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A NEW METHOD OF ASSESSMENT BASED ON FUZZY RANKING AND AGGREGATED WEIGHTS (AFRAW) FOR MCDM PROBLEMS UNDER TYPE-2 FUZZY ENVIRONMENT

Abstract. *Fuzzy multi-criteria decision-making (MCDM) methods and problems have increasingly been considered in the past years. Type-1 fuzzy sets are usually used by decision-makers (DMs) to express their evaluations in the process of decision-making. Interval type-2 fuzzy sets (IT2FSs), which are extensions of type-1 fuzzy sets, have more degrees of flexibility in modeling of uncertainty. In this research, a new ranking method to calculate the ranking values of interval type-2 fuzzy sets is proposed. A comparison is performed to show the efficiency of this ranking method. Using the proposed ranking method and the arithmetic operations of IT2FSs, a new method of* ***A****ssessment based on* ***F****uzzy* ***R****anking and* ***A****ggregated* ***W****eights (AFRAW)is developed for multi-criteria group decision-making. To obtain more realistic and practical weights for the criteria, the subjective weights expressed by DMs and objective weights calculated based on a deviation-based method are combined, and the aggregated weights are used in the proposed method. A numerical example related to assessment of suppliers in a supply chain and selecting the best one is used to illustrate the procedure of the proposed method. Moreover, a comparison and a sensitivity analysis are performed in this study. The results of these analyses show the validity and stability of the proposed method.*

Keywords: *MCDM, interval type-2 fuzzy sets, fuzzy ranking method, multi-criteria group decision-making, AFRAW.*

JELClassification: C02, C44, C61, C63, L6

1. Introduction

Multi-criteria decision-making (MCDM) has been one of the fastest growing problem areas during at least the last two decades. MCDM methods have been developed to support the decision-maker (DM) in their unique and personal decision process and to provide techniques for finding a compromise solution with respect to multiple criteria (Zavadskas *et al.*, 2009). MCDM methods provide mathematical methodology that incorporates the values of decision-makers and stakeholders as well as technical information to select the best solution for the problems (Chakraborty and Zavadskas, 2014). It allows for a more logical and scientifically defensible decision to be made, and has many applications in science and engineering fields such as reliability engineering, robotics, scheduling, manufacturing, etc. (Kumar and Gag, 2010; Keshavarz Ghorabaee *et al.*, 2015a, 2015c; Amiri *et al.*, 2014). Because of the characteristics of MCDM problems, decision-makers usually confront with many problems with vague and incomplete information (Cheng, 2013). Approaches which use the fuzzy set theory are appropriate when the modeling of human knowledge and human evaluations is needed in the decision-making process (Kahraman *et al*., 2013). Fuzzy set theory is recognized as an important theory in many problems and techniques. This theory, which was proposed by Zadeh (1965), has been studied extensively over the past 40 years.

Over the years there have been successful applications and implementations of fuzzy set theory in the field of multi-criteria decision-making. To deal with fuzziness in MCDM problems, the evaluations of decision-makers are usually described by type-1 fuzzy sets. Many researchers have studied fuzzy MCDM methods and problems, and applied type-1 fuzzy sets in their works. Sangaiah *et al.*(2015) developed a fuzzy approach by integrating the Decision-Making Trial and Evaluation Laboratory Model (DEMATEL) and the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) to evaluate partnership quality and team service climate aspects with respect to the global software development project outcomes. Keramati *et al.*(2013) proposed a fuzzy methodology based on the analytic hierarchy process (AHP) for evaluating the risk of customer relationship management (CRM) projects. Yeh *et al.*(204) presented a new hybrid multi-criteria decision-making based on fuzzy DEMATEL and fuzzy AHP to determine critical factors in new-product development. Peldschus and Zavadskas (2005) proposed a new multi-criteria decision-making method based on fuzzy sets and matrix games and applied it for evaluating and selecting water supply resources. Wadhwa *et al.*(2009) proposed a multi-criteria decision-making model based on fuzzy-set theory to determine a suitable reverse manufacturing option. The proposed model can help in designing effective and efficient flexible return policy with respect to various criteria. Lin *et al*. (2010) used the fuzzy analytic hierarchy process method as an analytical tool to determine a unique competitive marketing strategy for a small tourism venture. Nieto-Morote and Ruz-

Vila (2011) developed a fuzzy MCDM method based on the AHP method which considers both quantitative and qualitative criteria in the decision-making process. They applied the proposed method for evaluation of cooling, heating, and power production systems. Su (2011) developed a hybrid fuzzy multi-criteria group decision-making (MCGDM) method based on the VlsekriterijumskaOptimizacija I KompromisnoResenje (VIKOR) method and grey relational analysis (GRA), and applied it for some problems in reverse logistic management. Liou *et al.*(2011) introduced a new hybrid MCDM model based on the DEMATEL and analytic network process (ANP) methods for selection of an outsourcing provider. Kim and Chung (2013) developed a fuzzy VIKOR approach for assessing the vulnerability of the water supply to climate change and variability in South Korea. Roshandel *et al.*(2013) used a hierarchical fuzzy TOPSIS for evaluation of suppliers and selecting the best one in a detergent production industry. Tanselİç *et al*. (2013) developed a two-phase robot selection decision support system, which is named ROBSEL. In development of ROBSEL, an independent set of criteria and the fuzzy analytical hierarchy process are used to obtain the best alternative. Vinodh *et al.* (2013) developed a fuzzy MCDM approach based on the VIKOR method to evaluate and select the best concept in an agile environment. Rezaie *et al.*(2014) proposed a fuzzy MCDM method by integrating the VIKOR and AHP methods to evaluate performance of cement firms. Moghimi and Anvari (2014) proposed an integrated fuzzy MCDM approach and analysis based on the AHP and TOPSIS methods to evaluate the financial performance of Iranian cement companies. Mehlawat and Gupta 2015) presented a new fuzzy multi-criteria group decision-making method and applied it to determine the critical path in a project network.

Although type-1 fuzzy sets are efficient tools which have many applications in modeling of multi-criteria decision-making problems and extending methods to handle these problems, sometimes we confront with situations that more degrees of flexibility are needed to deal with MCDM problems. For example, finding out the exact membership function of a fuzzy set is possibly difficult for the decision-makers and/or analysts in the process of decision-making. Type-2 fuzzy set (T2FS) which was proposed by Zadeh (1975) can be used to handle this issue. T2FSs are the extension of type-1 fuzzy set, three-dimensional, and their membership function is represented by a fuzzy set on the interval [0, 1]. The membership function of T2FSs is delineated by both primary and secondary membership to provide more degrees of freedom and flexibility. Therefore, we can say that the accuracy of T2FSs in the modeling of uncertainty is more than type-1 fuzzy sets. In spite of this advantage, using type-2 fuzzy sets for solving problems requires a large amount of computations (Mendel *et al*., 2006). By considering some simplifying assumptions, interval type-2 fuzzy sets (IT2FSs) are introduced by researchers to deal with this difficulty (Mendel, 2009). The concept of IT2FSs is defined by an interval-valued membership function. Some basic definitions of IT2FSs were proposed by Mendel *et al.*(2006).

Recently, interval type-2 fuzzy sets have increasingly been considered by researchers in applications and extensions of multi-criteria decision-making methods. For example, Chen and Lee (2010) developed a new ranking method for interval type-2 fuzzy sets and used it in a new fuzzy MCDM method. Chen *et al*. (2012) proposed a new ranking method and a new multi-criteria decision-making method with interval type-2 fuzzy sets. Wang *et al.*(2012) introduced an MCGDM method in type-2 fuzzy environment, which can be used with incomplete information about criteria weights. Celik *et al*. (2013) proposed a novel interval type-2 fuzzy MCDM method based on TOPSIS and GRA to evaluate and improve customer satisfaction in Istanbul public transportation. Hu *et al*. (2013) developed a new ranking method based on the possibility degree for IT2FSs and applied it in multi-criteria decision-making process. Chen *et al.*(2013) introduced an extended QUALIFLEX (QUALItativeFLEXible) method for handling multi-criteria decision-making problems in the context of the interval type-2 fuzzy sets. Abdullah and Najib (2014) developed a fuzzy multi-criteria decision-making method based on the AHP method and IT2FSs and used it for evaluation of work safety. Celik *et al.*(2014) proposed an interval type-2 fuzzy MCDM method to identify and evaluate critical success factors for humanitarian relief logistics management. Kahraman *et al.*2014) introduced a new fuzzy ranking method and applied it for developing an AHP method with interval type-2 fuzzy sets. Keshavarz Ghorabaee *et al.*(2014) presented a new fuzzy ranking method and extended COPRAS (ComplexProportionalASsessment) method in the context of IT2FSs to evaluate suppliers in a supply chain. Wang *et al*. (2015) developed a new likelihood-based QUALIFLEX method with interval type-2 fuzzy sets for multi-criteria decision-making. Dymova *et al*. (2015) used alpha cuts to extend the TOPSIS method for multi-criteria decision-making with interval type-2 fuzzy sets. Chen (2015a) developed an interval type-2 fuzzy PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) method using a likelihood-based outranking comparison approach. Keshavarz Ghorabaee (2015) presented a multi-criteria decision-making method based on the VIKOR method and IT2FSs for evaluating and selecting industrial robots. Kiliç and Kaya (2015) developed a multi-criteria decision-making approach based on the type-2 fuzzy AHP and type-2 fuzzy TOPSIS methods to evaluate investment projects. Chen (2015b) proposed a new likelihood-based interval type-2 fuzzy MCDM method using the concepts of likelihood-based performance indices, likelihood-based comprehensive evaluation values, and signed distance-based evaluation values. Qin and Liu (2015) presented a new method to handle multi-criteria group decision-making problems based on a combined ranking value under interval type-2 fuzzy environment. Keshavarz Ghorabaee *et al.*(2015b) developed a multi-criteria decision-making approach for project selection based on the VIKOR method with interval type-2 fuzzy sets.

