reviews the literature related to the development of BWM in uncertain environments. In Section 3, the proposed approach, along with its steps and how to calculate the consistency rate, is explained. Several numerical examples are provided in Section 4. In Section 5, sensitivity analysis is performed to check the robustness of the results obtained from the proposed approach. In Section 6, the results obtained from the proposed approach are compared with existing methods. The significant outcomes of the proposed approach are discussed in Section 7, and finally, conclusions, limitations, and suggestions are presented in Section 8.

2. Literature Review

In this section, some previous studies regarding the development of BWM in uncertain environments are reviewed. To address many real-world decision-making issues and challenges, definitive, clear, and accurate data are not available. There is uncertainty and ambiguity in most of the input data needed for decision-making processes. To deal with uncertainty, various theories such as fuzzy set theory, rough theory, D-numbers, and grey theory have been developed [26,27]. Fuzzy BWM methods have been widely used in the literature and are able to optimize a decision based on multiple criteria. In addition, it has been recommended that the development of decision-making tools based on MCDM methods in various fields such as supply chain management and emerging economies can be attractive [28].

Today, a wide range of strategic and sensitive decisions in organizations are made in an uncertain environment, and decision makers are forced to use decision-making tools developed for uncertain conditions. Most of the methods and models developed for uncertain conditions have complex and time-consuming calculations that make it difficult for decision makers to make the right decision. On the other hand, MCDM methods are inherently developed to solve complex decision-making problems [29]. Using MCDM methods to solve complex decision-making problems, especially in uncertain environments, requires complex calculations, a lot of time, and the use of related software, which can lead to the problem becoming more complex [30]. To cover this research gap, simplified BWM was developed in this study in order to make decisions under uncertainty without the need for complex calculations, specialized skills, and software packages.

BWM is one of the most efficient multi-criteria decision-making methods based on pairwise comparisons. In this research, this method is extended to make decisions under uncertain conditions. The proposed approach uses the simplified form of BWM and does not need to solve complex mathematical problems or use optimization software to weight the criteria. It has been recommended that the best method for making decisions and solving management issues and challenges is the simplest method [31].

Guo and Zhao [32] suggested that the preferences of decision makers in BWM, which are expressed as numbers between 1 to 9, are not effective enough to solve complex problems in the real world, and it is necessary to consider ambiguities and uncertainties. For this purpose, they modified BWM using fuzzy logic and presented new fuzzy models using TFNs. The results of their study showed that the fuzzy BWM leads to better comparisons than the original BWM. Mou et al. [33] extended BWM using intuitionistic fuzzy preference relation (IFPR) in fuzzy environments. Several mathematical models were proposed to calculate the importance of each criterion, and finally the developed approach was analyzed using several numerical examples. Pamučar et al. [34] extended the BWM using interval-valued fuzzy-rough numbers (IVFRNs). They investigated the new fuzzy BWM model using an example in the aviation industry and suggested that their proposed approach has high efficiency for decision making in uncertain environments. In another study, BWM was extended based on Z-number. The proposed approach had the ability to include uncertainties in experts' preferences and help decision makers make better decisions in situations with ambiguity and lack of information. The proposed approach was evaluated using an example for supplier management [35].

In another study, BWM was extended based on hesitant fuzzy linguistic information. The proposed approach was able to consider the preferences of experts using several different sets with different values as input of BWM and thus deal with uncertainties in decision problems. The proposed approach was used to evaluate the performance of medical centers, and the results were analyzed [36]. BWM was developed using a fully fuzzy linear mathematical model. The presented model evaluated the importance of the criteria by making reference comparisons. The advantages of the proposed fully fuzzy linear model were investigated using an example in the field of repairs and maintenance [37]. To select a green supplier, BWM was extended based on gray numbers. The proposed method was able to consider the uncertainties and ambiguities in decision-making problems in the form of specific intervals of uncertainty, and it was also able to examine the interdependence of the criteria [38]. In another research, BWM was extended to deal with uncertainty in decision-making problems based on interval type-2 fuzzy sets (IT2FSs). The proposed approach was implemented to select a green supplier, and the results were analyzed. Sensitivity analysis was also performed to check the robustness of the weights obtained from the proposed approach [39].

BWM was extended based on intuitionistic fuzzy multiplicative preference relations (IFMPRs). The developed approach was able to include the uncertainty in the input parameters of the model, and it was also possible to make decisions based on a multi-criteria and group process. The proposed approach was tested using a practical example in the field of health management, and its applicability was examined [40]. Feng et al. [41] presented a new method to calculate the consistency rate in BWM. In their proposed approach, the compatibility rate was calculated based on the distance between the reference comparisons in BWM. They suggested that the differences of the preferences made by decision makers in both vectors (the best compared to the others and the others compared to the worst) directly affects the reliability of the decisions. Then, to deal with uncertainty, they extended BWM using linear programming models and gray numbers and provided different numerical examples to show the details of the proposed approach. Hasan et al. [42] proposed the multi-choice version of BWM. In their study, the reference comparisons in BWM had high flexibility, so that the decision maker could consider more than a single value for his preferences to make the decision-making process more realistic. Although the input data in the multi-choice model was not considered as fuzzy data, their method reduced the ambiguities and complexity in the decision-making process. They developed the BWM linear programming model based on the multi-choice programming model, conducted several experimental studies, and compared the results to show the quality, application, and performance of the proposed approach.

Abdel-Basset et al. [43] proposed an approach to evaluate sustainable supply chain financing in the gas industry. Their proposed approach prioritized decision criteria under uncertain conditions. They applied TODIM, TOPSIS, and BWM based on neutrosophic sets and suggested that their proposed approach is more stable under uncertain conditions. The results of their research showed that financial characteristics and criteria related to product or service management have the greatest impact on sustainable financing of the supply chain. In another study, the traditional BWM was extended based on the belief function theory (BFT). In this study, it was stated that there are usually ambiguities and uncertainties in the qualitative judgments of decision makers, and decision makers face uncertainty due to the lack of sufficient information and knowledge. They stated that the belief function theory is well capable of dealing with uncertainty, and extending BWM based on this theory can help decision makers in uncertain situations. Finally, to show the applicability of the proposed approach, a case study was conducted to evaluate hospital service quality, and the results were discussed [44].

Jafarzadeh-Ghoushchi et al. [45] developed a new fuzzy BWM based on the Importance-Necessity concept (G-number). They stated that despite its many advantages, BWM is not able to deal with uncertainties. They added some new features to the basic BWM using the concept of G-number theory. They conducted two case studies to validate the results of the proposed approach. To enable group decision making in non-deterministic space, the BWM was extended using hesitant fuzzy multiplicative preference relation. In the proposed approach, the decision makers' preferences were used as inputs for the model using hesitant fuzzy elements, and in this way, the proposed model was able to deal with ambiguities and lack of information in the decision-making process. To examine the capabilities of the proposed approach, several case studies were conducted, and the results showed that the proposed approach has more advantages than the original BWM due to its more consistent comparisons [46]. The gray approach of BWM was developed to evaluate criteria in uncertain environments. In the proposed approach, decision makers' opinions were collected based on gray linguistic variables, and the gray nature of the data maintained throughout the decision-making process. Finally, robustness of the results was evaluated using a case study and performing sensitivity analysis [47].

Amiri et al. [48] developed a linear programming model for BWM using possibilistic chance-constrained programming (PCCP) and trapezoidal fuzzy numbers. They analyzed three different approaches of Possibility, Necessity, and Credibility in the basic model of BWM. Their proposed approach had two main advantages: First, decision makers could consider uncertainties in their decisions, and second, they could make flexible decisions (optimistic and pessimistic) according to the level of uncertainty in the decision making. The Possibility approach reflected DM's optimistic view of the problem, the Necessity approach was used in situations where the decision maker preferred a pessimistic view to an optimistic view, and the Credibility approach was used in situations where the decision maker preferred an intermediate state between optimistic and pessimistic states. Finally, to examine the applicability of the approach, some numerical examples were provided and the results were analyzed. BWM was extended for use in uncertain environments. The proposed approach used the concept of generalized interval-valued trapezoidal fuzzy multiplicative preference relation and was able to help decision makers in situations where there is not enough and accurate information for decision making, and it implemented the group decision making as a decision support system. The proposed approach was called generalized interval-valued trapezoidal fuzzy BWM (GITrFBWM). A method to calculate the consistency rate was described, and applicability of the proposed model was investigated using three case studies [49]. Chen et al. [50] proposed a new multi-criteria group decision-making approach in a trapezoidal interval type-2 fuzzy (TrIT2F) environment. In the proposed approach, a combination of BWM and DEA was used, TrIT2F-BWM was used to calculate the importance of each criterion, and the alternatives were ranked using TrIT2F-DEA. The required operators as well as the method of calculating the consistency rate were fully explained. The capabilities of the model were investigated in a case study about choosing an appropriate location for makeshift hospitals. Given that the proposed approach was developed using TrIT2F information, it was able to include a higher level of uncertainty in the decision-making process.

BWM was extended based on TFNs for decision making in fuzzy environments. The mathematical programming model was developed to obtain the optimal triangular weights. Different mathematical models were formulated for optimistic, pessimistic, and neutral states of decision making. The efficiency and capabilities of the developed fuzzy BWM were investigated and tested using several practical examples [51]. In a study, a group decision framework was proposed based on BWM and intuitionist 2-tuple linguistic (I2TL) sets. In the proposed approach, it was claimed that because the nonlinear programming model in BWM may provide multiple optimal solutions, the linear and Euclidean programming models were used. Moreover, intuitionistic 2-tuple linguistic Hamacher weighted average (I2TLHWA) and intuitionistic 2-tuple linguistic Hamacher weighted geometric (I2TLHWG) operators were introduced in the proposed framework. In the proposed approach, preference vectors were created in the I2TL environment, and thus uncertainties were involved in the decision making. Finally, the efficiency and validity of the method were examined using numerical examples [52].

Seyfi-Shishavan et al. [53] developed a modified BWM based on intuitionistic fuzzy mode. Their proposed method was able to consider uncertainties in the decision-making space. The proposed method also had the ability to obtain the importance of each criterion without the need for de-fuzzification. The compatibility and applicability of the proposed approach was investigated in a practical example about the role of the banking sector in supply chain financing. In another study, BWM was developed in intuitionistic fuzzy environment and was used to evaluate the efficiency of a water treatment plant. In the proposed approach, the consistency of the comparisons was higher than the basic BWM. A comparative study and sensitivity analysis were conducted to confirm the validity of the model [54]. Hussain et al. [55] extended the basic BWM to solve various management problems in fuzzy environment. In their proposed approach, fuzzy reference comparisons were made between criteria, and after formulating a planning problem, the maximum importance of each criterion and alternative was determined. They also provided a consistency ratio to check the reliability of the results. Two case studies were conducted to clarify how to use the proposed approach. They found that the approach had higher performance than some of the other fuzzy/deterministic multi-criteria decision making methods. In another research, a multi-criteria group decision-making approach was proposed based on BWM and TFNs. In that research, it was stated that in most decision-making issues, a group of decision makers are involved, and a senior decision maker makes appropriate decisions by reviewing the opinions of other decision makers. Using fuzzy numbers, they extended the linear programming model of BWM and proposed an approach to combine the opinions of decision makers. The proposed approach was capable of examining decisions in two modes: democratic and autocratic. Finally, the applicability of the model was examined using two numerical examples [56].

