

Standardized RC beam tests for modeling the fiber bridging effect in SFRC

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ABSTRACT

Fiber reinforcement is a promising solution to cracking problems and improving the concrete's structural performance. The residual strength of the cracked concrete can characterize the reinforcement efficiency. However, quantifying the residual performance of steel fiber-reinforced concrete (SFRC) is challenging. The existing methodologies provide empirical formulas for estimating the residual strength using test results of SFRC elements in which a predominant crack governs the mechanical resistance. For instance, the 0.5 mm crack width determines the minimum value considered in the RILEM standard formulas. Thus, the SFRC strength evolution at earlier cracking stages remains unknown. At the same time, such a crack approximation is irrelevant to structural cases when reinforcement bars stimulate the formation of multiple cracks, and the 0.1–0.2 mm crack typically corresponds to the yielding of the steel bars. This study describes an alternative approach for quantifying the average residual stresses in SFRC elements with multiple cracks. It hypothesizes the possibility of separating the mechanical resistance components, corresponding to tension stiffening and fiber bridging effects characteristic of SFRC elements with bar reinforcement, using standardized small-scale specimens to estimate the fiber contribution. The laboratory tests of the plain concrete and SFRC beams with bar reinforcement illustrate the proposed technique. The RILEM standard three-point bending tests and the numerical simulation of full-scale beams verify the analysis's adequacy. The developed model is suitable for finite element simulations (employing the smeared crack model); the capability of separating the tension stiffening and fiber bridging effects ensures its versatility.

1. Introduction

Cracking is an intricate physical process intrinsic to concrete. Initiated at the microstructural level, it becomes apparent only at the mesoscale; the bar reinforcement further complicates the cracking trajectories [1,2]. Utilizing fibers can solve cracking problems and improve the mechanical resistance of structural elements under complex loading conditions [3–7]. The fiber efficiency results from the fibers' ability to carry a portion of the tensile stresses in a cracked section. The post-cracking residual strength of steel fiber-reinforced concrete (SFRC) can estimate the fiber bridging effect, which quantification defines the fundamental issue of the structural application of SFRC [3,4].

Structural engineering typically treats SFRC as a homogeneous material capable of resisting tensile stresses after cracking [5], and the existing methodologies provide empirical formulas to estimate the residual strength of SFRC. The RILEM recommendations [8] describe the typical case when the test results of SFRC elements, in which a predominant crack governs the mechanical resistance of the test sample,

define the residual strength model. A similar philosophy describes the SFRC classification in European standards [9]. Following this approach, a relatively small (150 × 150 mm cross-section and 550–700 mm long) pre-notched prism subjected to a three-point bending defines the object for the residual strength analysis. In these tests, the 0.5 mm crack represents the minimum value appearing in the residual strength formulas [8]; thus, the SFRC strength evolution at earlier cracking stages remains unspecified. However, this crack limitation is irrelevant to the structures where bar reinforcement stimulates the formation of multiple cracks [10–12]. Depending on the structural scheme, a crack within the range of 0.1–0.2 mm can correspond to the yielding of steel bars. At the same time, the stochastic fiber distribution in concrete alters the number and orientation of the fibers crossing the artificially formed crack in the notched prism [8,9], scattering the test results [13]. Furthermore, the fiber types' variety and ability to combine different fibers, e.g., [14–16], complicates the analytical predictions of mechanical performances of fiber-reinforced composites, making the experimental analysis intrinsic to the development of efficient materials.

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Alternative testing methodologies include the notched four-point bending prisms [17] and un-notched specimens, i.e., three-point bending prisms [18] and four-point flexural prismatic samples [19–21]. The tension tests can also determine the residual resistance of SFRC; however, the sophisticated complexity of such tests makes these setups rare and impractical [22]. Together with the empirical formulas (e.g., [8]), several inverse analysis procedures have been proposed to estimate the residual resistance of SFRC. For instance, Soranakom & Mobasher [23,24] proposed closed-form solutions to transform the flexural load–deflection relationship into a simplified stress–strain model applicable to various design applications. Park et al. [25] adapted this methodology to analyze the mechanical resistance of 3D textile reinforced concretes. Meskenas et al. [26] proposed the inverse analysis procedure for the constitutive modeling of SFRC, which employs the RILEM testing methodology [8].