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In this research, a new ranking method is proposed for calculating ranking values of interval type-2 fuzzy sets. A special kind of interval type-2 fuzzy sets, called trapezoidal IT2FSs, is used in this method. Although some useful ranking methods have been developed by researchers to handle IT2FSs in MCDM problems, most of them are computationally complicated when we confront with the practical decision-making situations. However, the proposed method in this study has relatively less computational complexity that makes it more suitable for dealing with MCDM problems. To show the efficiency of the proposed fuzzy ranking method, a comparison with some existing ranking methods is performed. Using the proposed ranking method,a new method of assessment based on fuzzy ranking and aggregated weights (AFRAW) is developedfor multi-criteria group decision-making problems in the interval type-2 fuzzy environment. To obtain more realistic weights for the criteria, the subjective and objective weights of criteria are combined in the decision-making process. The subjective weights are expressed by decision-makers, and a deviation-based method is used to calculate the objective weights of criteria. Unlike many developed methods which transform the non-beneficial (cost) criteria to beneficial criteria in their process, the proposed method keeps the characteristics of non-beneficial criteria in the decision-making process. The validity of the proposed method is demonstrated by comparing the results with some interval type-2 fuzzy MCDM methods. Also, a sensitivity analysis with different criteria weights is performed to represent the stability of the proposed method. It can be seen that the results of the proposed method are relatively consistent with the other methods, and the proposed method has good stability when the weights of criteria are changed.

The paper is organized as follows. Section 2 briefly introduces some basic concepts and arithmetic operations of IT2FSs. In Section 3, a new ranking method is presented for calculating ranking values of IT2FSs. The proposed ranking method is compared with some existing methods in this section. In Section 4, a new method is proposed for multi-criteria group decision-making with IT2FSs. Section 5 shows the procedure of using the proposed MCGDM method based on an illustrative example. A comparison and a sensitivity analysis are also presented in this section to show the validity and stability of the results. The conclusions are discussed in Section 6.

2.Preliminaries

Type-2 fuzzy sets (T2FSs) are one of the main extensions of the type-1 fuzzy sets. T2FSs are represented by primary and secondary membership values. These types of fuzzy sets could be very useful in many fields of sciences, especially decision-making theory. In this section, the basic concepts and arithmetic operations of this type of fuzzy sets are defined.

Definition 1. The following equation can be used to describe a T2FS ($\tilde{\tilde{A}}$) by a type-2 membership function (Mendel *et al*., 2006):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \quad (1)$$

In the above equation, X represents the domain of $\tilde{\tilde{A}}$,$J_X \subseteq [0,1]$ and $\mu_{\tilde{\tilde{A}}}$denote the primary membership function and secondary membership function of $\tilde{\tilde{A}}$, respectively, and $\int \int$ symbolizes the union over all admissible x and u.

Definition 2. If all values of $\mu_{\tilde{\tilde{A}}}(x,u)$ is equal to 1 in a T2FS $\tilde{\tilde{A}}$, this fuzzy set is called interval type-2 fuzzy set (IT2FS). An interval type-2 fuzzy set $\tilde{\tilde{A}}$ could be described by the following equation (Mendel *et al*., 2006):

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_X} 1/(x,u) \quad (2)$$

where $J_X \subseteq [0,1]$.

Definition 3. Uncertain bounded region of the primary membership function, which is the union of all primary memberships, is called footprint of uncertainty (FOU). Upper membership function (UMF) and lower membership function (LMF), which are type-1 fuzzy sets, are used to describe FOU (Mendel *et al*., 2006). If the UMF and the LMF are both trapezoidal fuzzy sets, an IT2FS is called trapezoidal interval type-2 fuzzy set (TIT2FS). A TIT2FS ($\tilde{\tilde{A}}$) can be expressed as follows (Keshavarz Ghorabaee, 2015):

$$\tilde{\tilde{A}} = \left(\tilde{A}^T : T \in \{U, L\}\right) = (a_i^T; H_1\left(\tilde{A}^T\right), H_2\left(\tilde{A}^T\right): T \in \{U, L\}, i = 1,2,3,4) \quad (3)$$

In the above equation, $\tilde{A}^U$ shows the UMF and $\tilde{A}^L$ represents the LMFof $\tilde{\tilde{A}}$. Moreover, $H_j\left(\tilde{A}^U\right) \in [0,1]$ $(j = 1,2)$ denotes the membership values of a_{j+1}^U element and $H_j\left(\tilde{A}^L\right) \in [0,1]$ $(j = 1,2)$ denotes the membership value of the a_{j+1}^L element of $\tilde{\tilde{A}}$. Fig. 1 represents an example of a TIT2FS.

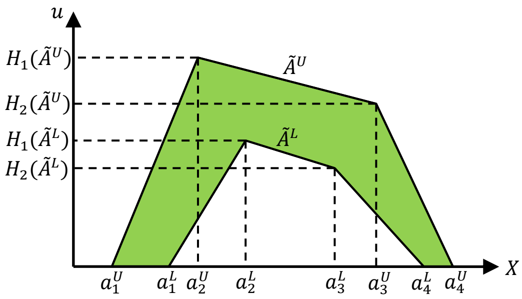

Figure 1. An example of a TIT2FS

Suppose that $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are two TIT2FSs as follows:

$$\tilde{\tilde{A}} = \left(\tilde{A}^T: T \in \{U, L\}\right) = (a_i^T; H_1\left(\tilde{A}^T\right), H_2\left(\tilde{A}^T\right): T \in \{U, L\}, i = 1,2,3,4)$$

$$\tilde{\tilde{B}} = \left(\tilde{B}^T: T \in \{U, L\}\right) = (b_i^T; H_1\left(\tilde{B}^T\right), H_2\left(\tilde{B}^T\right): T \in \{U, L\}, i = 1,2,3,4)$$

Definition 4. The addition operation is defined as follows (Chen and Lee, 2010):

$$\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = (a_i^T + b_i^T; \min(H_1\left(\tilde{A}^T\right), H_1\left(\tilde{B}^T\right)), \min(H_2\left(\tilde{A}^T\right), H_2\left(\tilde{B}^T\right)): T \in \{U, L\}, i = 1,2,3,4) \quad (4)$$

Definition 5. The following equation is used to subtract two TIT2FSs (Chen and Lee, 2010):

$$\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = (a_i^T - b_{5-i}^T;\ \min(H_1\left(\tilde{A}^T\right), H_1\left(\tilde{B}^T\right)), \min(H_2\left(\tilde{A}^T\right), H_2\left(\tilde{B}^T\right)): T \in \{U, L\}, i = 1,2,3,4) \quad (5)$$

Definition 6. The following equation is used to add a crisp number d to a TIT2FS(Chen and Lee, 2010):

$$\tilde{\tilde{A}} + d = \left(a_i^T + d; H_1\left(\tilde{A}^T\right), H_2\left(\tilde{A}^T\right): T \in \{U, L\}, i = 1,2,3,4\right) \quad (6)$$

Definition 7. The following equations is used to multiply two TIT2FSs (Keshavarz Ghorabaee*et al*., 2014):

$$\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = (X_i^T;\ \min(H_1\left(\tilde{A}^T\right), H_1\left(\tilde{B}^T\right)), \min(H_2\left(\tilde{A}^T\right), H_2\left(\tilde{B}^T\right)): T \in \{U, L\}, i = 1,2,3,4) \quad (7)$$

where

$$X_i^T = \begin{cases} \min\left(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T\right) & if\ i = 1,2 \\ \max\left(a_i^T b_i^T, a_i^T b_{5-i}^T, a_{5-i}^T b_i^T, a_{5-i}^T b_{5-i}^T\right) & if\ i = 3,4 \end{cases} \tag{8}$$

and $T \in \{U, L\}$.

Definition 8. The following equation is used for multiplication of a TIT2FS by a crisp number k(Keshavarz Ghorabaee*et al*., 2014):

$$k.\tilde{\tilde{A}} = \begin{cases} \left(k.a_i^T; H_1\left(\tilde{A}^T\right), H_2\left(\tilde{A}^T\right): T \in \{U, L\}, i = 1,2,3,4\right) & if\ k \geq 0 \\ \left(k.a_{5-i}^T; H_1\left(\tilde{A}^T\right), H_2\left(\tilde{A}^T\right): T \in \{U, L\}, i = 1,2,3,4\right) & if\ k \leq 0 \end{cases} \tag{9}$$

Definition 9. Definition 8 with $k = 1/l$ and $l \neq 0$ can be used for defining division of a TIT2FS by a crisp number l (Keshavarz Ghorabaee*et al*., 2014).