Guo and Qi [57] proposed a new fuzzy approach based on BWM method for group decision making. They stated that in most decision-making scenarios there is more than one decision maker, and in most cases, there are uncertainties throughout the decision-making process. Using multi-granularity linguistic term sets (LTS), they extended BWM for uncertain situations. In the proposed approach, the best and worst criteria were selected by each expert separately, and finally the experts' preferences were merged into two vectors. They proposed a new method to calculate the consistency of comparisons made by decision makers and finally provided different numerical examples to demonstrate the effectiveness and applicability of the proposed approach. In another research, axiomatic design (AD) and BWM were developed in an interval type-2 fuzzy (IT2F) environment. The proposed approach was based on the concepts of fuzzy logic and was used for MCDM under uncertain conditions. IT2F-BWM and IT2F-AD were used to prioritize blockchain deployment projects in a real case, and the results were discussed [58].

Zhou et al. [59] proposed a new model called hesitant fuzzy linguistic hybrid cloud (HFLHC). They stated that due to the complex uncertainties in linguistic evaluations in MCDM, it is necessary to develop new models to deal with these uncertainties. The proposed model considered more flexibility in linguistic terms by integrating normal cloud and trapezium cloud. Using the concepts of optimization programming, they extended BWM based on the HFLHC model to determine the optimal weights of the decision criteria. A method to calculate the consistency rate was also proposed. Finally, using a practical example in the field of sustainable supplier selection, they showed that the proposed approach is more reliable than the previous methods. In another research, BWM was extended in the rough-fuzzy environment to consider the uncertainty in the decision-making process. In the proposed approach, decision makers used rough-fuzzy numbers

(RFN) to express their preferences, and the rough-fuzzy vector of the best-to-others and others-to-worst comparisons was obtained. In this research, data envelopment analysis (DEA) was also introduced in the rough-fuzzy environment. The proposed approach took into account the uncertainties in the group decision environment while maintaining the previous advantages of BWM, such as fast calculation of the criteria optimal weights. The validity and applicability of the proposed approach were examined using a case study about smart vehicle service module selection [60].

Ögel et al. [61] developed a new fuzzy BWM approach called Dombi-Bonferroni operators-based fuzzy BWM (DB-FBWM). The proposed approach was based on the fuzzy approach introduced by Guo and Zhao [26], with the difference that Dombi and Bonferroni operators were added in the proposed model. They suggested that the proposed approach had higher flexibility and reliability and examined the applicability of the method in a real case study in the field of retail food waste. The BWM was extended based on TFNs and using α-cut analysis to solve decision-making problems in uncertain environments. The proposed approach was able to consider different levels of uncertainty for a particular parameter. By using the proposed approach, decision makers were able to determine a suitable amount of uncertainty in their decisions by setting the value of α parameter between 0.1 and 0.9. To investigate the applicability of the proposed approach, a real case study was conducted in the automotive industry and the results were analyzed and compared [62].

Che and Zhang [63] proposed a new MCDM framework for decision making in uncertain situations. They developed their approach based on the BWM-based ANP network analysis method in the 2-dimensional uncertain linguistic (2-DUL) environment. The results showed that the proposed approach increases the effectiveness and efficiency of the decision-making process as well as the stability of the matrix of pairwise comparisons. The use of 2-DUL in the proposed approach allowed decision makers to include vague and uncertain information in their final decision and improve the reliability of the results. A case study of municipal solid waste management in Beijing was provided, and the results were discussed. The BWM was developed based on generalized interval-valued trapezoidal fuzzy numbers (GITrFNs). In the proposed approach, the preferences of the experts and the optimal weights of the criteria were expressed and used in the form of GITrFNs. Moreover, an ideal planning model was proposed to obtain the optimal weights of the criteria. The consistency index and consistency ratio were also formulated based on the proposed approach. They also proposed an approach to improve consistency for comparisons that had high consistency rates. Finally, the effectiveness and performance of the proposed approach was demonstrated by solving three real sample problems [64]. In another study, BWM was extended based on interval type-2 fuzzy sets (IT2FS). The proposed approach determined the importance of each criterion and alternative by taking into account the ambiguities and the uncertain atmosphere governing the decision making. The proposed approach was a flexible method to consider and involve hesitant opinions of experts, especially in the field of group decision making. Some numerical examples were presented, and the results were compared with the results of some previous methods. The ability of the proposed model to solve problems in a group-manner was also investigated. The results showed that the proposed approach not only provides acceptable results, but also outperforms the traditional BWM and its type 1 fuzzy extension [65].

As seen, most of the studies that have extended BWM in non-deterministic environments have complex calculations and mathematical programming models. To deal with uncertainties, researchers have to use fuzzy multi-criteria decision-making methods to make decisions with greater certainty. On the other hand, the complexity of calculations in fuzzy methods requires more time and cost to achieve the goals. It will be useful to provide a method that can solve various problems in various fields with simple calculations and without the need for special software. Therefore, the main purpose of this research is to introduce a fuzzy simplified decision-making approach to solve various problems faced by managers and decision makers. Table 1 provides a summary of the reviewed studies.

Table 1. A summary of the developed BWM in an uncertain environment.

No.	Reference	Information Form	Technique(s)/Approach	Mathematical Model	Application
1	[32]	Triangular fuzzy numbers	Graded mean integration representation (GMIR)	√	Case studies
2	[33]	Intuitionistic fuzzy numbers (IFNs)	Intuitionistic fuzzy preference relation (IFPR)- intuitionistic fuzzy weighted averaging (IFWA)	√	Numerical examples
3	[34]	Interval-valued fuzzy-rough numbers (IVFRN)	Rough approach	√	Aviation industry
4	[35]	Z-numbers	BWM and z-numbers integration	√	Supplier development
5	[36]	Hesitant fuzzy linguistic information	Hesitant fuzzy linguistic BWM	√	Hospital performance evaluation
6	[37]	Triangular fuzzy number	Fully fuzzy linear mathematical programming	√	Maintenance problem
7	[38]	Grey numbers	Grey theory	√	Supplier selection
8	[39]	Interval type-2 fuzzy sets (IT2FSs)	Grey approach	√	Supplier selection
9	[40]	Intuitionistic fuzzy numbers (IFNs)	Intuitionistic fuzzy multiplicative preference relations (IFMPRs)	√	Health management
10	[41]	Grey numbers	Consistency measure and priority weights	√	Numerical example
11	[42]	Multi-choice information	Multi-choice programming	√	Experimental studies
12	[43]	Neutrosophic scale	Neutrosophic TODIM-TOPSIS-BWM	√	Gas industry evaluation
13	[44]	Belief information	Belief function theory (BFT)	√	Hospital service quality evaluation
14	[45]	G-number	theory G-number	√	Case studies
15	[46]	Hesitant fuzzy elements	Hesitant fuzzy multiplicative preference relation	√	Case studies
16	[47]	Grey numbers	Grey theory	√	Numerical example
17	[48]	Trapezoidal fuzzy numbers	Possibilistic chance-constrained programming (PCCP)	√	Numerical examples
18	[49]	Generalized interval-valued trapezoidal fuzzy number	GITrFBWM	√	Case studies
19	[50]	Trapezoidal interval type-2 fuzzy numbers	TrIT2F-BWM-DEA	√	Makeshift hospital selection
20	[51]	Triangular fuzzy number	Mathematical programming model	√	Numerical examples
21	[52]	Intuitionist 2-tuple linguistic information	Intuitionist 2-tuple linguistic BWM	√	Numerical examples
22	[53]	Intuitionistic fuzzy numbers (IFNs)	Intuitionistic fuzzy BWM	√	Banking industry performance
23	[54]	Intuitionistic fuzzy numbers (IFNs)	Intuitionistic fuzzy BWAHP	√	Water treatment plant
24	[55]	Triangular fuzzy number	Fuzzy technique for best–worst analysis (FTBWA)	√	Case studies
25	[56]	Triangular fuzzy number	Fuzzy linear programming-GDM	√	Numerical example
26	[57]	Multi-granularity linguistic information	Geometric averaging method	√	Numerical example
27	[58]	Interval type-2 fuzzy numbers	IT2F-BWM and IT2F-AD	√	Blockchain deployment projects
28	[59]	Hesitant fuzzy linguistic hybrid cloud information	HFLHC-based BWM	√	Sustainable supplier selection
29	[60]	Rough-fuzzy number (RFN)	Rough–fuzzy BWM-DEA	√	Smart vehicle service module selection
30	[61]	Triangular fuzzy number	DB-FBWM	√	Retail food waste
31	[62]	Triangular fuzzy number	α-cut analysis	√	Supplier selection
32	[63]	2-DUL information	BWM-based ANP	√	waste management
33	[64]	Generalized interval-valued trapezoidal fuzzy numbers (GITrFNs)	Goal programming	√	Case studies
34	[65]	Trapezoidal interval type-2 fuzzy number	Interval type-2 hesitant fuzzy BWM	√	Case studies
	This Study	**Triangular fuzzy numbers**	**Simplified BWM (SBWM)**	×	**Numerical Examples**

3. The Proposed Approach

The simplified best-worst method (SBWM) was introduced by Amiri et al. [25]. SBWM was able to solve multi-criteria decision problems using simple mathematical relationships based on expert's preferences. Reference comparisons in the BWM are seen in Figure 1. In SBWM, there was no mathematical programming model, and the calculations could be easily performed. In this research, we extend SBWM for use in a non-deterministic environment and we call it a fuzzy extension of simplified best-worst method (F-SBWM). For this purpose, we use TFNs and fuzzy operators. TFNs have been used in the development of the proposed fuzzy algorithm because it is more general than other fuzzy numbers, and it is easier to perform fuzzy calculations in TFNs. Moreover, TFNs are special cases of trapezoidal fuzzy numbers [66]. In the proposed approach, similar to the traditional BWM, a set of criteria are first determined to carry out the decision-making process. The decision maker selects the most important and the least important criteria. Then fuzzy reference comparisons (comparisons of the best criteria to the other criteria and the other criteria to the worst criteria) are made using linguistic terms and TFNs. The expert's fuzzy preferences vector is formed, and at this stage, the importance of each criterion is calculated only using simple calculations and fuzzy operators, without the need to formulate mathematical programming models. In the fuzzy comparisons of the best criterion to the other criteria, the importance of the best criterion is determined, and using the weight of the best criterion, the weights of other criteria is also calculated as the best-to-others vector. Moreover, in the fuzzy comparisons of the other criteria to the worst criterion, the importance of the worst criterion is determined, and using the weight of the worst criterion, the weights of other criteria is obtained as the others-to-worst vector. Then, both vectors are combined to obtain the final weights of the decision criteria. The proposed approach significantly reduces the time required for the decision-making process, and since there are no mathematical programming models in this method, the decision maker does not need to use software packages. Moreover, the numerical examples and the comparison of the results with other studies show that the proposed approach has high flexibility in using different fuzzy scales. The notations used in the proposed approach along with their descriptions are provided in Table 2.

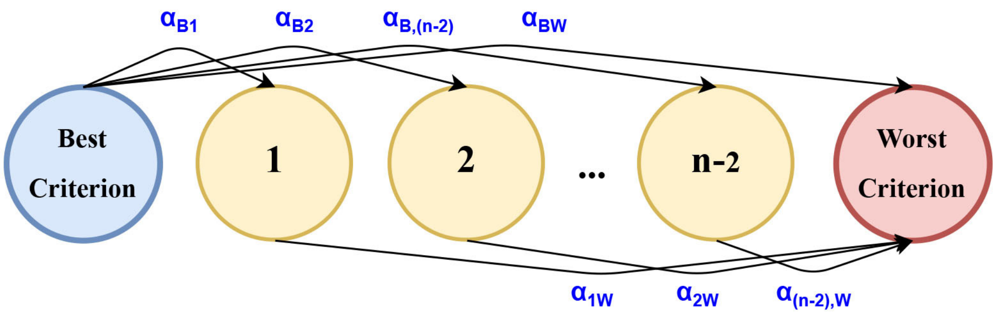

Figure 1. Reference comparisons in the best-worst method.