However, notwithstanding the seeming diversity of the existing SFRC analysis approaches, they have a similarity—straightforward application of the residual strength values, assessed using any standard technique, in numerical (finite element, FE) modeling of structural components with bar reinforcement leads to overestimating stiffness of the cracked members. Numerous studies have supported this inference. Among others, works by Stang & Aarre [27], Chao et al. [28], Gribniak et al. [29], Bernardi et al. [30], Mazaheripour et al. [31], and Meskenas et al. [26,32,33] could be mentioned in this regard.

Several interrelated effects, including tension softening of the concrete and bond behavior of fibers, accompany the cracking process. Fig. 1a sketches the main components of the residual strength of SFRC observed in standard tests (without bar reinforcement). This scheme represents a hypothetical situation when the black curve denotes the sum of ordinates corresponding to the tension softening and fiber bridging components, i.e., “A + F.” Noticeably, the fiber bridging effect becomes apparent only during the crack formation [34,35]. For instance, Li et al. [36] defined the 0.1–0.2 mm crack openings as the contribution boundary of the tensile concrete in Fig. 1a. However, at the lower opening range, essential for structural analysis (e.g., [31,33]), the tension softening effect can be predominant (regarding the fiber

contribution). Thus, the inverse SFRC models (e.g., [23–26]) include a substantial portion of the concrete softening stresses.

The bar reinforcement complicates the cracking process [1]. Following the Fig. 1a analogy and according to the superposition principles, Fig. 1b shows the structure of stresses in tension concrete in the reinforced concrete (RC) bending elements with different fiber content from the article [29] estimated through the inverse analysis procedure [37]. In this graph, the red diagram describes the stress distribution in tension concrete; it combines the tension softening stresses (because of the concrete fracture) and the tension stiffening stresses induced by the bar reinforcement interaction with concrete [1,38,39]. The fiber contribution is defined by the area between the black and red lines in Fig. 1b. Such a separation of the tension stiffening (“TS + A”) and fiber bridging (“F”) effects is nominally possible because of identical loading conditions, geometry, and bar reinforcement percentages of both beam samples [29]. Still, this hypothesis requires further verification, which is the object of the present study.

The simplified concept in Fig. 1b does not individually account for the fiber effect on the bond performance of bar reinforcement in concrete well documented in the literature [40,41], attributing it to the overall fiber bridging process. Supposing the superposition principle to the results contributing to the stress distribution in Fig. 1b, the subtraction of the black and red graphs describes the fiber bridging contribution to resisting the tensile stresses. Thus, Fig. 1c shows the normalized residual strength diagram related to the tensile strain of concrete, $\varepsilon_{cr}(= f_{ct}/E_c)$, and the theoretical tensile strength (f_{ct}) and modulus of elasticity (E_c) of the concrete. The increase of the residual stresses (so-called stress hardening) in Fig. 1c results from the formation and opening of the multiple cracks observed in [29] because of the bar reinforcement. In other words, each crack opening and further development generates an additional fiber contribution, continuously raising the aggregated stress over the beam sample's pure bending zone.

Considering the modeling aspects, the discrete cracking models, e.g., [13,22,26], are suitable for analytical calculations. Still, regarding commercial FE software, they introduce uncertainty related to the estimation of the crack width, having a functional dependence on finite