Definition 10. The defuzzified value of a TIT2FS is defined as follows (Keshavarz Ghorabaee*et al*., 2015b):

$$\kappa\left(\tilde{\tilde{A}}\right) = \frac{1}{2}\left(\sum_{T \in \{U,L\}} \frac{a_1^T + \left(1 + H_1\left(\tilde{A}^T\right)\right) a_2^T + \left(1 + H_2\left(\tilde{A}^T\right)\right) a_3^T + a_4^T}{4 + H_1\left(\tilde{A}^T\right) + H_2\left(\tilde{A}^T\right)}\right) \tag{10}$$

3. Ranking the TIT2FSs based on a new method

In this section, a new method is presented to obtain the ranking value of TIT2FSs. The method is designed based on the weighted distance between the elements of TIT2FSs. The membership values of elements are used to calculate weighted distance between them. The dominance degree of TIT2FSs over each other, which is defined in this section, is obtained using these distances. Some definitions are presented for illustrating this ranking method. Suppose that $\tilde{\tilde{A}}_s$ and $\tilde{\tilde{A}}_t$ be two TIT2FSs as shown in Fig. 2:

$$\tilde{\tilde{A}}_s = \left(\tilde{A}_s^T : T \in \{U, L\}\right) = \left(a_{si}^T; H_1\left(\tilde{A}_s^T\right), H_2\left(\tilde{A}_s^T\right): T \in \{U, L\}, i = 1,2,3,4\right)$$
$$\tilde{\tilde{A}}_t = \left(\tilde{A}_t^T : T \in \{U, L\}\right) = \left(a_{ti}^T; H_1\left(\tilde{A}_t^T\right), H_2\left(\tilde{A}_t^T\right): T \in \{U, L\}, i = 1,2,3,4\right)$$

Definition 11. The dominance degree of $\tilde{\tilde{A}}_s$ over $\tilde{\tilde{A}}_t$ is defined as follows:

$$\mathfrak{D}\left(\tilde{\tilde{A}}_s > \tilde{\tilde{A}}_t\right) = \frac{\sum_{T \in \{U,L\}}[\omega(D_1^T) + 3\omega(D_2^T) + 3\omega(D_3^T) + \omega(D_4^T)]}{8\sum_{T \in \{U,L\}}[\max(a_{s4}^T, a_{t4}^T) - \min(a_{s1}^T, a_{t1}^T)]} \tag{11}$$

where

$$D_i^T = \begin{cases} a_{si}^T.H_1\left(\tilde{A}_s^T\right) - a_{ti}^T.H_1\left(\tilde{A}_t^T\right) & i = 1,2 \\ a_{si}^T.H_2\left(\tilde{A}_s^T\right) - a_{ti}^T.H_2\left(\tilde{A}_t^T\right) & i = 3,4 \end{cases} \tag{12}$$

and

$$\omega(x) = max(0, x) \tag{13}$$

The values of D_i^T represent the weighted distances between the elements of $\tilde{\tilde{A}}_s$and $\tilde{\tilde{A}}_t$. Normal values of weighted distances are obtained when all membership values $(H_1(\tilde{A}_s^T)$,$H_2(\tilde{A}_s^T)$,$H_1(\tilde{A}_t^T)$ and $H_2(\tilde{A}_t^T))$ are equal to 1. These values of weighted distances are symbolized by nD_i^T and depicted in Fig. 2.

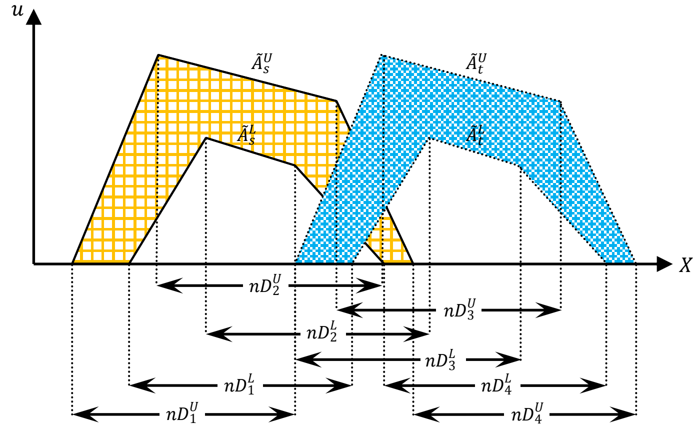

Figure 2. Two TIT2FSs and the normal values of weighted distances

If we need to compare n TIT2FSs, the following dominance degree matrix ($\mathfrak{D}_m$) can be used:

$$\mathfrak{D}_m = \begin{bmatrix} \mathfrak{D}\left(\tilde{\tilde{A}}_1 > \tilde{\tilde{A}}_1\right) & \mathfrak{D}\left(\tilde{\tilde{A}}_1 > \tilde{\tilde{A}}_2\right) & \cdots & \mathfrak{D}\left(\tilde{\tilde{A}}_1 > \tilde{\tilde{A}}_n\right) \\ \mathfrak{D}\left(\tilde{\tilde{A}}_2 > \tilde{\tilde{A}}_1\right) & \mathfrak{D}\left(\tilde{\tilde{A}}_2 > \tilde{\tilde{A}}_2\right) & \cdots & \mathfrak{D}\left(\tilde{\tilde{A}}_2 > \tilde{\tilde{A}}_n\right) \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{D}\left(\tilde{\tilde{A}}_n > \tilde{\tilde{A}}_1\right) & \mathfrak{D}\left(\tilde{\tilde{A}}_n > \tilde{\tilde{A}}_2\right) & \cdots & \mathfrak{D}\left(\tilde{\tilde{A}}_n > \tilde{\tilde{A}}_n\right) \end{bmatrix} \tag{14}$$

It should be noted that the dominance degree has two main properties as follows:

- $0 \leq \mathfrak{D}\left(\tilde{\tilde{A}}_s > \tilde{\tilde{A}}_t\right) \leq 1$
- $\mathfrak{D}\left(\tilde{\tilde{A}}_s > \tilde{\tilde{A}}_s\right) = 0$

Definition 12. Suppose that we have n trapezoidal interval type-2 fuzzy sets which represented as $\tilde{\tilde{A}}_i$ $(i = 1,2,\ldots,n)$. By calculating the elements of the dominance degree matrix based on the previous definitions, the ranking values (R_{value}) of each TIT2FS can be obtained by the following formula (Xu, 2001):

$$R_{value}\left(\tilde{\tilde{A}}_i\right) = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n}\mathfrak{D}\left(\tilde{\tilde{A}}_i > \tilde{\tilde{A}}_j\right) + \frac{n}{2} - 1\right) \tag{15}$$

where $1 \le i \le n$.

Table 1. Thirteen sets of fuzzy sets given by Bortolan and Degani (1985).

Sets of fuzzy sets		$\tilde{A}_i^T$ and $T \in \{U, L\}$					
		a_{1i}^T	a_{2i}^T	a_{3i}^T	a_{4i}^T	$H_1(\tilde{A}_i^T)$	$H_2(\tilde{A}_i^T)$
Set 1	$\tilde{\tilde{A}}_1$	0.35	0.4	0.4	1	1	1
	$\tilde{\tilde{A}}_2$	0.15	0.7	0.7	0.8	1	1
Set 2	$\tilde{\tilde{A}}_1$	0	0.1	0.5	1	1	1
	$\tilde{\tilde{A}}_2$	0.5	0.6	0.6	0.7	1	1
Set 3	$\tilde{\tilde{A}}_1$	0	0.1	0.5	1	1	1
	$\tilde{\tilde{A}}_2$	0.6	0.7	0.7	0.8	1	1
Set 4	$\tilde{\tilde{A}}_1$	0.4	0.9	0.9	1	1	1
	$\tilde{\tilde{A}}_2$	0.4	0.7	0.7	1	1	1
	$\tilde{\tilde{A}}_3$	0.4	0.5	0.5	1	1	1
Set 5	$\tilde{\tilde{A}}_1$	0.5	0.7	0.7	0.9	1	1
	$\tilde{\tilde{A}}_2$	0.3	0.7	0.7	0.9	1	1
	$\tilde{\tilde{A}}_3$	0.3	0.4	0.7	0.9	1	1
Set 6	$\tilde{\tilde{A}}_1$	0.3	0.5	0.8	0.9	1	1
	$\tilde{\tilde{A}}_2$	0.3	0.5	0.5	0.9	1	1
	$\tilde{\tilde{A}}_3$	0.3	0.5	0.5	0.7	1	1
Set 7	$\tilde{\tilde{A}}_1$	0.2	0.5	0.5	0.8	1	1
	$\tilde{\tilde{A}}_2$	0.4	0.5	0.5	0.6	1	1
Set 8	$\tilde{\tilde{A}}_1$	0	0.4	0.6	0.8	1	1
	$\tilde{\tilde{A}}_2$	0.2	0.5	0.5	0.9	1	1
	$\tilde{\tilde{A}}_3$	0.2	0.6	0.7	0.8	1	1
Set 9	$\tilde{\tilde{A}}_1$	0	0.2	0.2	0.4	1	1
	$\tilde{\tilde{A}}_2$	0.6	0.8	0.8	1	0.8	0.8
Set 10	$\tilde{\tilde{A}}_1$	0.4	0.6	0.6	0.8	1	1
	$\tilde{\tilde{A}}_2$	0.8	0.9	0.9	1	0.2	0.2
Set 11	$\tilde{\tilde{A}}_1$	0	0.2	0.2	0.4	0.2	0.2
	$\tilde{\tilde{A}}_2$	0.6	0.8	0.8	1	1	1

Set 12	$\tilde{\tilde{A}}_1$	0.2	0.6	0.6	1	1	1
	$\tilde{\tilde{A}}_2$	0.2	0.6	0.6	1	0.2	0.2
Set 13	$\tilde{\tilde{A}}_1$	0.6	1	1	1	1	1
	$\tilde{\tilde{A}}_2$	0.8	1	1	1	0.2	0.2

To compare the proposed ranking method with some existing methods, thirteen fuzzy sets provided by Bortolan and Degani (1985) are used. These fuzzy sets, which are shown in Table 1, are used in many studies for comparing ranking results. The methods proposed by Lee and Li (1988), Baas and Kwakernaak(1977), Chang *et al*. (2006), Chen and Lee (2010), Keshavarz Ghorabaee *et al.*(2014) and Hu *et al.*(2013) are considered for the comparison. The results obtained by each method are represented in Table 2. With respect to Table 2, some points are stated to compare the proposed method with these selected methods.