Definition 1. *Fuzzy sets [14]: Let* $X = \{x_1, x_2, \ldots, x_n\}$ *be the set of elements denoted by* x_i. *A fuzzy set* $\widetilde{A}$ *in X is represented as* $A = \{(x_i, \mu_{\widetilde{A}}(x_i)) | x_i \in X\}$, *where* $\mu_{\widetilde{A}}$ *is the membership function of the fuzzy set* $\widetilde{A}$, $\mu_{\widetilde{A}}(x_i)$ *denotes the grade of membership of element* x_i, *associated with* $\widetilde{A}$ *and* $\mu_{\widetilde{A}}(x_i) \in [0,1]$.

Definition 2. *A triangular fuzzy number (TFN) [51]: A fuzzy number* $\widetilde{A}$ *is called TFN and denoted by* $\widetilde{A} = (a^L, a^M, a^U)$, *where* $\widetilde{A}$ *is a fuzzy set defined on the set R of real numbers. The TFN membership function* $\mu_{\widetilde{A}}(x_i)$ *can be defined as follows:*

$$\mu_{\widetilde{A}}(x_i)\begin{cases} \frac{a^U - x}{a^U - a^M},\ if\ a^M \le x \le a^U \\ \frac{x - a^L}{a^M - a^L},\ if\ a^L \le x \le a^M \\ 0,\ if\ x < a^L\ or\ x > a^U \end{cases}$$

where a^L, a^M, *and* a^U *are called the lower bound, the modal, and the upper bound of the TFN* $\widetilde{A} = (a^L, a^M, a^U)$, *respectively, and* $a^L \le a^M \le a^U$. *If* $a^L \ge 0$, *then the TFN is called a positive TFN. If* $a^U \le 0$, *then the TFN is called a negative TFN.*

Definition 3. *The relationships and mathematical operators of TFNs used in the proposed approach are as follows [67]:*

$$\widetilde{A} = \left(a^L, a^M, a^U\right)$$

$$\widetilde{B} = \left(b^L, b^M, b^U\right)$$

$$\widetilde{A} + \widetilde{B} = \left(a^L + b^L, a^M + b^M, a^U + b^U\right)$$

$$\widetilde{A} - \widetilde{B} = \left(a^L - b^U, a^M - b^M, a^U - b^L\right)$$

$$\widetilde{A} \times \widetilde{B} \cong \left(a^L b^L, a^M b^M, a^U b^U\right)$$

$$\widetilde{A} \div \widetilde{B} \cong \left(\frac{a^L}{b^U}, \frac{a^M}{b^M}, \frac{a^U}{b^L}\right)$$

$$\frac{1}{\widetilde{A}} \cong \left(\frac{1}{a^U}, \frac{1}{a^M}, \frac{1}{a^L}\right)$$

$$\widetilde{A} \times \lambda = \left(a^L \lambda, a^M \lambda, a^U \lambda\right), \text{ where } \lambda \geq 0.$$

Table 2. Notations used in the proposed approach and their descriptions.

Notation	Description
C_i	The i-th criterion defined by the decision maker
$\widetilde{a}_{Bj}$	The priority of the best criterion over the j-th criterion in the form of TFN
$\widetilde{a}_{jW}$	The priority of j-th criterion over the worst criterion in the form of TFN
a^L_{Bj}	The lower limit of the TFN related to the priority of the best criterion over the j-th criterion
a^M_{Bj}	The moderate limit of the TFN related to the priority of the best criterion over the j-th criterion
a^U_{Bj}	The upper limit of the TFN related to the priority of the best criterion over the j-th criterion
a^L_{jW}	The lower limit of the TFN related to the priority of the j-th criterion over the worst criterion
a^M_{jW}	The moderate limit of the TFN related to the priority of the j-th criterion over the worst criterion
a^U_{jW}	The upper limit of the TFN related to the priority of the j-th criterion over the worst criterion
$\widetilde{w}'_j$	The weight of the j-th criterion in the reference comparisons of the best-to-others
$\widetilde{w}''_j$	The weight of the j-th criterion in the reference comparisons of the others-to-worst
w^L_j	The lower limit of the TFN related to j-th criterion weight
w^M_j	The moderate limit of the TFN related to j-th criterion weight
w^U_j	The upper limit of the TFN related to j-th criterion weight
$\widetilde{w}'_B$	The best criterion weight in the reference comparisons of the best-to-others
$\widetilde{w}''_W$	The worst criterion weight in the reference comparisons of the others-to-worst
$\widetilde{w}^*_j$	Final weight of the j-th criteria
CR	Consistency Ratio

The steps of the F-SBWM are as follows:

Step 1: Determining decision criteria in the form of $C_i = \{c_1, c_2, \ldots, c_n\}$ and choosing the most important and least important criteria.

Step 2: Determining the priority of the best criterion over each of the other criteria using linguistic terms and TFNs in the form of $\widetilde{a}_{Bj} = \left(a^L_{Bj}, a^M_{Bj}, a^U_{Bj}\right)$.

Step 3: Determining the priority of each of the criteria over the least important criteria using linguistic terms and TFNs in the form of $\widetilde{a}_{jW} = \left(a^L_{jW}, a^M_{jW}, a^U_{jW}\right)$.

Step 4: Calculating the priority of each criteria using reference comparisons of the best criterion to the other criteria in the form of $\widetilde{w}'_j = \left(w^L_j, w^M_j, w^U_j\right)$. Using Equation (1), the priority of the best criterion over each of the criteria is calculated. Then the weight of the best criterion is calculated. By substituting the weight of the best criterion in Equation (2), the weights of the other criteria is also calculated.

$$\sum_j \frac{1}{\widetilde{a}_{Bj}} \widetilde{w}'_B = 1 \Rightarrow \widetilde{w}'_B = \frac{1}{\sum_j \frac{1}{\widetilde{a}_{Bj}}} \tag{1}$$

$$\widetilde{w}'_B - \widetilde{a}_{Bj} \widetilde{w}'_j = 0 \Rightarrow \widetilde{a}_{Bj} \widetilde{w}'_j = \widetilde{w}'_B \Rightarrow \widetilde{w}'_j = \frac{\widetilde{w}'_B}{\widetilde{a}_{Bj}}, \ \forall j \tag{2}$$

Step 5: Calculating the priority of each criteria using reference comparisons of each criteria to the worst criterion in the form of $\widetilde{w}''_j = \left(w^L_j, w^M_j, w^U_j\right)$. The priority of each criterion over the worst criterion is calculated using Equation (3), and the weight of the worst criterion is calculated. By substituting the weight of the worst criterion in Equation (4), the weights of the other criteria are also calculated.

$$\sum_j \widetilde{a}_{jW} \widetilde{w}''_W = 1 \Rightarrow \widetilde{w}''_W = \frac{1}{\sum_j \widetilde{a}_{jW}} \tag{3}$$

$$\widetilde{w}''_j - \widetilde{a}_{jW} \widetilde{w}''_W = 0 \Rightarrow \widetilde{w}''_j = \widetilde{a}_{jW} \widetilde{w}''_W, \quad \forall j \tag{4}$$

Step 6: Calculating the final weights of the decision criteria as $\left(\widetilde{w}^*_1, \widetilde{w}^*_2, \ldots, \widetilde{w}^*_n\right)$ using the final weights obtained from the reference comparisons of the best criterion to the other criteria $\left(\widetilde{w}'_j\right)$ and all criteria to the worst criterion $\left(\widetilde{w}''_j\right)$, using Equation (5).

$$\widetilde{w}^*_j = \frac{\widetilde{w}'_j + \widetilde{w}''_j}{2} \tag{5}$$

The reliability of the results obtained from MCDM approaches is very important for managers and decision makers because the results of multi-criteria analysis are directly used to make key and strategic decisions in organizations. If the results have low reliability, relying on them may cause many problems for the organization [68]. For this purpose, the consistency rate in the proposed approach is calculated using Equation (6). In decision-making methods based on pairwise comparisons, it is inevitable to calculate the consistency rate of the comparisons. If the value of the consistency rate is not appropriate, the results will not be reliable, and the experts should be asked to revise their preferences and make their comparisons more carefully until the consistency rate reaches an acceptable range. In the proposed approach, if the weights obtained through pairwise comparisons made by experts in two cases (comparison of the best criterion to the other criteria and comparison of the other criteria to the worst criterion) have the same values, the comparisons will be "completely consistent". It is obvious that it is difficult to make fully consistent pairwise comparisons, especially when there are a large number of decision criteria. Therefore, the closer the value obtained from Equation (6) is to zero, the more consistent the comparisons are. The proposed approach framework is shown in Figure 2.

$$\mathrm{CR} = \sum_j \left| \widetilde{w}'_j - \widetilde{w}''_j \right|^2 \tag{6}$$

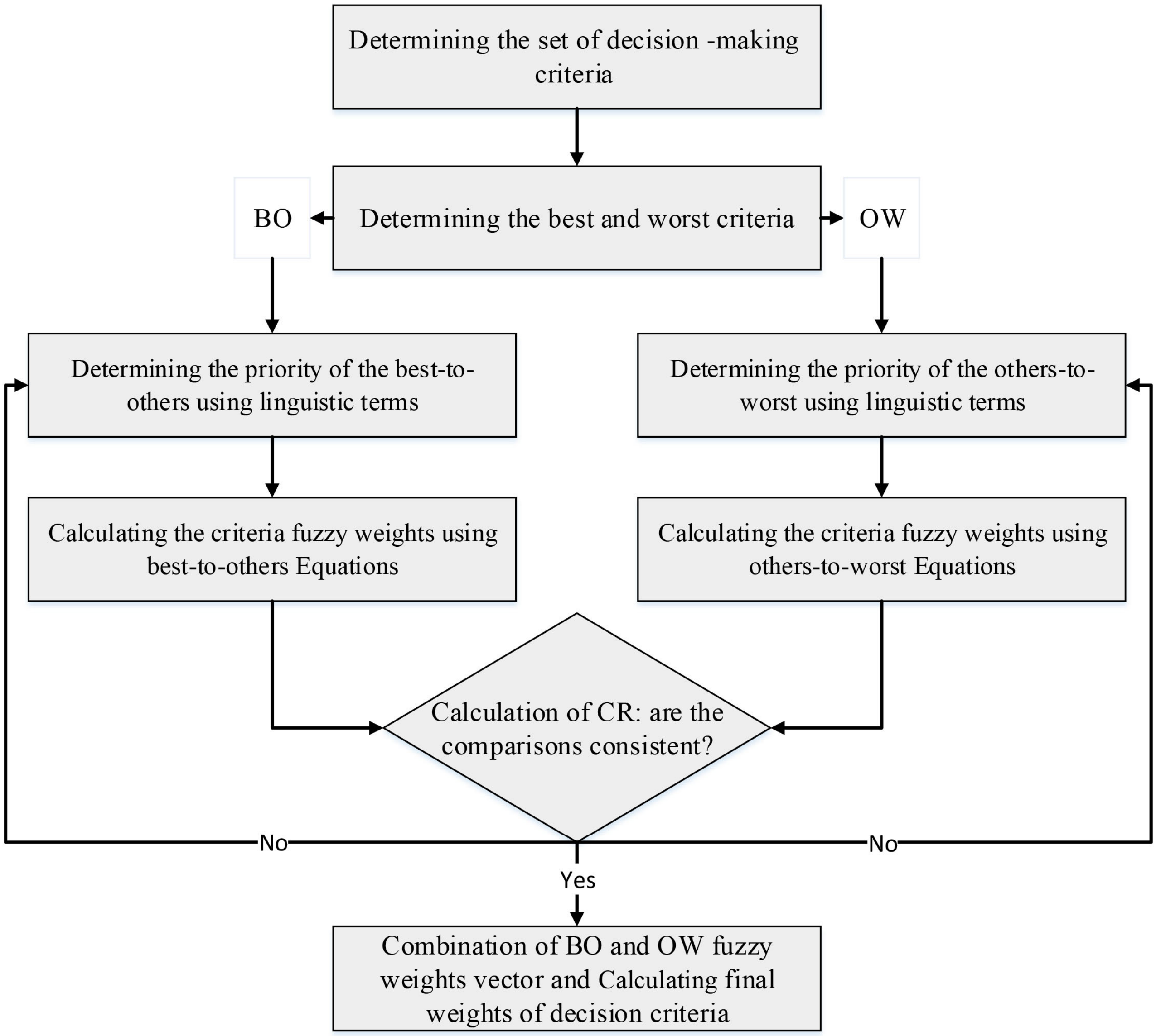

Figure 2. The F-SBWM framework.