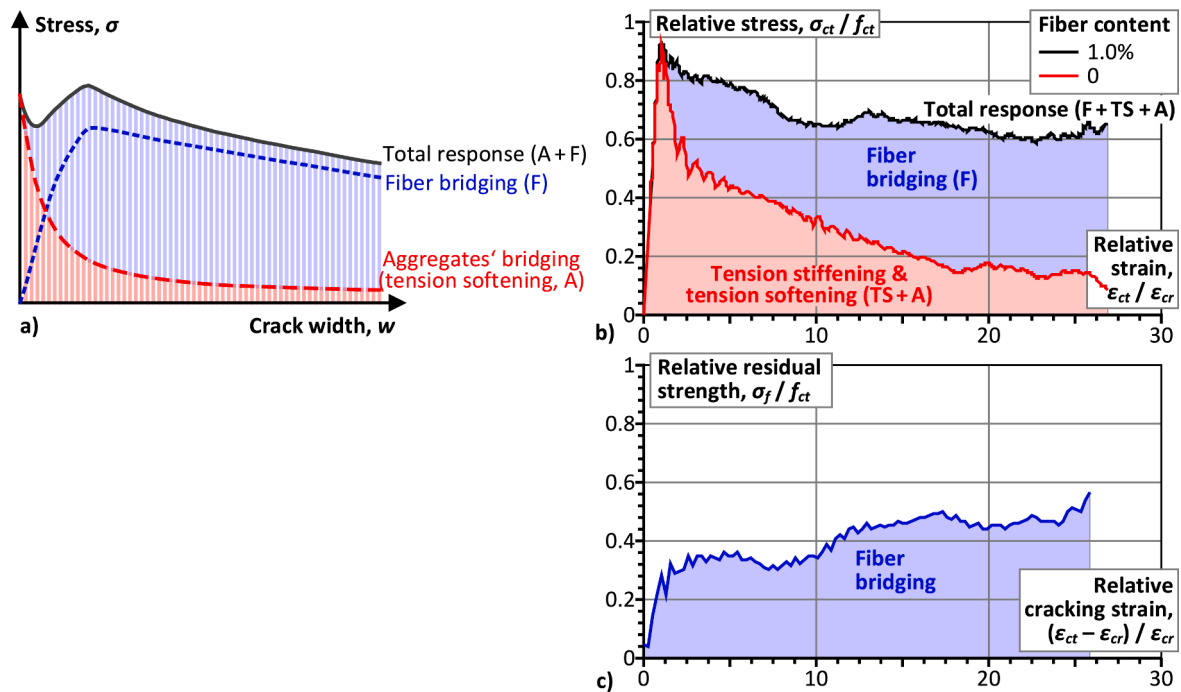


Fig. 1. Residual strength of SFRC: a) mechanical resistance components of bending SFRC tests (the drawing concept was taken from [36]); b) inverse analysis outcome of RC beams with different fiber volumes (the drawing concept was taken from [29]); c) the fiber bridging effect estimated using the test results of SFRC beams with bar reinforcement.

element mesh size [38]. Therefore, the smeared cracking concept defines a much more robust alternative in this study than the discrete crack model. Moreover, most commercial FE software tools employ the average cracking approach, which makes it efficient for engineering applications, e.g., [29,33,37,38].

Remarkably, both black diagrams in Fig. 1a and 1b include the tension-softening stress component. Therefore, their combination, typical for numerical simulation of SFRC RC elements, duplicates the aggregates' bridging effect from the tension stiffening and SFRC constitutive models. Thus, the tension softening overestimation increases the theoretical stiffness assessments of RC elements identified in the literature [26–33]. To overcome the modeling inadequacy, the numerical (finite element, FE) simulations must use the SFRC models shown in Fig. 1c. Moreover, such a separation would allow the development of a versatile modeling approach, tailoring the tension stiffening and fiber models separately. However, the full-scale beam tests analyzed in [29] are expensive and require specific test setups.

This study develops an alternative approach for determining the residual stresses of SFRC, employing a new flexural testing layout designed to form multiple cracks in a small-size laboratory specimen. The tests of composite reinforcement systems [42] and biological concrete [43] proved the adequacy of the proposed experimental setup. The analysis hypothesizes the possibility of separating the discussed mechanical resistance components, corresponding to tension stiffening and fiber bridging effects. The comparison with the exact inverse analysis solution [37], the RILEM bending [8] test results, and the numerical simulation of full-scale beams from the previous study [29] check the proposed analysis procedure.

2. The proposed testing procedure

The studies [44–46] indicated that the number of cracks primarily affects the constitutive modeling results. Since laboratory conditions limit the test specimen dimensions, increasing the potential crack number was the design focus of the testing layout. Williams [47], Purainer [48], and Gilbert [49] revealed that the slab-shaped geometry of RC elements ensures the formation of multiple cracks with narrow crack spacing. Thus, this study also utilizes such specimen geometry, which was tailored through the preliminary tests [50,51]. Fig. 2 shows the specimen geometry and testing scheme. The developed slab-shaped beam specimen has a rectangular cross-section reinforced with three 8 mm deformed steel bars with a total area of 150.8 mm². In this study, the reinforcement ratio is constant but can vary, as shown in [42].