- According toSet 1 inTable 2, the same ranking order is obtained from the methods of Baas and Kwakernaak (1977), Chang *et al.*(2006), Lee and Li (1988) (in Proportional mode), Hu *et al.*(2013) and the proposed method.
- As can be seen in Table 2, the ranking result of the proposed method inSet 2, Set 3, Set 4, Set 9 and Set 11 is completely consistent with the results of the other methods in the comparison.
- As shown in Table 2, according to Set 5, Set 6 and Set 8, the same results of the methods of Chen and Lee (2010), Lee and Li (1988), Chang *et al.*(2006), Keshavarz Ghorabaee *et al.*(2014) and Hu *et al.*(2013) are obtained by the proposed method. However, the method of Baas and Kwakernaak (1977) cannot make a distinction between the ranking values of fuzzy sets.
- According toSet 7 in Table 2, except the method of Chang *et al.*(2006) in $\alpha = 0.1$ and $\beta = 0.9$, the other methods in comparison cannot get an order of fuzzy sets. The proposed method is also unable to obtain an order in this set.
- As can be seen in Table 2, the ranking results of Set 10 obtained by the methods of Chang *et al.* (2006) in$\alpha = 0.5$ and $\beta = 0.5$ and Hu *et al.*(2013) are consistent with the result of the proposed method. However, the other methods get different results in this set.
- According toSet 12 and Set 13 in Table 2, it can be seenthat the ranking results of the proposed method and the methods of Chang *et al.*(2006), Chen and Lee (2010), Keshavarz Ghorabaee *et al.*(2014) and Hu *et al.*(2013) are the same.

Table 2.A comparison of the ranking results with different methods

Sets of fuzzy sets		Lee and Li, 1988		Baas and Kwakernaak,1977	Chang *et al.*,2006		Chen and Lee,2010	Keshavarz Ghorabaee *et al.*, 2014	Hu *et al.*,2013	The proposed method
		Uniform	Proportional		α=0.1, β=0.9	α=0.5, β=0.5				
Set 1	$\tilde{\tilde{A}}_1$	0.58	0.54	0.84	0.417	0.519	0.52	1.000	0.423	0.029
	$\tilde{\tilde{A}}_2$	0.55	0.59	1	0.462	0.544	0.48	0.988	0.576	0.132
Set 2	$\tilde{\tilde{A}}_1$	0.41	0.38	0.82	0.158	0.45	0.4	0.927	0.25	0.019
	$\tilde{\tilde{A}}_2$	0.6	0.60	1	0.554	0.55	0.6	0.988	0.75	0.144
Set 3	$\tilde{\tilde{A}}_1$	0.41	0.38	0.66	0.158	0.45	0.36	0.897	0.375	0.013
	$\tilde{\tilde{A}}_2$	0.70	0.70	1	0.644	0.6	0.64	0.988	0.625	0.188
Set 4	$\tilde{\tilde{A}}_1$	0.77	0.80	1	0.878	0.65	0.39	0.583	0.431	0.208
	$\tilde{\tilde{A}}_2$	0.70	0.70	0.74	0.788	0.6	0.33	0.577	0.292	0.125
	$\tilde{\tilde{A}}_3$	0.63	0.60	0.6	0.698	0.55	0.28	0.564	0.277	0.083
Set 5	$\tilde{\tilde{A}}_1$	0.70	0.70	1	0.752	0.6	0.4	0.579	0.487	0.128
	$\tilde{\tilde{A}}_2$	0.63	0.65	1	0.743	0.575	0.32	0.572	0.333	0.115
	$\tilde{\tilde{A}}_3$	0.58	0.57	1	0.73	0.538	0.28	0.564	0.18	0.083
Set 6	$\tilde{\tilde{A}}_1$	0.62	0.63	1	0.775	0.563	0.39	0.583	0.487	0.153
	$\tilde{\tilde{A}}_2$	0.57	0.55	1	0.653	0.525	0.34	0.572	0.333	0.090
	$\tilde{\tilde{A}}_3$	0.50	0.50	1	0.572	0.5	0.27	0.556	0.18	0.083
Set 7	$\tilde{\tilde{A}}_1$	0.50	0.50	1	0.608	0.5	0.5	1	0.5	0.021
	$\tilde{\tilde{A}}_2$	0.50	0.50	1	0.536	0.5	0.5	1	0.5	0.021
Set 8	$\tilde{\tilde{A}}_1$	0.44	0.46	1	0.635	0.475	0.28	0.555	0.294	0.090
	$\tilde{\tilde{A}}_2$	0.53	0.53	0.88	0.649	0.513	0.35	0.575	0.337	0.100
	$\tilde{\tilde{A}}_3$	0.56	0.58	1	0.694	0.538	0.37	0.583	0.369	0.139
Set 9	$\tilde{\tilde{A}}_1$	0.20	0.20	0	0.158	0.35	0.28	0.76	0	0.000
	$\tilde{\tilde{A}}_2$	0.80	0.80	0.8	0.688	0.6	0.72	0.989	1	0.220
Set 10	$\tilde{\tilde{A}}_1$	0.60	0.60	0	0.518	0.55	0.49	0.92	0.59	0.350
	$\tilde{\tilde{A}}_2$	0.90	0.90	0.2	0.784	0.5	0.51	0.933	0.41	0.000
Set 11	$\tilde{\tilde{A}}_1$	0.20	0.20	0	0.118	0.15	0.25	0.693	0	0.000
	$\tilde{\tilde{A}}_2$	0.80	0.80	0.2	0.698	0.65	0.75	1	1	0.380
Set 12	$\tilde{\tilde{A}}_1$	0.60	0.60	0.2	0.446	0.55	0.63	1	0.75	0.300
	$\tilde{\tilde{A}}_2$	0.60	0.60	0.2	0.406	0.35	0.37	0.933	0.25	0.000
Set 13	$\tilde{\tilde{A}}_1$	0.87	0.90	0.2	0.932	0.7	0.63	0.985	0.82	0.944
	$\tilde{\tilde{A}}_2$	0.95	0.95	0.2	0.901	0.525	0.37	0.933	0.18	0.000

4. A new method of assessment based on fuzzy ranking and aggregated weights (AFRAW)

The conflictive expression of DMs about their preferences is one of the important issues in the process of decision-making. This issue is usually due to different backgrounds, different level of knowledge and different expertise of DMs. To handle this challenge, group decision-making could be used as an effective way. In a group decision-making process, the assessments and evaluations of all decision-makers are used and this could lead to a more precise decision. Sometimes the decision-makers faced with an uncertain environment for making a decision. Fuzzy sets and linguistic terms are efficient tools for DMs to express their preferences. In this section, anew method of assessment based on fuzzy ranking and aggregated weights (AFRAW)is proposed for multi-criteria group decision-making with interval type-2 fuzzy sets. In an uncertain environment, interval type-2 fuzzy sets enable decision-makers to express their preferences with more degrees of flexibility.

DMs usually express the weights of criteria in a subjective manner. This subjective evaluation of criteria weights from different DMs can lead to different weights for one criterion. To obtain more realistic weights for criteria of the problem, a procedure is designed for combining the subjective weights expressed by DMs and objective weights calculated based on a deviation-based method. Using the combination of subjective and objective weights of criteria can help us to reduce the sensitivity of the decision-making process to changing the weights by DMs. The framework for using the proposed method is represented in Fig. 3.

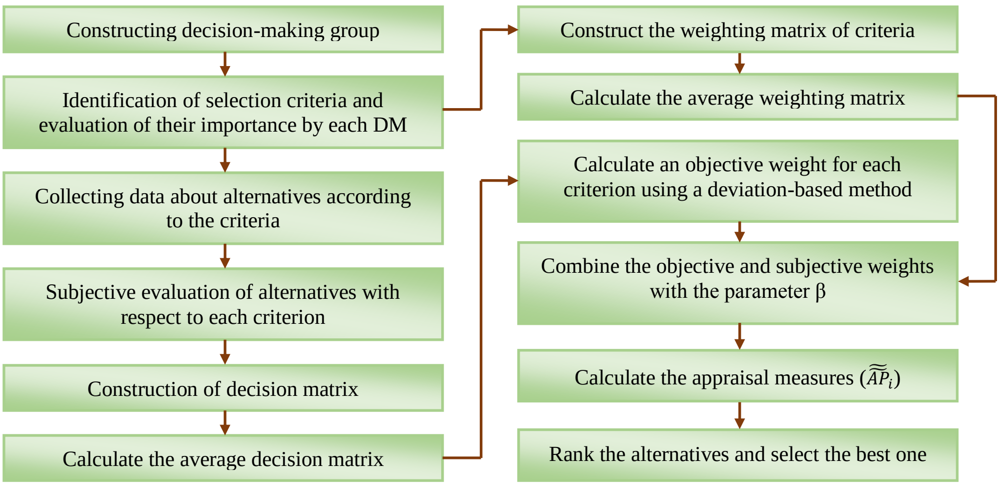

Figure 3. The framework for using the proposed method

Although this research only uses subjective evaluations for alternatives, the proposed method can be used in the situations with both subjective and objective evaluations. The basic concepts and the ranking method, which presented in the previous sections, are used to develop the AFRAW method with TIT2FSs.In this section, the proposed MCGDM method is introduced in detail to handle multi-criteria group decision-making problems. Suppose that we have a set of n alternatives $(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n)$, a set of m criteria $(\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_m)$ and k decision-makers $(\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_k)$. The proposed method is presented as follows.