Total deviation (TD) is calculated in MCDM methods to examine the acceptability of the results. TD is the Euclidean difference of the weight ratios of the decision criteria ($\frac{W_i}{W_j}$) and their priorities (a_{ij}) in the process of pairwise comparisons between the criteria. A lower value of TD means that the weights of the decision criteria obtained from the MCDM approach are more consistent with the decision makers' preferences, and this is what we seek in multi-criteria decision tools based on pairwise comparisons and decision maker knowledge. In this study, Equation (7) is used to calculate TD. In BWM, the final value of TD is divided by 2n, where n is the number of decision criteria [24]. To facilitate TD calculation, the fuzzy values can be de-fuzzified using appropriate methods.

$$TD = \sum_j \left(\widetilde{a}_{Bj} - \frac{\widetilde{w}_B}{\widetilde{w}_j} \right)^2 + \sum_j \left(\widetilde{a}_{jW} - \frac{\widetilde{w}_j}{\widetilde{w}_W} \right)^2 \tag{7}$$

4. Numerical Examples

In this section, two numerical examples are described to show the applicability of the proposed approach. The examples have a different number of criteria to more clearly show how the proposed approach works. The results of the proposed approach are compared with the results of the triangular fuzzy approach of BWM available in the literature [32]. Next, in Section 4.3, a real example is provided about choosing the right medical mask to prevent the transmission of the COVID-19. The weights and priorities of the criteria obtained using the proposed approach are compared with the results of the base research. These examples will help understand that the proposed approach provides results similar to those of the complex fuzzy models available in the literature, with only simple calculations and without the need for linear programming models, and hence it can be an effective simplified fuzzy method to address management challenges. Table 3 shows the linguistic terms and their corresponding values in the form of TFNs for numerical examples 1 and 2.

Table 3. Linguistic terms used for evaluating the criteria in examples 1 and 2 [69].

Linguistic Terms	Fuzzy Scales
Equally importance (EI)	(1,1,1)
Weakly important (WI)	(1,2,3)
Moderate importance (MI)	(2,3,4)
Moderate plus importance (MP)	(3,4,5)
Strong importance (SI)	(4,5,6)
Strong plus importance (SP)	(5,6,7)
Very strong importance (VS)	(6,7,8)
Extreme importance (EX)	(7,8,9)

4.1. Numerical Example 1

Suppose we are facing a decision problem in which there are four criteria. The decision maker seeks to determine the importance of each criterion to make appropriate decisions. In the first step, the criteria are specified precisely, and C_1 is chosen as the most important criterion and C_4 as the least important criterion. The priority of the best criterion over each of the other criteria (BO) and the priority of each criterion over the worst criterion (OW) are determined by experts and are provided in Table 4.

Table 4. The priorities of the criteria in numerical example 1.

BO & OW		C_1	C_2	C_3	C_4
BO	Best: C_1	EI	MP	SI	VS
OW	Worst: C_4	VS	MP	MI	EI

After obtaining the fuzzy vector of the priority of the best criterion over the other criteria, the weight of the first criterion is obtained using Equation (1). Then, by substituting the weight of the best criterion in Equation (2), weights of the other criteria are obtained. As can be seen from Equations (8) and (9), the weights of the decision criteria are obtained using the vector of the priority of the best criterion over the other criteria ($\widetilde{w}'_j$).

$$\sum_j \frac{1}{\widetilde{a}_j}\widetilde{w}'_1 = 1 \Rightarrow \widetilde{w}'_1 = \frac{1}{\sum_j \frac{1}{\widetilde{a}_{1j}}} = \frac{1}{\frac{1}{(1,1,1)} + \frac{1}{(3,4,5)} + \frac{1}{(4,5,6)} + \frac{1}{(6,7,8)}} = \frac{1}{(1.492,\ 1.593,\ 1.750)} = (0.571,\ 0.628,\ 0.670) \quad (8)$$

$$\begin{aligned}
\widetilde{w}'_1 - \widetilde{a}_{12}\widetilde{w}'_2 = 0 \Rightarrow \widetilde{a}_{12}\widetilde{w}'_2 = \widetilde{w}'_1 \Rightarrow \widetilde{w}'_2 = \frac{\widetilde{w}'_1}{\widetilde{a}_{12}} = \frac{(0.571,\ 0.628,\ 0.670)}{(3,4,5)} = (0.114,\ 0.157,\ 0.223) \\
\widetilde{w}'_1 - \widetilde{a}_{13}\widetilde{w}'_3 = 0 \Rightarrow \widetilde{a}_{13}\widetilde{w}'_3 = \widetilde{w}'_1 \Rightarrow \widetilde{w}'_3 = \frac{\widetilde{w}'_1}{\widetilde{a}_{13}} = \frac{(0.571,\ 0.628,\ 0.670)}{(4,5,6)} = (0.095,\ 0.126,\ 0.167) \\
\widetilde{w}'_1 - \widetilde{a}_{14}\widetilde{w}'_4 = 0 \Rightarrow \widetilde{a}_{14}\widetilde{w}'_4 = \widetilde{w}'_1 \Rightarrow \widetilde{w}'_4 = \frac{\widetilde{w}'_1}{\widetilde{a}_{14}} = \frac{(0.571,\ 0.628,\ 0.670)}{(6,7,8)} = (0.071,\ 0.090,\ 0.112)
\end{aligned} \quad (9)$$

Next, using the fuzzy vector of the priorities of all criteria over the worst criterion, the weight of the worst criterion are calculated according to Equation (3), and by substituting the weight of the worst criterion in Equation (4), the weights of the other criteria are also calculated. Equations (10) and (11) show how to calculate the fuzzy weight of the worst criterion and the other criteria, respectively.

$$\sum_j \widetilde{a}_{j4}\widetilde{w}''_4 = 1 \Rightarrow \widetilde{w}''_4 = \frac{1}{\sum_j \widetilde{a}_{j4}} = \frac{1}{(6,7,8)+(3,4,5)+(2,3,4)+(1,1,1)} = \frac{1}{(12,15,18)} = \left(\frac{1}{18}, \frac{1}{15}, \frac{1}{12}\right) = (0.055, 0.067, 0.083) \quad (10)$$

$$\begin{aligned}
\widetilde{w}''_1 - \widetilde{a}_{14}\widetilde{w}''_4 = 0 \Rightarrow \widetilde{w}''_1 = \widetilde{a}_{14}\widetilde{w}''_4 = (6,7,8) \times (0.055, 0.067, 0.083) = (0.330, 0.469, 0.664) \\
\widetilde{w}''_2 - \widetilde{a}_{24}\widetilde{w}''_4 = 0 \Rightarrow \widetilde{w}''_2 = \widetilde{a}_{24}\widetilde{w}''_4 = (3,4,5) \times (0.055, 0.067, 0.083) = (0.165, 0.268, 0.415) \\
\widetilde{w}''_3 - \widetilde{a}_{34}\widetilde{w}''_4 = 0 \Rightarrow \widetilde{w}''_3 = \widetilde{a}_{34}\widetilde{w}''_4 = (2,3,4) \times (0.055, 0.067, 0.083) = (0.110, 0.201, 0.332)
\end{aligned} \quad (11)$$

After obtaining the weights of all criteria and giving the ($\widetilde{w}'_j$) and ($\widetilde{w}''_j$), the final weights of the decision criteria are achieved. Equation (12) shows the final fuzzy weights of the decision criteria. Table 5 compares the results of the proposed approach to the results of fuzzy BWM developed by [32]. Using Equation (6), the value of the consistency rate in the numerical example 1 is calculated as 0.042. For defuzzification of the final weights, the relationship $w^*_j = \frac{L+4M+U}{6}$ or other defuzzification methods can be used. As the results show, the proposed approach is able to calculate the final weights of decision criteria in uncertain environments by using simple calculations and without the need to formulate mathematical programming models or use software packages. Figure 3 shows the comparison of final weights obtained from the proposed approach with FBWM [32] in numerical example 1.

$$\begin{aligned}
\widetilde{w}^*_1 = \frac{\widetilde{w}'_1 + \widetilde{w}''_1}{2} = \frac{(0.571,\ 0.628,\ 0.670) + (0.330, 0.469, 0.664)}{(2,\ 2,\ 2)} = (0.450, 0.548, 0.667) \\
\widetilde{w}^*_2 = \frac{\widetilde{w}'_2 + \widetilde{w}''_2}{2} = \frac{(0.114,\ 0.157,\ 0.223) + (0.165, 0.268, 0.415)}{(2,\ 2,\ 2)} = (0.139, 0.212, 0.319) \\
\widetilde{w}^*_3 = \frac{\widetilde{w}'_3 + \widetilde{w}''_3}{2} = \frac{(0.095,\ 0.126,\ 0.167) + (0.110, 0.201, 0.332)}{(2,\ 2,\ 2)} = (0.102, 0.163, 0.249) \\
\widetilde{w}^*_4 = \frac{\widetilde{w}'_4 + \widetilde{w}''_4}{2} = \frac{(0.071,\ 0.090,\ 0.112) + (0.055, 0.067, 0.083)}{(2,\ 2,\ 2)} = (0.063, 0.078, 0.097)
\end{aligned} \quad (12)$$

Table 5. The final fuzzy weights obtained using the proposed approach and comparison with the results of [32].