The 1000 mm beam sample resisted a four-point-loading with a 600 mm pure bending zone and two 150 mm shear spans. The shear span, a , was reduced to increase the monitoring length; such shear span also fits the Eurocode 2 [52] condition for elements not requiring the shear reinforcement, i.e., $a \leq 2d$, where d is the effective depth of the cross-section (Fig. 2). Fig. 3 shows the test setup. The specimens were loaded using a 5 MN servohydraulic apparatus under displacement control and a 0.4 mm/min loading speed. The monitoring parameters included vertical displacements and surface deformations of the pure

bending zone, and a load cell measured the applied load. Nine LVDT (L_1 – L_9 , Fig. 2) measured the vertical displacements. Figs. 2 and 3 show the linear variable displacement transducers (LVDT L_{10} – L_{15}) attached in two continuous lines to the side surface of the specimen, measuring the surface deformation. In addition, a stereoscopic digital image correlation system (DIC) monitored the deformation and crack formation on the opposite specimen's side. Two cameras, incorporating a charge-coupled device (CCD) detector with a 2456 × 2085 pixels resolution at the 12.2 fps rate, were placed on a tripod at a 3.0 m distance from the test sample; the distance between the cameras was equal to 0.4 m. A data logger collected the outputs of the load cell and LVDT devices every second.

Further analysis considers the moment–curvature response of the pure bending zone. The instrumentation in Fig. 2 enables determining the moment–curvature diagrams in three ways by using:

- **Vertical displacements** measured by LVDT L_1 – L_9 placed beneath the soffit of the beam specimen at the mid-span and loading points, as Fig. 2 shows. Assuming a circular arc shape of the deflection curve along the pure bending zone, average curvature over this zone can be obtained from the deflection readings by:

$$\kappa = \frac{2 \cdot \delta}{(l_b/2)^2 + \delta^2} = \frac{8 \cdot \delta}{l_b^2 + 4 \cdot \delta^2} \cdot \delta$$

$$= \frac{\delta_4 + \delta_5 + \delta_6}{3} - \frac{\delta_1 + \delta_2 + \delta_3 + \delta_7 + \delta_8 + \delta_9}{6} \quad (1)$$

Here δ is the deflection over the pure bending zone; δ_i is the vertical displacement obtained by the i -th LVDT ($i = 1 \dots 9$); l_b is the length of the bending zone (in our case, $l_b = 0.6$ m).

- **Average strains of the concrete surface**, $\bar{\epsilon}_i$ ($i = 1, 2$), obtained along each monitoring line as a mean value of three LVDT (i.e., L_{10} – L_{12} and L_{13} – L_{15} , Fig. 2) estimated values ϵ_i ($i = 10 \dots 15$). The following equation defines the averaged curvature:

$$\kappa = (\bar{\epsilon}_2 - \bar{\epsilon}_1) / \Delta h = [(\epsilon_{13} + \epsilon_{14} + \epsilon_{15}) - (\epsilon_{10} + \epsilon_{11} + \epsilon_{12})] / 3 \Delta h \quad (2)$$

Here Δh is the distance between the strain monitoring lines (in our case, $\Delta h \approx 0.055$ m).

- **Relative displacement profiles identified by the DIC system.** Following this approach, the image-processing algorithm transforms the relative displacements into deformation vectors. This study uses two displacement profiles located at the same levels as the LVDT shown in Fig. 2 but on the opposite surface of the beam (side “@”). These profiles determine the average strains of the monitoring lines, $\bar{\epsilon}_i$ ($i = 1, 2$), and eq. (2) defines the corresponding average curvature.

The DIC system can construct any number of relative displacement profiles of the pixels recognized at the monitored surface, securing a robust assessment of the deformation response. However, the DIC results are highly susceptible to illumination and camera position changes [53]. Thus, alternative curvature determination procedures using LVDT devices help check DIC outcomes, as shown in [54].

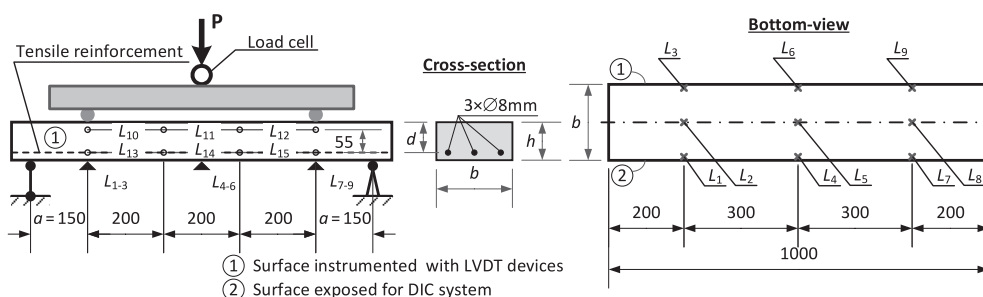


Fig. 2. Distribution of the monitoring devices and loading scheme of the beam samples.