Step 1. Construct the decision matrix X_p of the pth decision-maker, shown as follows:

$$X_p = \left[\tilde{\tilde{X}}_{ijp}\right]_{n\times m} = \begin{bmatrix} \tilde{\tilde{X}}_{11p} & \tilde{\tilde{X}}_{12p} & \cdots & \tilde{\tilde{X}}_{1mp} \\ \tilde{\tilde{X}}_{21p} & \tilde{\tilde{X}}_{22p} & \cdots & \tilde{\tilde{X}}_{2mp} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{X}}_{n1p} & \tilde{\tilde{X}}_{n2p} & \cdots & \tilde{\tilde{X}}_{nmp} \end{bmatrix} \tag{16}$$

where$\tilde{\tilde{X}}_{ijp}$ denotes the performance value of alternative $\mathcal{A}_i$ on the criterion $\mathcal{C}_j$ assigned by the pth decision-maker, $1 \le i \le n$, $1 \le j \le m$, $1 \le p \le k$.

Step 2. Construct the average decision matrix $\bar{X}$, shown as follows:

$$\tilde{\tilde{X}}_{ij} = \left(\left(\tilde{\tilde{X}}_{ij1} \oplus \tilde{\tilde{X}}_{ij2} \oplus \ldots \oplus \tilde{\tilde{X}}_{ijk}\right)/k\right) \tag{17}$$

$$\bar{X} = \left[\tilde{\tilde{X}}_{ij}\right]_{n\times m} \tag{18}$$

where$\tilde{\tilde{X}}_{ij}$ denotes the average performance value of alternative $\mathcal{A}_i$ on the criterion $\mathcal{C}_j$, $1 \le i \le n$, $1 \le j \le m$.

Step 3. Calculate the average performance value of each criterion as follows:

$$\tilde{\tilde{X}}_j^a = \left(\left(\tilde{\tilde{X}}_{1j} \oplus \tilde{\tilde{X}}_{2j} \oplus \ldots \oplus \tilde{\tilde{X}}_{nj}\right)/n\right) \tag{19}$$

$$\bar{\bar{X}} = \left[\tilde{\tilde{X}}_j^a\right]_{1\times m} \tag{20}$$

Step 4. Calculate an objective weight (w_j^o) for each criterion using a deviation-based method as follows:

$$s_j = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\kappa\left(\tilde{\tilde{X}}_{ij} \ominus \tilde{\tilde{X}}_j^a\right)\right]^2} \tag{21}$$

$$w_j^o = \frac{s_j}{\sum_{i=1}^{n} s_j} \tag{22}$$

wheres_j and w_j^odenote the deviation measure and the objective weight related to jth criterion, respectively.

Step 5. Construct the subjective weighting matrix (W_p^s) of the pth decision-maker, shown as follows:

$$W_p^s = \left[\tilde{\tilde{w}}_{jp}^s\right]_{m\times 1} = \begin{bmatrix} \tilde{\tilde{w}}_{1p}^s \\ \tilde{\tilde{w}}_{2p}^s \\ \vdots \\ \tilde{\tilde{w}}_{mp}^s \end{bmatrix} \tag{23}$$

where$\tilde{\tilde{w}}_{jp}^s$ denotes the subjective weight of the criterion C_j assigned by the pth decision-maker, $1 \le j \le m$, $1 \le p \le k$.

Step 6. Calculate the average subjective weight ($\tilde{\tilde{w}}_j^s$) for each criterion, shown as follows:

$$\tilde{\tilde{w}}_j^s = \left(\left(\tilde{\tilde{w}}_{j1}^s \oplus \tilde{\tilde{w}}_{j2}^s \oplus \ldots \oplus \tilde{\tilde{w}}_{jk}^s\right)/k\right) \tag{24}$$

Step 7. Combine the subjective and objective weights of each criterion and compute the aggregated weight of criteria ($\tilde{\tilde{w}}_j$), shown as follows:

$$\tilde{\tilde{w}}_j = \beta\tilde{\tilde{w}}_j^s + (1-\beta)w_j^o \tag{25}$$

whereβ is the aggregating coefficient which could be changed in the range of 0 to 1.

Step 8. Calculate the appraisal measure of each alternative as follows:

$$\widetilde{\widetilde{AP}}_i = \left(\sum_{j\in B}\frac{\tilde{\tilde{w}}_j \otimes \tilde{\tilde{X}}_{ij}}{\kappa\left(\tilde{\tilde{X}}_j^a\right)}\right) \ominus \left(\sum_{j\in N}\frac{\tilde{\tilde{w}}_j \otimes \tilde{\tilde{X}}_{ij}}{\kappa\left(\tilde{\tilde{X}}_j^a\right)}\right) \tag{26}$$

where B and N denote the sets of beneficial and non-beneficial criteria, respectively.

Step 9. Rank the alternatives with respect to decreasing ranking values of $\widetilde{\widetilde{AP}}_i$ ($R_{value}\left(\widetilde{\widetilde{AP}}_i\right)$).

5. Illustrative example

In this section, a numerical example is used to represent the procedure of the proposed multi-criteria group decision-making method. The example is related to assessment of suppliers in a supply chain and selecting the best one. Suppose that a company wants to select a supplier from some alternatives. Seven alternatives

($\mathcal{A}_1$to $\mathcal{A}_7$) remain for further assessment after initial screening. A group of three decision-makers ($\mathcal{D}_1$,$\mathcal{D}_2$and $\mathcal{D}_3$) is formed from the members of company's board of directors by the chief executive officer of the company. After a survey, five criteria ($\mathcal{C}_1$ to $\mathcal{C}_5$) are defined by this group of decision-makers to appraise the alternatives. These criteria and their definitions are represented as follows:

- **Defect rate** ($\mathcal{C}_1$): Supplier defect rate measures the percentage of materials or products received from suppliers that do not meet required quality or compliance specifications.
- **Cost** ($\mathcal{C}_2$): This criterion is related to estimated costs of selecting a supplier in a supply chain.
- **Delivery reliability** ($\mathcal{C}_3$): This criterion measures the supplier's ability to complete processes as promised.
- **Responsiveness** ($\mathcal{C}_4$): This criterion can be defined as the ability to react purposefully and within an appropriate time-scale to customer demand or changes in the marketplace, to bring about or maintain a competitive advantage.
- **Flexibility** ($\mathcal{C}_5$): Supplier flexibility is defined as the extent to which the supplier is willing and capable of making changes to accommodate the customer's changing needs.

The defect rate ($\mathcal{C}_1$) and cost ($\mathcal{C}_2$) are non-beneficial criteria, and the other criteria ($\mathcal{C}_3$to $\mathcal{C}_5$) are beneficial. Decision-makers use the linguistic terms shown in Table 3 and the data collected from experts to appraise the importance of the criteria and assess the performance values of alternatives with respect to each criterion. The performance values of the seven alternatives given by the decision-makers under the various criteria are presented in Table 4 and the subjective weights of the criteria determined by these decision-makers are shown in Table 5.

Table 3. Linguistic terms and their corresponding interval type-2 fuzzy sets

Linguistic terms	Interval type-2 fuzzy sets
Very low (VL)	[(0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9)]
Low (L)	[(0,0.1,0.15,0.3;1,1),(0.05,0.1,0.15,0.2;0.9,0.9)]
Medium low(ML)	[(0.1,0.3,0.35,0.5;1,1),(0.2,0.3,0.35,0.4;0.9,0.9)]
Medium (M)	[(0.3,0.5,0.55,0.7;1,1),(0.4,0.5,0.55,0.6;0.9,0.9)]
Medium high (MH)	[(0.5,0.7,0.75,0.9;1,1),(0.6,0.7,0.75,0.8;0.9,0.9)]
High (H)	[(0.7,0.85,0.9,1;1,1),(0.8,0.85,0.9,0.95;0.9,0.9)]
Very high (VH)	[(0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9)]

Table 4. Performance values of alternatives with respect to different criteria and decision-makers