Criteria	Proposed Approach			FBWM [32]		
	Fuzzy Weights ($\tilde{w}_j^*$)	Crisp	Rank	Fuzzy Weights ($\tilde{w}_j^*$)	Crisp	Rank
C_1	(0.450, 0.548, 0.667)	0.551	1	(0.529,0.594,0.594)	0.583	1
C_2	(0.139, 0.212, 0.319)	0.218	2	(0.137,0.208,0.250)	0.203	2
C_3	(0.102, 0.163, 0.249)	0.167	3	(0.118,0.135,0.185)	0.140	3
C_4	(0.063, 0.078, 0.097)	0.079	4	(0.065,0.073,0.074)	0.072	4
TD	0.923			0.861		
CR	0.042			0.091		

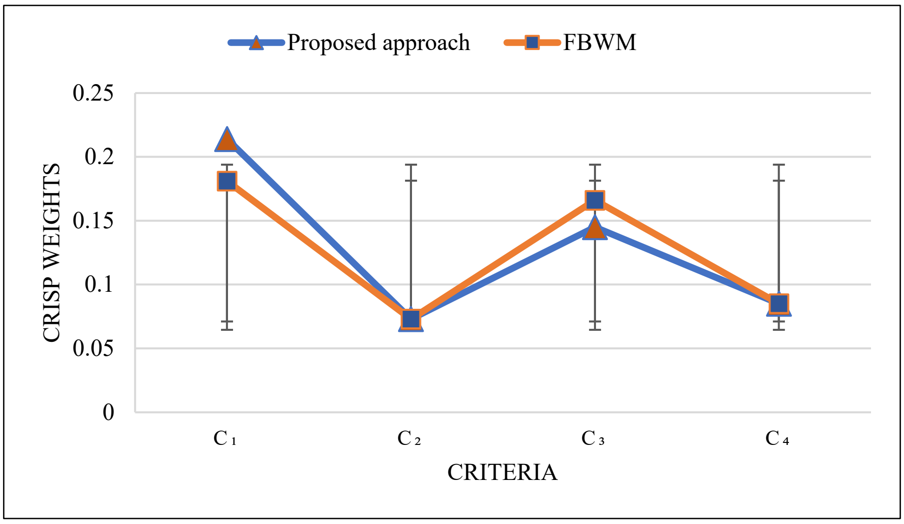

Figure 3. Comparison of final weights in numerical example 1.

4.2. Numerical Example 2

In this section, a decision-making problem regarding the selection of a location for construction of a warehouse for distribution of products is examined [70]. Choosing the right place to build warehouses is a strategic decision because it has a direct impact on the performance of the supply chain. Reducing costs and responding quickly to customers requires choosing the right location for warehouses [71]. Companies need to develop effective solutions for the problem of warehouse location selection because it has a significant impact on logistics and supply chain efficiency [72]. Identifying appropriate criteria for choosing the right place to build a warehouse is very important because of its direct impact on decision-making processes.

Although the criteria for choosing a warehouse location may be different for each type of organization with different types of manufactured products, the selected criteria should create an appropriate balance between costs and effectiveness of processes for the potential location. The criteria for choosing the right location to build a warehouse for proper distribution of products are as follows [70]:

- Distance to the main market (C_1)
- Distance to the busiest container port (C_2)
- Distance to highways (C_3)
- Land cost in the proposed location (C_4)
- Distance to cargo airport (C_5)

After determining the criteria for choosing the warehouse location, the decision maker chooses the best and worst criteria. In this step, the priority of the best criterion (the most important criterion) over the other criteria and the priorities of the other criteria over the worst criterion (the least important criterion) are determined using the linguistic terms provided in Table 3. The priorities of the criteria are provided in Table 6.

Table 6. The priorities of the criteria in numerical example 2.

BO & OW		C_1	C_2	C_3	C_4	C_5
BO	Best: C_1	EI	MP	SI	VS	EX
OW	Worst: C_4	EX	SI	MP	MI	EI

By substituting the priorities of the criteria in Equation (1), the fuzzy importance of the best criterion (C_1) is obtained. Equation (13) calculates the fuzzy weight of the best criterion. Then, by substituting the fuzzy weight of the best criterion in Equation (2), the fuzzy weights of the other criteria are obtained. Equation (14) calculates the fuzzy weights of the other decision criteria using the fuzzy weight of the best criterion. In this way, the vector of fuzzy weights resulting from reference comparisons of the best criterion to the other criteria are obtained.

$$\sum_j \frac{1}{\tilde{a}_j}\tilde{w}'_1 = 1 \Rightarrow \tilde{w}'_1 = \frac{1}{\sum_j \frac{1}{\tilde{a}_{1j}}} = \frac{1}{\frac{1}{(1,1,1)} + \frac{1}{(3,4,5)} + \frac{1}{(4,5,6)} + \frac{1}{(6,7,8)} + \frac{1}{(7,8,9)}} = \frac{1}{(1.603,\ 1.718,\ 1.893)} = (0.528,\ 0.582,\ 0.624) \quad (13)$$

$$\begin{aligned}
\tilde{w}'_1 - \tilde{a}_{12}\tilde{w}'_2 = 0 \Rightarrow \tilde{a}_{12}\tilde{w}'_2 = \tilde{w}'_1 \Rightarrow \tilde{w}'_2 = \frac{\tilde{w}'_1}{\tilde{a}_{12}} = \frac{(0.528,\ 0.582,\ 0.624)}{(3,4,5)} = (0.106, 0.145, 0.208)\\
\tilde{w}'_1 - \tilde{a}_{13}\tilde{w}'_3 = 0 \Rightarrow \tilde{a}_{13}\tilde{w}'_3 = \tilde{w}'_1 \Rightarrow \tilde{w}'_3 = \frac{\tilde{w}'_1}{\tilde{a}_{13}} = \frac{(0.528,\ 0.582,\ 0.624)}{(4,5,6)} = (0.088,\ 0.116,\ 0.156)\\
\tilde{w}'_1 - \tilde{a}_{14}\tilde{w}'_4 = 0 \Rightarrow \tilde{a}_{14}\tilde{w}'_4 = \tilde{w}'_1 \Rightarrow \tilde{w}'_4 = \frac{\tilde{w}'_1}{\tilde{a}_{14}} = \frac{(0.528,\ 0.582,\ 0.624)}{(6,7,8)} = (0.066,\ 0.083,\ 0.104)\\
\tilde{w}'_1 - \tilde{a}_{15}\tilde{w}'_5 = 0 \Rightarrow \tilde{a}_{15}\tilde{w}'_5 = \tilde{w}'_1 \Rightarrow \tilde{w}'_5 = \frac{\tilde{w}'_1}{\tilde{a}_{15}} = \frac{(0.528,\ 0.582,\ 0.624)}{(7,8,9)} = (0.059,\ 0.073,\ 0.089)
\end{aligned} \quad (14)$$

After obtaining the fuzzy weights vector using the reference comparisons of the best criterion to the other criteria ($\tilde{w}'_j$), the fuzzy weights vector using the reference comparisons of all criteria to the worst criterion ($\tilde{w}''_j$) is calculated. For this purpose, after calculating Equation (3) and putting the preferences of the decision maker in the equation, the fuzzy weight of the worst criterion is obtained. Then, by substituting the fuzzy weight of the worst criterion in Equation (4), the fuzzy weights of the other criteria is also calculated. The fuzzy weights of the worst and the other criteria are calculated using Equations (15) and (16), respectively.

$$\sum_j \tilde{a}_{j5}\tilde{w}''_5 = 1 \Rightarrow \tilde{w}''_5 = \frac{1}{\sum_j \tilde{a}_{j5}} = \frac{1}{(7,8,9) + (4,5,6) + (3,4,5) + (2,3,4) + (1,1,1)} = \frac{1}{(17,21,25)} = \left(\frac{1}{25}, \frac{1}{21}, \frac{1}{17}\right) = (0.040, 0.048, 0.059) \quad (15)$$

$$\begin{aligned}
\tilde{w}''_1 - \tilde{a}_{15}\tilde{w}''_5 = 0 \Rightarrow \tilde{w}''_1 = \tilde{a}_{15}\tilde{w}''_5 = (7,8,9) \times (0.040, 0.048, 0.059) = (0.280, 0.384, 0.531)\\
\tilde{w}''_2 - \tilde{a}_{25}\tilde{w}''_5 = 0 \Rightarrow \tilde{w}''_2 = \tilde{a}_{25}\tilde{w}''_5 = (4,5,6) \times (0.040, 0.048, 0.059) = (0.160, 0.240, 0.354)\\
\tilde{w}''_3 - \tilde{a}_{35}\tilde{w}''_5 = 0 \Rightarrow \tilde{w}''_3 = \tilde{a}_{35}\tilde{w}''_5 = (3,4,5) \times (0.040, 0.048, 0.059) = (0.120, 0.192, 0.295)\\
\tilde{w}''_4 - \tilde{a}_{45}\tilde{w}''_5 = 0 \Rightarrow \tilde{w}''_4 = \tilde{a}_{45}\tilde{w}''_5 = (2,3,4) \times (0.040, 0.048, 0.059) = (0.080, 0.144, 0.236)
\end{aligned} \quad (16)$$

Then, using Equation (5), the fuzzy weights obtained from the reference comparisons of the best criterion to the other and the other to the worst are combined to calculate final fuzzy weight ($\tilde{w}^*_j$) of each criterion. Equation (17) calculates the final fuzzy weights of the criteria. Table 7 provides the defuzzified values of the weights along with the results of the fuzzy BWM presented by [32] (for comparison). The consistency rate in numerical example 2 is calculated as 0.056 based on Equation (6). As the results show, the proposed approach has the ability to calculate the weights of the decision criteria in uncertain conditions, without the need to formulate mathematical programming models and only through simple numerical calculations. Figure 4 shows the comparison of final weights obtained from the proposed approach with FBWM [32] in numerical example 2.

$$\begin{aligned}
\widetilde{w}_1^* &= \frac{\widetilde{w}_1' + \widetilde{w}_1''}{2} = \frac{(0.528,\ 0.582,\ 0.624) + (0.280, 0.384, 0.531)}{(2,\ 2,\ 2)} = (0.404, 0.483, 0.577)\\
\widetilde{w}_2^* &= \frac{\widetilde{w}_2' + \widetilde{w}_2''}{2} = \frac{(0.106, 0.145, 0.208) + (0.160, 0.240, 0.354)}{(2,\ 2,\ 2)} = (0.160, 0.192, 0.281)\\
\widetilde{w}_3^* &= \frac{\widetilde{w}_3' + \widetilde{w}_3''}{2} = \frac{(0.088,\ 0.116,\ 0.156) + (0.120, 0.192, 0.295)}{(2,\ 2,\ 2)} = (0.104, 0.154, 0.225)\\
\widetilde{w}_4^* &= \frac{\widetilde{w}_4' + \widetilde{w}_4''}{2} = \frac{(0.066,\ 0.083,\ 0.104) + (0.080, 0.144, 0.236)}{(2,\ 2,\ 2)} = (0.073, 0.113, 0.170)\\
\widetilde{w}_5^* &= \frac{\widetilde{w}_5' + \widetilde{w}_5''}{2} = \frac{(0.059,\ 0.073,\ 0.089) + (0.040, 0.048, 0.059)}{(2,\ 2,\ 2)} = (0.049, 0.060, 0.074)
\end{aligned} \tag{17}$$

Table 7. Final fuzzy weights obtained from the proposed approach and comparison with the results of [32].