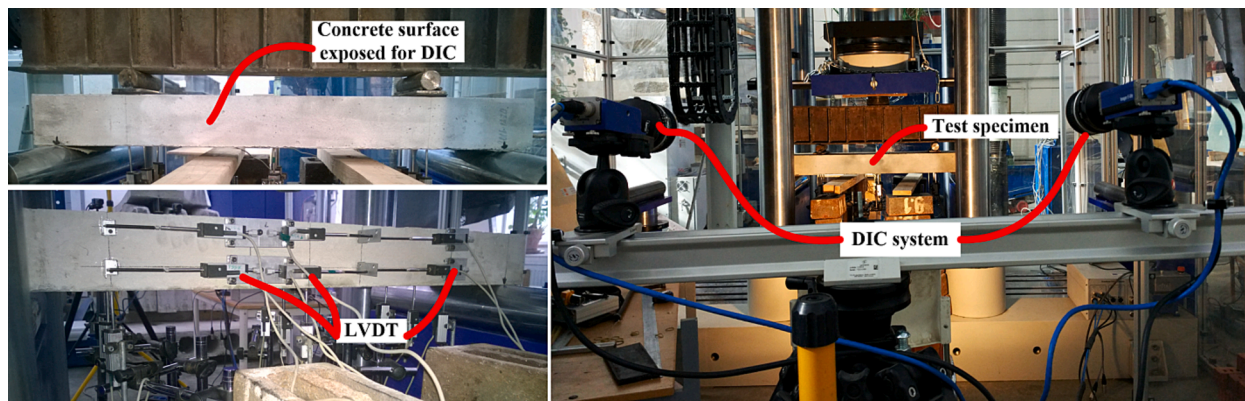


Fig. 3. Experimental setup.

3. Experimental data

This illustrative analysis employs the results of four standardized beams (Fig. 2) tested in the Laboratory of Innovative Building Structures at Vilnius Gediminas Technical University. The beams B1 and B2 were produced from ordinary concrete, and the remaining samples (B1F and B2F) were from SFRC. Five RILEM prismatic specimens [8] were also manufactured to determine the SFRC characteristics for comparison purposes. In addition, four full-scale beams (ordinary concrete S3-2-3, S3-2-6, and SFRC S3-1-F10, S3-2-F10) from previous work [29] illustrate the applicability of the proposed modeling concept.

3.1. Characteristics and material properties of the test specimens

In this study, the samples were formed in two batches. The reference beams (B1 and B2) were made from ordinary concrete with the following proportions: 465.8 kg/m³ of cement (CEM II/A-LL 42.5 N), 197.6 l/m³ of water, 845.3 kg/m³ of 0/4 mm sand, and 860.7 kg/m³ of 4/11 mm crushed granite aggregates. The concrete also contained 0.6 % (by the cement weight) of superplasticizer. The same concrete proportions were used for the SFRC beams B1F and B2F, including 71.8 kg/m³ of hooked-end steel fibers and 0.9 kg/m³ of synthetic micro-fibers. The previous studies [42,50,51] indicated the absence of the synthetic fibers' contribution to the mechanical performance of the considered RC beams (Fig. 2). Simultaneously, these fibers stabilize the cracking process, reducing the scatter of the inverse analysis results. The

manufacturer-specified mechanical properties of the fibers (Fig. 4) are the following:

- Steel fibers: the tensile strength = 1345 MPa; the modulus of elasticity = 210 GPa; the length = 35 mm; the diameter = 0.55 mm.
- Synthetic micro-fibers: the tensile strength = 380 MPa; the length = 13 mm; the diameter = 22 μ m.

The 100 mm cubes were used to determine the compressive strength of concrete; the RILEM [8] prisms (150 × 150 × 600 mm) were used to assess the SFRC residual strength. All specimens were cast using steel forms and demolded within 24 h of age. To diminish the shrinkage effect, the samples were stored in a water tank at 18–20 °C until the test day. The specimens were tested after 19 days. The average compressive strength of the reference concrete and SFRC was 50.1 MPa and 55.5 MPa, respectively. Fig. 2 shows the cross-section geometry of the beams with the actual parameters as follows:

- The height (h) equals 105 mm (B1), 102 mm (B2), 101 mm (B1F), and 104 mm (B2F).
- The width (b) equals 199 mm (B1), 200 mm (B2), 199 mm (B1F), and 202 mm (B2F).
- The effective depth (d) equals 78 mm (B1), 77 mm (B2), 76 mm (B1F), and 78 mm (B2F).