DMs	Alternatives	Criteria				
		$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$
$\mathcal{D}_1$	$\mathcal{A}_1$	L	ML	VH	M	MH
	$\mathcal{A}_2$	L	VL	VH	H	VH
	$\mathcal{A}_3$	H	MH	M	MH	ML
	$\mathcal{A}_4$	MH	VH	MH	L	VL
	$\mathcal{A}_5$	M	VH	M	ML	MH
	$\mathcal{A}_6$	VH	M	L	MH	VH
	$\mathcal{A}_7$	MH	M	VL	VH	H
$\mathcal{D}_2$	$\mathcal{A}_1$	VL	L	H	MH	M
	$\mathcal{A}_2$	ML	VL	VH	H	VH
	$\mathcal{A}_3$	MH	M	MH	MH	M
	$\mathcal{A}_4$	MH	MH	H	ML	ML
	$\mathcal{A}_5$	M	H	M	M	MH
	$\mathcal{A}_6$	H	ML	ML	H	H
	$\mathcal{A}_7$	MH	M	L	H	MH
$\mathcal{D}_3$	$\mathcal{A}_1$	VL	M	H	MH	H
	$\mathcal{A}_2$	VL	VL	VH	H	VH
	$\mathcal{A}_3$	M	MH	M	M	M
	$\mathcal{A}_4$	M	VH	M	VL	L
	$\mathcal{A}_5$	ML	H	MH	ML	H
	$\mathcal{A}_6$	MH	MH	ML	MH	VH
	$\mathcal{A}_7$	M	M	ML	MH	MH

Table 5. Weights of the criteria evaluated by the decision-makers

Criteria	Decision-makers		
	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$
$\mathcal{C}_1$	VH	VH	H
$\mathcal{C}_2$	MH	MH	M
$\mathcal{C}_3$	VH	H	VH
$\mathcal{C}_4$	H	MH	MH
$\mathcal{C}_5$	H	H	MH

The process of using the proposed MCGDM method is presented as follows.

Step 1. The decision matrices X_1, X_2 and X_3 of the seven alternatives with respect to the five criteria of the problem are constructed based on Table 4 and Eq.(16):

$$X_1 = \begin{bmatrix} \mathrm{L} & \mathrm{ML} & \mathrm{VH} & \mathrm{M} & \mathrm{MH} \\ \mathrm{L} & \mathrm{VL} & \mathrm{VH} & \mathrm{H} & \mathrm{VH} \\ \mathrm{H} & \mathrm{MH} & \mathrm{M} & \mathrm{MH} & \mathrm{ML} \\ \mathrm{MH} & \mathrm{VH} & \mathrm{MH} & \mathrm{L} & \mathrm{VL} \\ \mathrm{M} & \mathrm{VH} & \mathrm{M} & \mathrm{ML} & \mathrm{MH} \\ \mathrm{VH} & \mathrm{M} & \mathrm{L} & \mathrm{MH} & \mathrm{VH} \\ \mathrm{MH} & \mathrm{M} & \mathrm{VL} & \mathrm{VH} & \mathrm{H} \end{bmatrix},$$

$$X_2 = \begin{bmatrix} \mathrm{VL} & \mathrm{L} & \mathrm{H} & \mathrm{MH} & \mathrm{M} \\ \mathrm{ML} & \mathrm{VL} & \mathrm{VH} & \mathrm{H} & \mathrm{VH} \\ \mathrm{MH} & \mathrm{M} & \mathrm{MH} & \mathrm{MH} & \mathrm{M} \\ \mathrm{MH} & \mathrm{MH} & \mathrm{H} & \mathrm{ML} & \mathrm{ML} \\ \mathrm{M} & \mathrm{H} & \mathrm{M} & \mathrm{M} & \mathrm{MH} \\ \mathrm{H} & \mathrm{ML} & \mathrm{ML} & \mathrm{H} & \mathrm{H} \\ \mathrm{MH} & \mathrm{M} & \mathrm{L} & \mathrm{H} & \mathrm{MH} \end{bmatrix},$$

$$X_3 = \begin{bmatrix} \mathrm{VL} & \mathrm{M} & \mathrm{H} & \mathrm{MH} & \mathrm{H} \\ \mathrm{VL} & \mathrm{VL} & \mathrm{VH} & \mathrm{H} & \mathrm{VH} \\ \mathrm{M} & \mathrm{MH} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ \mathrm{M} & \mathrm{VH} & \mathrm{M} & \mathrm{VL} & \mathrm{L} \\ \mathrm{ML} & \mathrm{H} & \mathrm{MH} & \mathrm{ML} & \mathrm{H} \\ \mathrm{MH} & \mathrm{MH} & \mathrm{ML} & \mathrm{MH} & \mathrm{VH} \\ \mathrm{M} & \mathrm{M} & \mathrm{ML} & \mathrm{MH} & \mathrm{MH} \end{bmatrix}.$$

Step 2. The average decision matrix $\bar{X}$ can be calculated based on the results of Step 1, Table 3 and Eqs. (17) and (18), shown as follows:

$$\bar{X} = \begin{bmatrix} \tilde{\tilde{X}}_{11} & \tilde{\tilde{X}}_{12} & \tilde{\tilde{X}}_{13} & \tilde{\tilde{X}}_{14} & \tilde{\tilde{X}}_{15} \\ \tilde{\tilde{X}}_{21} & \tilde{\tilde{X}}_{22} & \tilde{\tilde{X}}_{23} & \tilde{\tilde{X}}_{24} & \tilde{\tilde{X}}_{25} \\ \tilde{\tilde{X}}_{31} & \tilde{\tilde{X}}_{32} & \tilde{\tilde{X}}_{33} & \tilde{\tilde{X}}_{34} & \tilde{\tilde{X}}_{35} \\ \tilde{\tilde{X}}_{41} & \tilde{\tilde{X}}_{42} & \tilde{\tilde{X}}_{43} & \tilde{\tilde{X}}_{44} & \tilde{\tilde{X}}_{45} \\ \tilde{\tilde{X}}_{51} & \tilde{\tilde{X}}_{52} & \tilde{\tilde{X}}_{53} & \tilde{\tilde{X}}_{54} & \tilde{\tilde{X}}_{55} \\ \tilde{\tilde{X}}_{61} & \tilde{\tilde{X}}_{62} & \tilde{\tilde{X}}_{63} & \tilde{\tilde{X}}_{64} & \tilde{\tilde{X}}_{65} \\ \tilde{\tilde{X}}_{71} & \tilde{\tilde{X}}_{72} & \tilde{\tilde{X}}_{73} & \tilde{\tilde{X}}_{74} & \tilde{\tilde{X}}_{75} \end{bmatrix}$$

The interval type-2 fuzzy sets related to the elements of $\bar{X}$ matrix are shown in Table 6.

Table 6. The average decision matrix ($\bar{X}$)

	$\tilde{X}^U_{ij}$						$\tilde{X}^L_{ij}$					
	x^U_{1ij}	x^U_{2ij}	x^U_{3ij}	x^U_{4ij}	$H_1(\tilde{X}^U_{ij})$	$H_2(\tilde{X}^U_{ij})$	x^L_{1ij}	x^L_{2ij}	x^L_{3ij}	x^L_{4ij}	$H_1(\tilde{X}^L_{ij})$	$H_2(\tilde{X}^L_{ij})$
$\tilde{\tilde{X}}_{11}$	0.00	0.03	0.05	0.17	1	1	0.02	0.03	0.05	0.10	0.9	0.9
$\tilde{\tilde{X}}_{21}$	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
$\tilde{\tilde{X}}_{31}$	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
$\tilde{\tilde{X}}_{41}$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\tilde{\tilde{X}}_{51}$	0.23	0.43	0.48	0.63	1	1	0.33	0.43	0.48	0.53	0.9	0.9
$\tilde{\tilde{X}}_{61}$	0.70	0.85	0.88	0.97	1	1	0.78	0.85	0.88	0.92	0.9	0.9
$\tilde{\tilde{X}}_{71}$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\tilde{\tilde{X}}_{12}$	0.13	0.30	0.35	0.50	1	1	0.22	0.30	0.35	0.40	0.9	0.9
$\tilde{\tilde{X}}_{22}$	0	0	0	0.10	1	1	0	0	0	0.05	0.9	0.9
$\tilde{\tilde{X}}_{32}$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\tilde{\tilde{X}}_{42}$	0.77	0.90	0.92	0.97	1	1	0.83	0.90	0.92	0.93	0.9	0.9
$\tilde{\tilde{X}}_{52}$	0.77	0.90	0.93	1	1	1	0.85	0.90	0.93	0.97	0.9	0.9
$\tilde{\tilde{X}}_{62}$	0.30	0.50	0.55	0.7	1	1	0.40	0.50	0.55	0.60	0.9	0.9
$\tilde{\tilde{X}}_{72}$	0.30	0.50	0.55	0.7	1	1	0.40	0.50	0.55	0.60	0.9	0.9
$\tilde{\tilde{X}}_{13}$	0.77	0.90	0.93	1	1	1	0.85	0.90	0.93	0.97	0.9	0.9
$\tilde{\tilde{X}}_{23}$	0.90	1	1	1	1	1	0.95	1	1	1	0.9	0.9
$\tilde{\tilde{X}}_{33}$	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
$\tilde{\tilde{X}}_{43}$	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
$\tilde{\tilde{X}}_{53}$	0.37	0.57	0.62	0.77	1	1	0.47	0.57	0.62	0.67	0.9	0.9
$\tilde{\tilde{X}}_{63}$	0.07	0.23	0.28	0.43	1	1	0.15	0.23	0.28	0.33	0.9	0.9
$\tilde{\tilde{X}}_{73}$	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
$\tilde{\tilde{X}}_{14}$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\tilde{\tilde{X}}_{24}$	0.70	0.85	0.90	1	1	1	0.80	0.85	0.90	0.95	0.9	0.9
$\tilde{\tilde{X}}_{34}$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\tilde{\tilde{X}}_{44}$	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
$\tilde{\tilde{X}}_{54}$	0.17	0.37	0.42	0.57	1	1	0.27	0.37	0.42	0.47	0.9	0.9
$\tilde{\tilde{X}}_{64}$	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
$\tilde{\tilde{X}}_{74}$	0.70	0.85	0.88	0.97	1	1	0.78	0.85	0.88	0.92	0.9	0.9
$\tilde{\tilde{X}}_{15}$	0.50	0.68	0.73	0.87	1	1	0.60	0.68	0.73	0.78	0.9	0.9
$\tilde{\tilde{X}}_{25}$	0.90	1	1	1	1	1	0.95	1	1	1	0.9	0.9
$\tilde{\tilde{X}}_{35}$	0.23	0.43	0.48	0.63	1	1	0.33	0.43	0.48	0.53	0.9	0.9
$\tilde{\tilde{X}}_{45}$	0.03	0.13	0.17	0.30	1	1	0.08	0.13	0.17	0.22	0.9	0.9
$\tilde{\tilde{X}}_{55}$	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
$\tilde{\tilde{X}}_{65}$	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
$\tilde{\tilde{X}}_{75}$	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9

Step 3. The average performance values of the criteria are calculated based on Table 6 and Eqs. (19) and (20). The results of this step are represented in Table 7.