	Proposed Approach			FBWM [32]		
Criteria	**Fuzzy Weights ($\tilde{w}_j^*$)**	**Crisp**	**Rank**	**Fuzzy Weights ($\tilde{w}_j^*$)**	**Crisp**	**Rank**
C_1	(0.404, 0.483, 0.577)	0.485	1	(0.537,0.537,0.537)	0.537	1
C_2	(0.160, 0.192, 0.281)	0.201	2	(0.156,0.190,0.223)	0.190	2
C_3	(0.104, 0.154, 0.225)	0.157	3	(0.089,0.134,0.172)	0.133	3
C_4	(0.073, 0.113, 0.170)	0.116	4	(0.064,0.078,0.122)	0.083	4
C_5	(0.049, 0.060, 0.074)	0.060	5	(0.051,0.056,0.062)	0.056	5
TD	1.990			1.516		
CR	0.056			0.115		

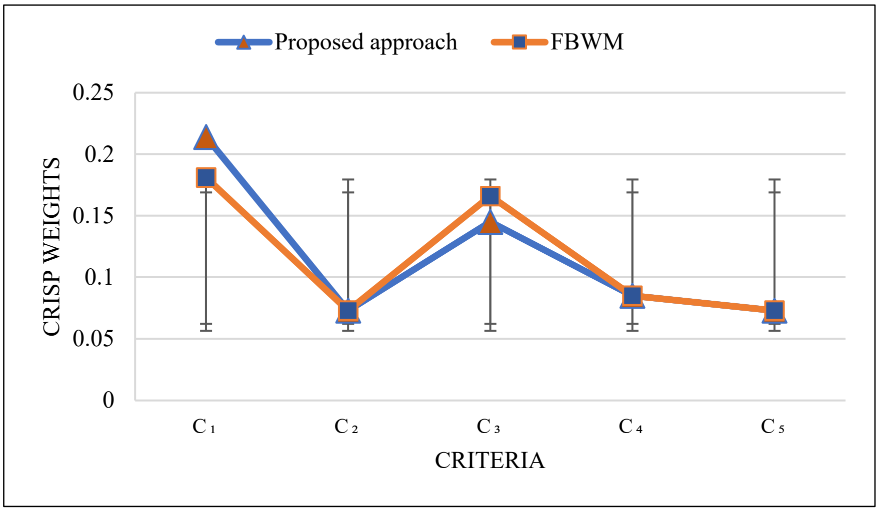

Figure 4. Comparison of final weights in numerical example 2.

4.3. Computational Experiment

In the previous sections, the performance of the proposed approach was discussed using hypothetical data, just to illustrate how to implement it. In this section, the proposed approach is tested and validated using real data for a case of choosing a suitable mask to prevent the transmission of the COVID-19 virus. The decision-making problem and its data (the set of criteria and the fuzzy preferences of experts regarding the criteria) are adapted from Kaya et al. [73]. Eight criteria were determined to choose the appropriate antivirus mask to prevent the transmission of the virus. The fuzzy weights of the decision criteria in the basic study were obtained using the triangular fuzzy BWM presented by Guo and Zhao [32]. Here we are going to examine eight decision criteria using the proposed approach and compare and analyze the results of prioritization and weighting.

With the emergence of the corona pandemic and the increasing number of people infected with this virus, the use of antivirus masks in addition to the other health protocols and vaccines has become popular as an important prevention tool. Using a suitable mask is one of the most reliable measures to stay safe from the corona virus suspended in the air [74]. In this way, what criteria should be considered to choose a suitable mask is very important. The criteria for choosing a suitable antivirus mask were identified as follows [73]:

- Proper covering of the face (C_1)
- Quality of the mask ingredients (C_2)
- Reusability (C_3)
- Breathing comfort while using the mask (C_4)
- Anti-allergenic materials in the mask (C_5)
- Ease of wearing and removing the mask from the face (C_6)
- Ability of the mask to filter viruses (C_7)
- Resistance to damage and tearing (C_8)

To determine the weights of the mask selection criteria, the decision maker expresses his preferences using linguistic terms and TFNs. These linguistic terms are provided in Table 8. C_7 and C_6 were selected as the most important and the least important criteria, respectively. Moreover, the fuzzy preferences of the decision maker regarding the criteria are provided in Table 9.

Table 8. Linguistic terms used for evaluating the mask selection criteria [73].

Linguistic Terms	Fuzzy Scales
Equally importance (EI)	(1,1,1)
Weakly important (WI)	(0.67,1,1.5)
Fairy important (FI)	(1.5,2,2.5)
Very important (VI)	(2.5,3,3.5)
Absolutely important (AI)	(3.5,4,4.5)

Table 9. The priorities of the criteria in the computational experiment.

BO & OW		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
BO	Best: C_7	WI	AI	FI	VI	AI	AI	EI	VI
OW	Worst: C_6	FI	WI	FI	WI	WI	EI	AI	WI

After determining the priorities of the criteria by the decision maker, the importance of the best criterion is calculated using Equation (1). Equation (18) shows how to calculate the importance of the best criterion in the best-to-others comparisons vector. Then, by substituting the weight of the best criterion in Equation (2), the weights of the other decision criteria in the best-to-others comparisons vector are obtained. Equation (19) shows how to calculate the weights of the other criteria using the weight of the best criterion. Here, the fuzzy weights ($\widetilde{w}'_j$) are obtained from the fuzzy reference comparisons of the best-to-others.

$$\sum_j \frac{1}{\tilde{a}_{7j}}\tilde{w}'_7 = 1 \Rightarrow \tilde{w}'_7 = \frac{1}{\sum_j \frac{1}{\tilde{a}_{7j}}}$$
$$= \frac{1}{\frac{1}{(0.67,1,1.5)}+\frac{1}{(3.5,4,4.5)}+\frac{1}{(1.5,2,2.5)}+\frac{1}{(2.5,3,3.5)}+\frac{1}{(3.5,4,4.5)}+\frac{1}{(3.5,4,4.5)}+\frac{1}{(1,1,1)}+\frac{1}{(2.5,3,3.5)}} \quad (18)$$
$$= \frac{1}{(3.305,3.917,4.816)} = (0.208, 0.255,\ 0.303)$$

$$\begin{aligned}
\tilde{w}'_7 - \tilde{a}_{71}\tilde{w}'_1 = 0 \Rightarrow \tilde{a}_{71}\tilde{w}'_1 = \tilde{w}'_7 \Rightarrow \tilde{w}'_1 = \frac{\tilde{w}'_7}{\tilde{a}_{71}} = \frac{(0.208,0.255,\ 0.303)}{(0.67,1,1.5)} = (0.138,0.255,0.452)\\
\tilde{w}'_7 - \tilde{a}_{72}\tilde{w}'_2 = 0 \Rightarrow \tilde{a}_{72}\tilde{w}'_2 = \tilde{w}'_7 \Rightarrow \tilde{w}'_2 = \frac{\tilde{w}'_7}{\tilde{a}_{72}} = \frac{(0.208,0.255,\ 0.303)}{(3.5,4,4.5)} = (0.046,\ 0.064,\ 0.086)\\
\tilde{w}'_7 - \tilde{a}_{73}\tilde{w}'_3 = 0 \Rightarrow \tilde{a}_{73}\tilde{w}'_3 = \tilde{w}'_7 \Rightarrow \tilde{w}'_3 = \frac{\tilde{w}'_7}{\tilde{a}_{73}} = \frac{(0.208,0.255,\ 0.303)}{(1.5,2,2.5)} = (0.083,\ 0.128,\ 0.201)\\
\tilde{w}'_7 - \tilde{a}_{74}\tilde{w}'_4 = 0 \Rightarrow \tilde{a}_{74}\tilde{w}'_4 = \tilde{w}'_7 \Rightarrow \tilde{w}'_4 = \frac{\tilde{w}'_7}{\tilde{a}_{74}} = \frac{(0.208,0.255,\ 0.303)}{(2.5,3,3.5)} = (0.059,\ 0.085,\ 0.121)\\
\tilde{w}'_7 - \tilde{a}_{75}\tilde{w}'_5 = 0 \Rightarrow \tilde{a}_{75}\tilde{w}'_5 = \tilde{w}'_7 \Rightarrow \tilde{w}'_5 = \frac{\tilde{w}'_7}{\tilde{a}_{75}} = \frac{(0.208,0.255,\ 0.303)}{(3.5,4,4.5)} = (0.046,\ 0.064,\ 0.086)\\
\tilde{w}'_7 - \tilde{a}_{76}\tilde{w}'_6 = 0 \Rightarrow \tilde{a}_{76}\tilde{w}'_6 = \tilde{w}'_7 \Rightarrow \tilde{w}'_6 = \frac{\tilde{w}'_7}{\tilde{a}_{76}} = \frac{(0.208,0.255,\ 0.303)}{(3.5,4,4.5)} = (0.046,\ 0.064,\ 0.086)\\
\tilde{w}'_7 - \tilde{a}_{78}\tilde{w}'_8 = 0 \Rightarrow \tilde{a}_{78}\tilde{w}'_8 = \tilde{w}'_7 \Rightarrow \tilde{w}'_8 = \frac{\tilde{w}'_7}{\tilde{a}_{78}} = \frac{(0.208,0.255,\ 0.303)}{(2.5,3,3.5)} = (0.059,\ 0.085,\ 0.121)
\end{aligned} \quad (19)$$

Then, we obtain the fuzzy weights ($\tilde{w}''_j$) of the decision criteria by performing the fuzzy reference comparisons of all criteria to the worst criterion. For this purpose, by using the decision maker's fuzzy preferences for all criteria over the worst criterion, Equation (20) is formed based on Equation (3) and by performing simple calculations, the importance of the worst criterion is calculated. By substituting the fuzzy weight of the worst criterion in Equation (4), the fuzzy weights of other criteria are calculated. Equation (21) shows how to calculate the fuzzy weights of other criteria in the others-to-worst comparisons.

$$\sum_j \tilde{a}_{j6}\tilde{w}''_6 = 1 \Rightarrow \tilde{w}''_6 = \frac{1}{\sum_j \tilde{a}_{j6}}$$
$$= \frac{1}{(1.5,2,2.5)+(0.67,1,1.5)+(1.5,2,2.5)+(0.67,1,1.5)+(0.67,1,1.5)+(1,1,1)+(3.5,4,4.5)+(0.67,1,1.5)} \quad (20)$$
$$= \frac{1}{(10.18,13,16.5)} = \left(\frac{1}{16.5}, \frac{1}{13}, \frac{1}{10.18}\right) = (0.061,0.077,0.098)$$

$$\begin{aligned}
\tilde{w}''_1 - \tilde{a}_{16}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_1 = \tilde{a}_{16}\tilde{w}''_6 = (1.5,2,2.5) \times (0.061,0.077,0.098) = (0.092,0.154,0.245)\\
\tilde{w}''_2 - \tilde{a}_{26}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_2 = \tilde{a}_{26}\tilde{w}''_6 = (0.67,1,1.5) \times (0.061,0.077,0.098) = (0.041,0.077,0.147)\\
\tilde{w}''_3 - \tilde{a}_{36}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_3 = \tilde{a}_{36}\tilde{w}''_6 = (1.5,2,2.5) \times (0.061,0.077,0.098) = (0.092,0.154,0.245)\\
\tilde{w}''_4 - \tilde{a}_{46}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_4 = \tilde{a}_{46}\tilde{w}''_6 = (0.67,1,1.5) \times (0.061,0.077,0.098) = (0.041,0.0.077,0.159)\\
\tilde{w}''_5 - \tilde{a}_{56}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_5 = \tilde{a}_{56}\tilde{w}''_6 = (0.67,1,1.5) \times (0.061,0.077,0.098) = (0.041,0.0.077,0.159)\\
\tilde{w}''_7 - \tilde{a}_{76}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_7 = \tilde{a}_{76}\tilde{w}''_6 = (3.5,4,4.5) \times (0.061,0.077,0.098) = (0.213,0.308,0.441)\\
\tilde{w}''_8 - \tilde{a}_{86}\tilde{w}''_6 = 0 \Rightarrow \tilde{w}''_8 = \tilde{a}_{86}\tilde{w}''_6 = (0.67,1,1.5) \times (0.061,0.077,0.098) = (0.041,0.0.077,0.159)
\end{aligned} \quad (21)$$

After obtaining ($\tilde{w}'_j$) and ($\tilde{w}''_j$), the final weights of the decision criteria ($\tilde{w}^*_j$) are calculated. Using Equation (5) and performing simple calculations, the final weight of the criteria are obtained. The final weights of the criteria in the example of choosing a suitable medical mask are obtained using Equation (22). In this example, eight decision criteria were determined, and the method of calculating the final weights was described. As seen, the proposed approach has the ability to calculate the fuzzy weights of the criteria using simple calculations and without the need to formulate mathematical programming models.