The 8 mm bars from S500 steel reinforced the cross-section (Fig. 2). The tensile tests of three 500 mm samples resulted in the yielding strength $f_y = 567$ MPa and the modulus of elasticity $E_s = 196$ GPa.

3.2. Test results

Section 2 described the testing methodology and equipment of the beam samples. Fig. 5 shows the cracking patterns captured with the DIC system. This figure indicates the cracking sequence and the loading moments corresponding to the first crack formation and the stabilized cracking stage. However, the illustrative nature of these patterns does not allow straightforward crack measurement. On the contrary, the DIC technique ensures a dynamic estimation of surface deformations (including the crack opening), identifying the relative displacement of the pixels recognized at the flat surface through the correlation analysis of a sequential series of digital images. Gribniak et al. [46] presented a detailed example of crack assessment with DIC. At the same time, Fig. 5 establishes the proposed loading scheme's ability to form multiple (10–12) cracks in the pure bending zone, which aligns with the preliminary tests [50,51].

The proposed analysis technique employs the moment–curvature response to estimate the residual strength of concrete. Fig. 6 exemplifies the moment–curvature diagrams constructed using the alternative



Fig. 4. Fibers used for SFRC.

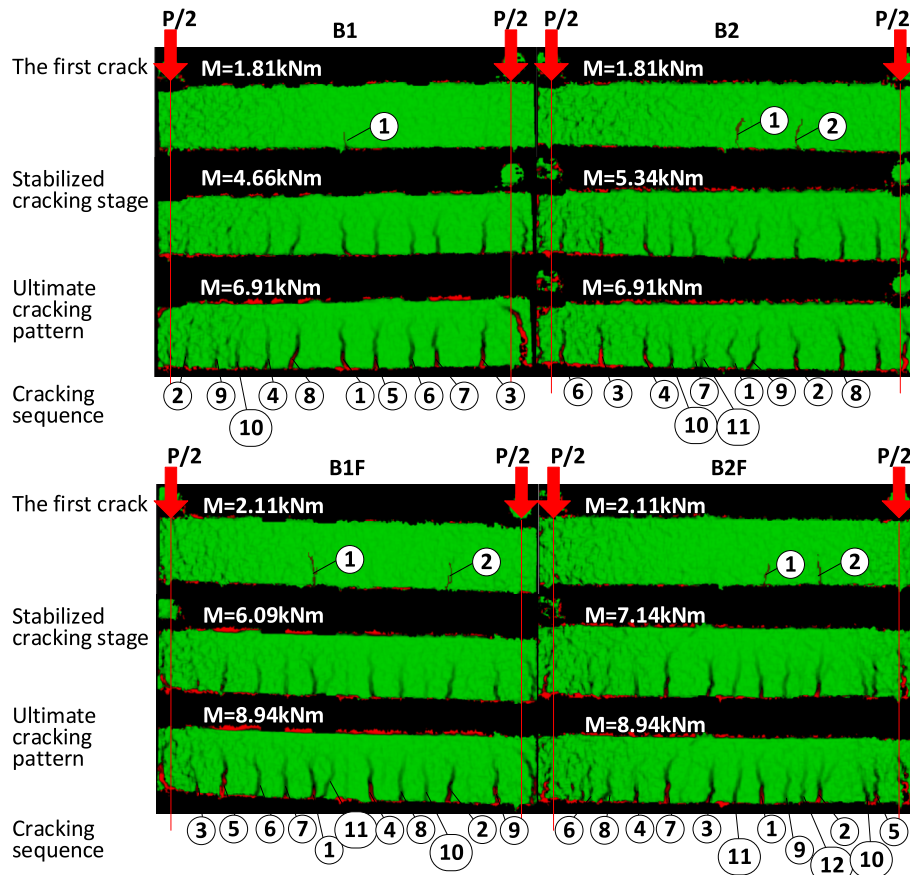


Fig. 5. Cracking schemes identified by the DIC system.