Table 7. The average performance value of each criterion

	$\tilde{X}_j^{aU}$						$\tilde{X}_j^{aL}$					
	X_{1j}^{aU}	X_{2j}^{aU}	X_{3j}^{aU}	X_{4j}^{aU}	$H_1(\tilde{X}_j^{aU})$	$H_2(\tilde{X}_j^{aU})$	X_{1j}^{aL}	X_{2j}^{aL}	X_{3j}^{aL}	X_{4j}^{aL}	$H_1(\tilde{X}_j^{aL})$	$H_2(\tilde{X}_j^{aL})$
$\tilde{\tilde{X}}_1^a$	0.33	0.49	0.53	0.66	1	1	0.41	0.49	0.53	0.57	0.9	0.9
$\tilde{\tilde{X}}_2^a$	0.39	0.53	0.57	0.69	1	1	0.46	0.53	0.57	0.61	0.9	0.9
$\tilde{\tilde{X}}_3^a$	0.43	0.58	0.62	0.73	1	1	0.51	0.58	0.62	0.66	0.9	0.9
$\tilde{\tilde{X}}_4^a$	0.43	0.60	0.65	0.78	1	1	0.52	0.60	0.65	0.70	0.9	0.9
$\tilde{\tilde{X}}_5^a$	0.52	0.67	0.71	0.81	1	1	0.60	0.67	0.71	0.75	0.9	0.9

Step 4. Based on Tables 6 and 7 and Eqs. (21) and (22), the deviation measures ($\mathscr{s}_j$) and objective weights (w_j^o) of all criteria are calculated. The following results are obtained in this step:

$\mathscr{s}_1$=0.277, $\mathscr{s}_2$=0.294, $\mathscr{s}_3$=0.287, $\mathscr{s}_4$=0.244 and $\mathscr{s}_5$=0.271.

w_1^o=0.202, w_2^o=0.214, w_3^o=0.209, w_4^o=0.178 and w_5^o=0.197.

Step 5. The subjective weighting matrices (W_1^s, $W_2^s$and W_3^s) are obtained based on Table 5 and Eq. (23), show as follows:

$$W_1^s = \begin{bmatrix} \mathrm{VH} \\ \mathrm{MH} \\ \mathrm{VH} \\ \mathrm{H} \\ \mathrm{H} \end{bmatrix}, W_2^s = \begin{bmatrix} \mathrm{VH} \\ \mathrm{MH} \\ \mathrm{H} \\ \mathrm{MH} \\ \mathrm{H} \end{bmatrix} \text{and } W_3^s = \begin{bmatrix} \mathrm{H} \\ \mathrm{M} \\ \mathrm{VH} \\ \mathrm{MH} \\ \mathrm{MH} \end{bmatrix}.$$

Step 6. The average subjective weight of all criteria are calculated based on Step 5 and Eq. (24). The results are represented in Table 8.

Table 8. The average subjective weights of criteria

	$\widetilde{w}_j^{sU}$						$\widetilde{w}_j^{sL}$					
	w_{1j}^{sU}	w_{2j}^{sU}	w_{3j}^{sU}	w_{4j}^{sU}	$H_1(\widetilde{w}_j^{sU})$	$H_2(\widetilde{w}_j^{sU})$	w_{1j}^{sL}	w_{2j}^{sL}	w_{3j}^{sL}	w_{4j}^{sL}	$H_1(\widetilde{w}_j^{sL})$	$H_2(\widetilde{w}_j^{sL})$
$\widetilde{\widetilde{w}}_1^s$	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
$\widetilde{\widetilde{w}}_2^s$	0.43	0.63	0.68	0.83	1	1	0.53	0.63	0.68	0.73	0.9	0.9
$\widetilde{\widetilde{w}}_3^s$	0.83	0.95	0.97	1	1	1	0.90	0.95	0.97	0.98	0.9	0.9
$\widetilde{\widetilde{w}}_4^s$	0.57	0.75	0.80	0.93	1	1	0.67	0.75	0.80	0.85	0.9	0.9
$\widetilde{\widetilde{w}}_5^s$	0.63	0.80	0.85	0.97	1	1	0.73	0.80	0.85	0.90	0.9	0.9

Step 7. Based on the results of Step 4, Table 8 and Eq. (25), the aggregated weights of criteria (with β=0.5) are calculated. Table 9 shows the results of this step.

Table 9. The aggregated weights of criteria

	$\tilde{w}_j^U$						$\tilde{w}_j^L$					
	w_{1j}^U	w_{2j}^U	w_{3j}^U	w_{4j}^U	$H_1(\tilde{w}_j^U)$	$H_2(\tilde{w}_j^U)$	w_{1j}^L	w_{2j}^L	w_{3j}^L	w_{4j}^L	$H_1(\tilde{w}_j^L)$	$H_2(\tilde{w}_j^L)$
$\tilde{\tilde{w}}_1$	0.52	0.58	0.58	0.60	1	1	0.55	0.58	0.58	0.59	0.9	0.9
$\tilde{\tilde{w}}_2$	0.32	0.42	0.45	0.52	1	1	0.37	0.42	0.45	0.47	0.9	0.9
$\tilde{\tilde{w}}_3$	0.52	0.58	0.59	0.60	1	1	0.55	0.58	0.59	0.60	0.9	0.9
$\tilde{\tilde{w}}_4$	0.37	0.46	0.49	0.56	1	1	0.42	0.46	0.49	0.51	0.9	0.9
$\tilde{\tilde{w}}_5$	0.42	0.50	0.52	0.58	1	1	0.47	0.50	0.52	0.55	0.9	0.9

Step 8 and 9. The appraisal measures of the alternatives ($\tilde{\tilde{AP}}_i$) are calculated based on Tables 6, 7 and 9 and Eq. (26). Table 10 represents the appraisal measures and the corresponding ranking values. According to this table, the ranking order of alternatives is $\mathcal{A}_2 \succ \mathcal{A}_1 \succ \mathcal{A}_7 \succ \mathcal{A}_5 \succ \mathcal{A}_6 \succ \mathcal{A}_3 \succ \mathcal{A}_4$. Therefore, $\mathcal{A}_2$ is the best alternative.

Table 10. Appraisal measures of alternatives and the corresponding ranking values

	$\widetilde{AP}_i^U$						
	AP_{i1}^U	AP_{i2}^U	AP_{i3}^U	AP_{i4}^U	$H_1(\widetilde{AP}_i^U)$	$H_2(\widetilde{AP}_i^U)$	
$\tilde{\tilde{AP}}_1$	0.56	1.50	1.75	2.42	1	1	
$\tilde{\tilde{AP}}_2$	1.30	2.15	2.31	2.73	1	1	
$\tilde{\tilde{AP}}_3$	-1.11	-0.07	0.24	1.29	1	1	
$\tilde{\tilde{AP}}_4$	-1.45	-0.69	-0.44	0.50	1	1	
$\tilde{\tilde{AP}}_5$	-0.95	0.04	0.36	1.39	1	1	
$\tilde{\tilde{AP}}_6$	-0.92	0.00	0.29	1.23	1	1	
$\tilde{\tilde{AP}}_7$	-0.87	0.07	0.36	1.34	1	1	
	$\widetilde{AP}_i^L$						$R_{value}(\tilde{\tilde{AP}}_i)$
	AP_{i1}^L	AP_{i2}^L	AP_{i3}^L	AP_{i4}^L	$H_1(\widetilde{AP}_i^L)$	$H_2(\widetilde{AP}_i^L)$	
$\tilde{\tilde{AP}}_1$	1.10	1.50	1.75	2.04	0.9	0.9	
$\tilde{\tilde{AP}}_2$	1.78	2.15	2.31	2.50	0.9	0.9	0.352
$\tilde{\tilde{AP}}_3$	-0.54	-0.07	0.24	0.68	0.9	0.9	0.472
$\tilde{\tilde{AP}}_4$	-1.00	-0.69	-0.44	-0.02	0.9	0.9	0.040
$\tilde{\tilde{AP}}_5$	-0.40	0.04	0.36	0.79	0.9	0.9	0.001
$\tilde{\tilde{AP}}_6$	-0.39	0.00	0.29	0.70	0.9	0.9	0.058
$\tilde{\tilde{AP}}_7$	-0.32	0.07	0.36	0.80	0.9	0.9	0.049

To validate the results of the proposed method and represent the stability of it, a comparison and a sensitivity analysis are performed. The methods of Keshavarz Ghorabaee *et al.*(2014),Wang*et al.*(2012), Hu *et al.*(2013), Balezentisand Zeng (2013), Chen *et al.*(2012) and Keshavarz Ghorabaee (2015) are used in the comparison. To compare the results, the Spearman's rank correlation coefficients (r) are utilized to test the association between the ranking obtained by the proposed method and the ranking obtained by the other methods in the comparison. Table 11 shows the interpretation of different values of r(Walters, 2009). To perform this comparison, the above-mentioned example is solved using these methods. Table 12 represents the ranking results obtained by different methods and the correlation between them and the results of the proposed method.