The flexibility of the proposed approach in using linguistic terms and different fuzzy scales is another advantage of the proposed approach.

$$\begin{aligned}
\widetilde{w}_1^* &= \frac{\widetilde{w}_1' + \widetilde{w}_1''}{2} = \frac{(0.138, 0.255, 0.452) + (0.092, 0.154, 0.245)}{(2,\ 2,\ 2)} = (0.115, 0.205, 0.348) \\
\widetilde{w}_2^* &= \frac{\widetilde{w}_2' + \widetilde{w}_2''}{2} = \frac{(0.046,\ 0.064,\ 0.086) + (0.041, 0.077, 0.147)}{(2,\ 2,\ 2)} = (0.044, 0.070, 0.117) \\
\widetilde{w}_3^* &= \frac{\widetilde{w}_3' + \widetilde{w}_3''}{2} = \frac{(0.083,\ 0.128,\ 0.201) + (0.092, 0.154, 0.245)}{(2,\ 2,\ 2)} = (0.087, 0.141, 0.223) \\
\widetilde{w}_4^* &= \frac{\widetilde{w}_4' + \widetilde{w}_4''}{2} = \frac{(0.059,\ 0.085,\ 0.121) + (0.041, 0.0.077, 0.159)}{(2,\ 2,\ 2)} = (0.050, 0.081, 0.140) \\
\widetilde{w}_5^* &= \frac{\widetilde{w}_5' + \widetilde{w}_5''}{2} = \frac{(0.046,\ 0.064,\ 0.086) + (0.041, 0.0.077, 0.159)}{(2,\ 2,\ 2)} = (0.044, 0.070, 0.123) \\
\widetilde{w}_6^* &= \frac{\widetilde{w}_6' + \widetilde{w}_6''}{2} = \frac{(0.046,\ 0.064,\ 0.086) + (0.061, 0.077, 0.098)}{(2,\ 2,\ 2)} = (0.054, 0.070, 0.092) \\
\widetilde{w}_7^* &= \frac{\widetilde{w}_7' + \widetilde{w}_7''}{2} = \frac{(0.208, 0.255,\ 0.303) + (0.213, 0.308, 0.441)}{(2,\ 2,\ 2)} = (0.211, 0.282, 0.372) \\
\widetilde{w}_8^* &= \frac{\widetilde{w}_8' + \widetilde{w}_8''}{2} = \frac{(0.059,\ 0.085,\ 0.121) + (0.041, 0.0.077, 0.159)}{(2,\ 2,\ 2)} = (0.050, 0.081, 0.140)
\end{aligned} \tag{22}$$

The consistency rate of pairwise comparisons in the above decision-making problem is equal to 0.017 (calculated using Equation (6)), which shows that the comparisons are consistent. The final fuzzy weights are provided in Table 10 and the results of prioritization of the decision criteria are compared with the results of [73]. As can be seen from the results, the proposed approach provides results similar to those of other complex fuzzy methods available in the literature. Even though the preferences determined by the decision maker for the criteria have values close to each other, we see that the ranks calculated by the proposed approach for the decision criteria are similar to the values obtained by other methods. This indicates the validity of the results and the accuracy of the proposed approach. Figure 5 shows the final weights of the decision criteria compared to the results of the previous study.

Table 10. Comparing the results of the proposed approach with FBWM [32] in the medical mask selection problem.

	Proposed Approach			FBWM [32]		
Criteria	**Fuzzy Weights ($\tilde{w}_j^*$)**	**Crisp**	**Rank**	**Fuzzy Weights ($\tilde{w}_j^*$)**	**Crisp**	**Rank**
C_1	(0.115, 0.205, 0.348)	0.214	2	(0.156, 0.179, 0.215)	0.181	2
C_2	(0.044,0.070,0.117)	0.073	5	(0.064, 0.073, 0.079)	0.073	5
C_3	(0.087,0.141,0.223)	0.145	3	(0.154, 0.167, 0.174)	0.166	3
C_4	(0.050,0.081,0.140)	0.085	4	(0.080, 0.082, 0.100)	0.085	4
C_5	(0.044,0.070,0.123)	0.073	5	(0.064, 0.073, 0.079)	0.073	5
C_6	(0.054, 0.070, 0.092)	0.072	6	(0.073, 0.073, 0.079)	0.072	6
C_7	(0.211, 0.282, 0.372)	0.284	1	(0.241, 0.259, 0.298)	0.262	1
C_8	(0.050,0.081,0.140)	0.085	4	(0.080, 0.082, 0.100)	0.085	4
TD	0.084			0.085		
CR	0.017			0.055		

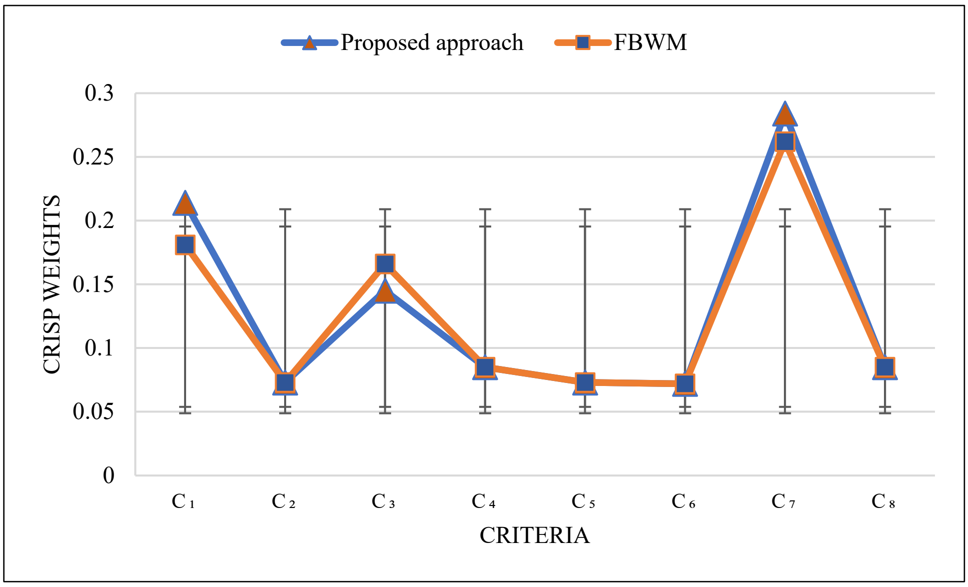

Figure 5. Comparing the final weights obtained from the proposed method with the results of FBWM [32] (for the medical mask selection problem).

5. Sensitivity Analysis

After prioritizing the decision criteria and obtaining their importance in the previous examples, sensitivity analysis was performed to evaluate the reliability and robustness of the results. In the development of MCDM approaches, it is essential to ensure the effectiveness of the results obtained from the proposed approach [75]. In the proposed approach, the fuzzy final weight of the criteria was determined by combining the weights obtained from two vectors of fuzzy pairwise comparisons (best-to-others and others-to-worst). Equation (23) shows how to combine the fuzzy weights ($\widetilde{w}'_j$ and $\widetilde{w}''_j$) obtained from the reference comparisons of the best-to-others and others-to-worst. The δ parameter is selected as the sensitivity analysis parameter, and using it, the weights obtained from both BO and OW fuzzy vectors are combined. If the value of the δ parameter is considered to be 0.5, it means that the importance of the weights obtained from both vectors of fuzzy reference comparisons is equal and the final weight of each criterion is equally affected by both vectors. If there is doubt over the decision maker filling out the initial questions of the questionnaire more accurately, the value of the δ parameter can be considered higher than 0.5, so that the final fuzzy weight of each criterion can be obtained with more attention to BO comparisons [25].

In Table 11, the final weights of the decision criteria are calculated for different values of the δ parameter (0≤ δ ≤1). As can be seen from the results, when δ is changed, the final weight of each criterion is also changed, but the prioritization of the decision criteria does not change, which confirms the reliability and robustness of the F-SBWM results.

$$w_j^* = \delta\widetilde{w}'_j + (1-\delta)\widetilde{w}''_j,\ \forall j \tag{23}$$

Table 11. Sensitivity analysis of weights for 0≤ δ ≤1.

Cases	Criteria	δ										
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Example 1	C_1	0.478	0.493	0.508	0.522	0.537	0.552	0.567	0.581	0.596	0.611	0.626
	C_2	0.275	0.264	0.252	0.241	0.230	0.218	0.207	0.195	0.184	0.172	0.161
	C_3	0.208	0.200	0.192	0.184	0.176	0.168	0.160	0.152	0.144	0.136	0.128
	C_4	0.068	0.070	0.072	0.075	0.077	0.079	0.081	0.084	0.086	0.088	0.091

Table 11. *Cont.*

Cases	Criteria	δ										
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Example 2	C_1	0.391	0.410	0.429	0.448	0.467	0.486	0.504	0.523	0.542	0.561	0.580
	C_2	0.246	0.236	0.226	0.217	0.207	0.197	0.188	0.178	0.168	0.159	0.149
	C_3	0.197	0.189	0.181	0.173	0.166	0.158	0.150	0.142	0.134	0.126	0.118
	C_4	0.149	0.142	0.136	0.129	0.123	0.116	0.110	0.103	0.097	0.090	0.084
	C_5	0.049	0.051	0.053	0.056	0.058	0.061	0.063	0.066	0.068	0.071	0.073
Computational Experiment	C_1	0.159	0.170	0.181	0.192	0.203	0.214	0.225	0.235	0.246	0.257	0.268
	C_2	0.083	0.081	0.079	0.077	0.075	0.074	0.072	0.070	0.068	0.066	0.065
	C_3	0.159	0.156	0.154	0.151	0.148	0.146	0.143	0.141	0.138	0.135	0.133
	C_4	0.085	0.085	0.086	0.087	0.087	0.088	0.089	0.090	0.090	0.091	0.092
	C_5	0.085	0.083	0.081	0.079	0.077	0.075	0.073	0.071	0.069	0.067	0.065
	C_6	0.078	0.077	0.075	0.074	0.073	0.071	0.070	0.069	0.067	0.066	0.065
	C_7	0.314	0.308	0.303	0.297	0.291	0.285	0.279	0.273	0.267	0.261	0.255
	C_8	0.085	0.085	0.085	0.085	0.085	0.086	0.086	0.086	0.086	0.086	0.087

6. Comparison of Methods

In this section, the results obtained from the proposed approach are compared with some existing fuzzy MCDM approaches [32,62]. These approaches have extended the BWM for decision making under uncertainty using TFNs, and since the proposed approach in this study has developed simplified BWM using TFNs, the results will be clearly comparable. The purpose of this comparison is the feasibility and effectiveness of the results obtained from the proposed approach. Table 12 shows the final weights and ranking of the decision criteria obtained from F-SBWM compared to the fuzzy approaches available in the literature. As can be seen from the results, the proposed approach provides the same results compared to the existing complex approaches without the need to formulate a mathematical programming model, complex calculations, and software packages. F-SBWM is a simplified and approximate approach that enables decision makers to solve complex decision-making problems under uncertainty by using only fuzzy operators and simple calculations.