Table 11. Interpretation of the correlation values (r)

Range	Relationship
$r \geq 0.8$	Very strong
$0.6 \leq r < 0.8$	Strong
$0.4 \leq r < 0.6$	Moderate
$0.2 \leq r < 0.4$	Weak
$r < 0.2$	Very weak

Table 12. Ranking of the alternatives with different methods and the corresponding correlation (r)

Alternatives	Keshavarz Ghorabaee *et al.*, 2014	Wang *et al.*, 2012	Hu *et al.*, 2013	Balezentis and Zeng, 2013	Chen *et al.*, 2012	Keshavarz Ghorabae, 2015	The proposed method
$\mathcal{A}_1$	2	2	2	2	2	2	2
$\mathcal{A}_2$	1	1	1	1	1	1	1
$\mathcal{A}_3$	6	4	6	6	5	4	6
$\mathcal{A}_4$	7	7	7	7	7	7	7
$\mathcal{A}_5$	4	3	4	3	4	3	4
$\mathcal{A}_6$	5	6	5	5	6	6	5
$\mathcal{A}_7$	3	5	3	4	3	5	3
r	1	0.82	1	0.96	0.96	0.82	—

As can be seen in Table 12, all correlation coefficients are greater than 0.8; therefore, the results of the proposed method is consistent with the other methods.

To show stability of the proposed method, a sensitivity analysis is also performed with different sets of criteria weights. Five sets are chosen for this analysis, which is represented in Fig. 4. With respect to this figure, one criterion has the highest and one criterion has the lowest weight in each set. Using this pattern helps us to consider a wide extent of weights for all criteria in the sensitivity analysis.

We also consider three values for β parameter in this analysis. Changing β parameter could demonstrate the effect of moving from the subjective weights to objective weights. The ranking results with β=0.1, 0.5 and 0.9 are shown in Fig. 5, Fig. 6 and Fig 7, respectively. Also, Table 13 represents the correlation between the ranking results in different sets of criteria weights and different values of β parameter. With respect to these results, we can say that increasing β parameter leads to more sensitivity in ranking of alternatives. This fact shows that using a combination of the subjective and objective weights can increase the stability of the decision-making process.

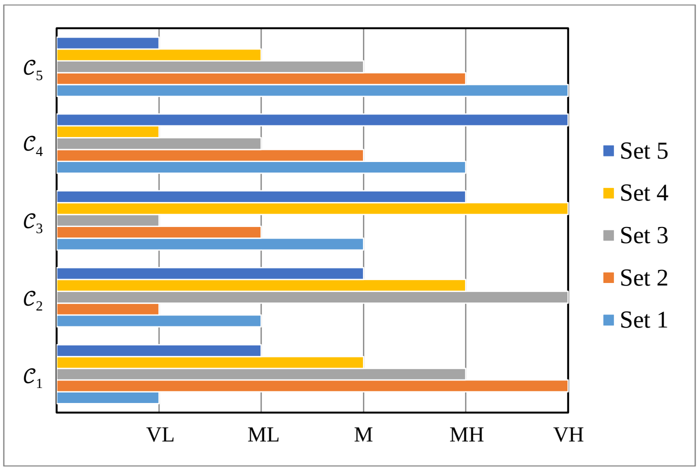

Figure 4. Five sets of the criteria weights for sensitivity analysis

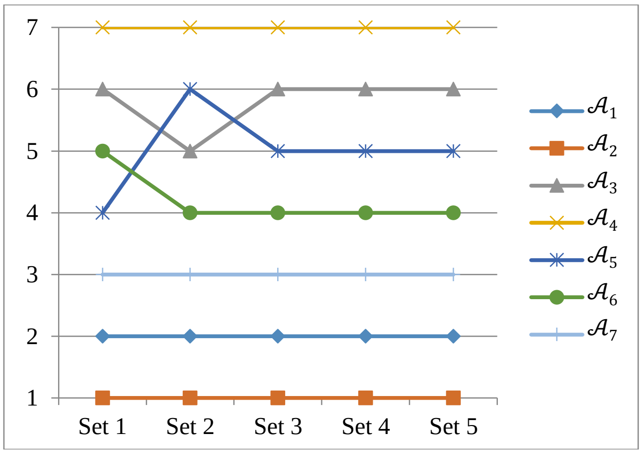

Figure 5. Ranking result in different sets of criteria weight and β=0.1

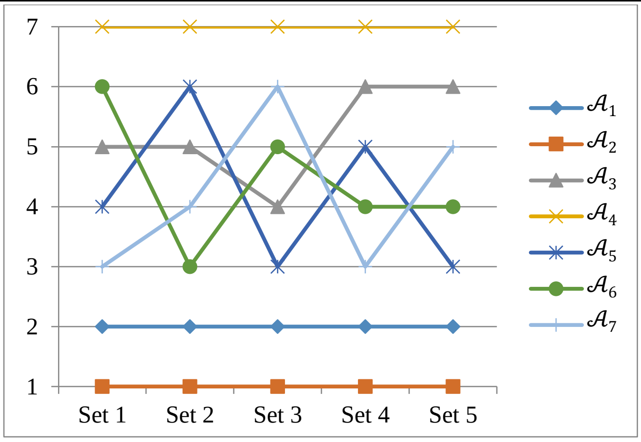

Figure 6. Ranking result in different sets of criteria weight and β=0.5

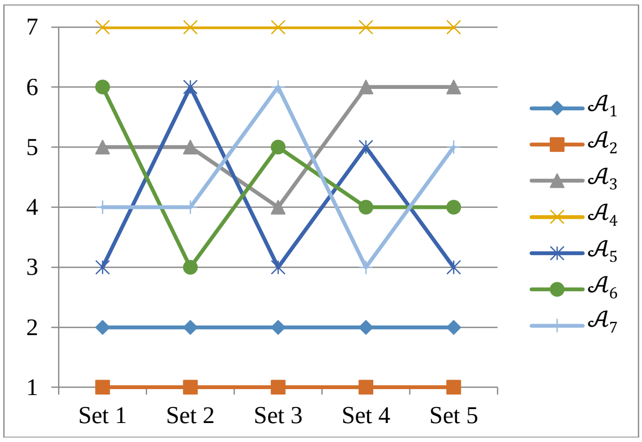

Figure 7. Ranking result in different sets of criteria weight and β=0.9

As can be seen in Table 13, all of the correlation values are greater than 0.6; therefore, the proposed method has a good stability in all values of β.

Table 13. Correlation (r) between results with different sets and different values of β

		Set 2	Set 3	Set 4	Set 5
$\beta = 0.1$	Set 1	0.89	0.96	0.96	0.96
	Set 2	—	0.96	0.96	0.96
	Set 3	—	—	1.00	1.00
	Set 4	—	—	—	1
	Set 5	—	—	—	—
$\beta = 0.5$	Set 1	0.75	0.79	0.89	0.82
	Set 2	—	0.68	0.93	0.79
	Set 3	—	—	0.68	0.89
	Set 4	—	—	—	0.86
	Set 5	—	—	—	—
$\beta = 0.9$	Set 1	0.68	0.89	0.82	0.89
	Set 2	—	0.68	0.93	0.79
	Set 3	—	—	0.68	0.89
	Set 4	—	—	—	0.86
	Set 5	—	—	—	—

6. Conclusion

Multi-criteria decision-making methods have many applications in science and engineering fields. In an uncertain environment, type-1 fuzzy sets are efficient tools to model and solve the MCDM problems. An extended form of a type-1 fuzzy set is interval type-2 fuzzy set. Interval type-2 fuzzy sets help decision-makers to express their preferences and evaluations with more degrees of flexibility. This study has proposed a new ranking method for calculating the ranking values of IT2FSs. The proposed method has less computational process and the comparison shows that it is efficient in ranking interval type-2 fuzzy sets. Using this fuzzy ranking method, a new method of assessment based on fuzzy ranking and aggregated weights (AFRAW) has been developed to deal with the multi-criteria group decision-making problems in the interval type-2 fuzzy environment. A combination of the subjective criteria weights expressed by DMs and objective weights calculated by a deviation-based method has been used in the process of decision-making. An illustrative example has been utilized for showing the procedure of the proposed approach. A comparison and a sensitivity analysis have been used to demonstrate the validity and stability of the method. The results of the comparison and sensitivity analysis show that the proposed method is consistent with the other method and using aggregated weights of criteria leads to more degrees of stability.

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