Table 12. Comparison of results obtained from the proposed approach and existing methods.

Cases	Criteria	Method (i) [32]		Method (ii) [62]		Proposed Method	
		Crisp Weights	**Rank**	**Crisp Weights**	**Rank**	**Crisp Weights**	**Rank**
Example 1	C_1	0.583	1	0.595	1	0.551	1
	C_2	0.203	2	0.187	2	0.218	2
	C_3	0.140	3	0.145	3	0.167	3
	C_4	0.072	4	0.071	4	0.079	4
	TD	0.861	—	0.754	—	0.923	—
	CR	0.091	—	0.013	—	0.042	—
Example 2	C_1	0.537	1	0.538	1	0.485	1
	C_2	0.190	2	0.172	2	0.201	2
	C_3	0.133	3	0.134	3	0.157	3
	C_4	0.083	4	0.092	4	0.116	4
	C_5	0.056	5	0.063	5	0.060	5
	TD	1.516	—	1.442	—	1.990	—
	CR	0.115	—	0.012	—	0.056	—

Table 12. *Cont.*

Cases	Criteria	Method (i) [32]		Method (ii) [62]		Proposed Method	
		Crisp Weights	Rank	Crisp Weights	Rank	Crisp Weights	Rank
Computational Experiment	C_1	0.181	2	0.183	2	0.214	2
	C_2	0.073	5	0.072	5	0.073	5
	C_3	0.166	3	0.154	3	0.145	3
	C_4	0.085	4	0.098	4	0.085	4
	C_5	0.073	5	0.072	5	0.073	5
	C_6	0.072	6	0.072	5	0.072	6
	C_7	0.262	1	0.249	1	0.284	1
	C_8	0.085	4	0.098	4	0.085	4
	TD	0.085	—	0.079	—	0.084	—
	CR	0.055	—	0.009	—	0.017	—

In the computational experiment, where the problem of medical mask selection was investigated with real data, the values of TD and CR obtained in the proposed approach were 0.084 and 0.017, respectively, which are suitable values compared to the other approaches. Moreover, the final ranking, $C_7 > C_1 > C_3 > C_4 = C_8 > C_2 = C_5 > C_6$, shows the accuracy of the proposed approach in calculating weights and prioritizing criteria.

7. Discussion

In many studies, the BWM has been extended for dealing with uncertainties, and each study has tried to deal with ambiguities and the lack of information in the decision-making process. All the articles that have extended BWM to deal with uncertain conditions have complex calculations and mathematical programming models with a large number of variables and constraints [53,65]. This complexity creates significant challenges for decision makers when making decisions in various organizations [76]. On the other hand, many decision-making problems such as selection, sorting, and ranking can be solved by MCDM approaches, and decision makers need to use them [77]. In addition to spending a long time, complex computational approaches require highly specialized skills, and in many cases, decision makers may prefer to use simpler approaches to perform tasks [78,79]. However, the simplified approaches of MCDM methods are limited, especially in uncertain environments, and the existence of a simplified fuzzy method that helps decision makers with high accuracy and reliability can be useful [80].

The proposed approach is able to solve MCDM problems by only using simple algebraic operations and fuzzy operators, and the decision maker does not need to spend a long time, use a computer, have specialized skills, and perform complex calculations. As can be seen from the results of the numerical examples, F-SBWM can prioritize the decision criteria because it is less complex than other weighting methods of MCDM. The steps of the proposed approach are well described in the first and second numerical examples. In these examples, pairwise comparisons between criteria were made by the authors. In each example, for simplicity, the first criterion was chosen as the best criterion, and the last criterion was chosen as the worst criterion. In the first numerical example, there were four criteria. After performing reference comparisons and determining the priorities of the criteria, the final fuzzy weights of the decision criteria were obtained as follows:

$$\widetilde{W}_j^* = \{(0.450, 0.548, 0.667), (0.139, 0.212, 0.319), (0.102, 0.163, 0.249), (0.063, 0.078, 0.097)\}$$

After defuzzification of the obtained weights, the criteria were prioritized as $C_1 > C_2 > C_3 > C_4$. Since in the proposed approach, calculations are performed using TFNs, the results of the proposed approach were compared with the results of the triangular fuzzy BWM approach available in the literature [32]. The results show that the proposed approach, without formulating a mathematical programming model and without using software packages, provides results similar to other complex fuzzy methods. The consistency rate in the first example is 0.042; it is less than 10% and shows the consistency of the comparisons. To examine the final weights (obtained using the F-SBWM approach) and the preferences of the criteria in pairwise comparisons (determined by the decision maker), the TD index was also calculated. Its value is acceptable compared to the results of other approaches. The second numerical example was presented with a larger number of decision criteria to compare the proposed approach with previous studies. The results of the second numerical example indicate that the proposed approach is able to prioritize decision criteria with high accuracy. The consistency rate in the second numerical example was equal to 0.056. Examining the results of numerical examples shows that although the proposed approach is an approximate and simplified method, it has high validity and applicability for solving decision-making problems and prioritizing decision criteria.

A real-world decision-making problem was also analyzed using the proposed approach. In the computational experiment section, the issue of choosing a suitable medical mask to prevent the transmission of the COVID-19 virus suspended in the air was addressed. Eight criteria were identified for choosing the right mask, and the decision makers determined the importance of each criterion. The required data, including decision criteria, the best and worst criteria, priorities of the best criterion over the other criteria, and priorities of the other criteria over the worst criterion, were adapted from the study [73]. The purpose of the computational experiment was to demonstrate the applicability of the proposed approach in solving real decision-making problems. Using the steps of the proposed approach and the provided mathematical relations, the mask selection problem was solved. The final fuzzy and deterministic weights obtained from the proposed approach were compared with the results of the base study. In this example, which was solved using the proposed approach and real data, the consistency rate was 0.017 and the TD value was 0.084, which showed better results than the approach used in the original study, without using a mathematical programming model. Figure 6 shows the values of the consistency rate and TD obtained from the proposed approach compared to the FBWM [32] in the illustrated examples. Additionally, the use of linguistic terms and different triangular fuzzy scales in the proposed approach makes it more flexible in dealing with different forms of data.

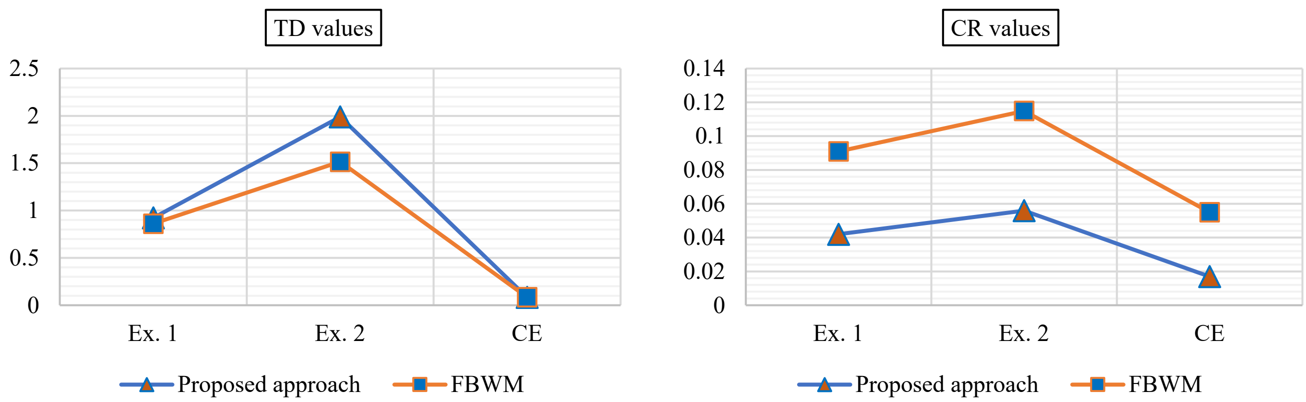

Figure 6. Values of the CR and TD in the illustrated examples.

8. Conclusions

The MCDM method as a practical and decision-supporting tool helps managers and decision makers of organizations to examine decision-making issues and make decisions with higher reliability in situations where there are several criteria and alternatives. The output of the decision-making process may directly or indirectly affect the organization's general policies, strategic goals, and productivity. In real conditions, complete and accurate information is not always available for the decision-making process, and due to the complexity of management issues and challenges in organizations, most of the time, decision makers are faced with a lack of information, ambiguities, and inaccurate data. Although MCDM methods are effective for decision makers in certain conditions in which accurate data are available, these methods are not effective under uncertain conditions and may lead to incorrect and imprecise decisions. However, the use of MCDM methods under uncertainty has its own challenges. The complexity of calculations, the need to use software packages, and the time-consuming nature of calculations are some of the challenges faced by decision makers when using decision-making methods under uncertain conditions.

In this study, the simplified best-worst MCDM method was developed using fuzzy logic theory and TFNs for uncertain conditions, and it is called the fuzzy extension of simplified best-worst method (F-SBWM). The F-SBWM, in addition to having the advantages of the basic BWM, has other advantages such as a reduced number of comparisons, higher consistency of comparisons, easier calculations, and no need for mathematical programming models and software packages. The proposed approach is able to take into account the uncertainties in the decision-making process and allow decision makers to make appropriate decisions based on the real conditions. In the proposed approach, first, the decision criteria are identified, and then the most important criterion and the least important criterion are selected by the decision maker. Then, using linguistic terms and their corresponding values in the form of TFNs, the decision maker determines the priority of the best criterion over the other criteria, and the vector of fuzzy weights resulting from the comparisons of the best criterion to the other criteria is obtained. Then, the decision maker determines the priorities of the other criteria over the worst criterion, and the vector of fuzzy weights resulting from the comparisons of all criteria to the worst criterion is obtained. By combining the weights obtained from both vectors, the final weight of each criterion is determined. In addition, the consistency rate is calculated to examine the consistency of paired comparisons made by the decision maker. To check the validity and applicability of the proposed approach, two numerical examples and a computational experiment with different number of criteria were described, and the results of the proposed approach were compared with the results of the triangular fuzzy approach available in the literature. The results showed that the proposed approach, in addition to simplifying calculations and efficiency in solving complex decision-making problems in uncertain environments, prioritizes the criteria just like the complex methods available in the literature. However, there have been some limitations in the development of the proposed fuzzy algorithm in this study. The selection of suitable linguistic terms and triangular fuzzy numbers corresponding to them to present the proposed approach as well as the selection of triangular fuzzy approaches available in the literature to compare the results obtained from the proposed approach and evaluate the robustness of the results are the most important limitations of this study.

There are suggestions for future research. The fuzzy algorithm developed in this study can be evaluated and used in various scenarios, such as supplier evaluation and selection, investment project prioritization, public transportation evaluation, performance evaluation, airlines evaluation, and blockchain platform selection. The development of the proposed approach to carry out the multi-criteria group decision-making process can also be one of the interesting aspects of future research. Comparing the results of the proposed approach with the results of other multi-criteria decision-making methods in uncertain environments can demonstrate the effectiveness of the proposed approach more clearly. Moreover, it can be useful to combine other multi-criteria decision-making methods with the proposed approach to simplify the process of weighting the criteria and prioritizing the alternatives. The feasibility of using different forms of data in the form of different linguistic terms in the proposed approach can also be a subject for future research.

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