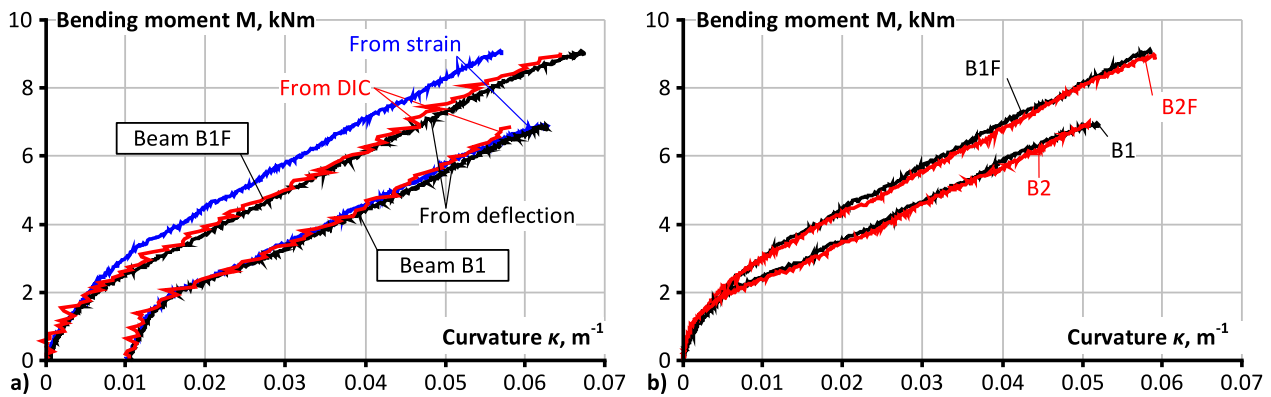


Fig. 6. Moment-curvature diagrams: a) diagrams identified using different approaches from Section 2; b) curves estimated using the vertical displacement monitoring results.

procedures described in Section 2. Fig. 6a shows the moment–curvature diagrams of two selected beams B1 and B1F, obtained using three alternative methods—all curves of beam B1 coincided; in contrast, the graphs of the beam B1F determined using the LVDT readings of the surface deformations (designated as “From strain”) differs from other two curves. Such a result is the often consequence of the crack formation in the LVDT attachment place. In the considered case, the moment–curvature diagrams identified by the DIC system (designated “From DIC” in Fig. 6a) and using the vertical displacements (“From deflections”) coincide for all specimens. Therefore, the vertical displacement diagrams are used for further analysis. Fig. 6b shows these diagrams for all the tested beams. The agreement between identical specimens indicates the reliability of the proposed testing methodology.

Yielding of the reinforcement steel localized outside the pure bending zone has terminated the tests in all considered cases; the fibers increased the shear resistance of the specimens by almost 30 %.

The RILEM methodology [8] was also used to identify the residual strength of SFRC: five $150 \times 150 \times 600$ mm prisms (P1...P5) were subjected to the three-point bending. The distance between the supports was equal to 500 mm. The prisms were notched at the mid-span on the surface perpendicular to the pouring direction; the notch width and depth were 5 mm and 25 mm. The load was applied at the 0.2 mm/min rate under displacement control. An LVDT device measured the crack-mouth opening displacement (CMOD). Fig. 7 shows the CMOD results and the corresponding residual strength values. This figure demonstrates a substantial scatter of the measured responses unavoidable to

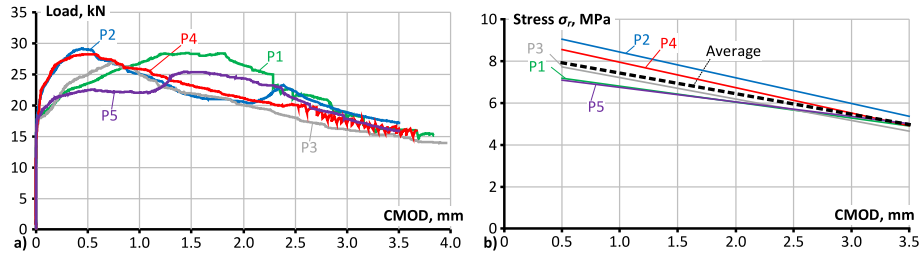


Fig. 7. Residual strength analysis following the RILEM methodology reported in [8]: a) load-CMOD diagrams; b) the corresponding residual stress values.

the simplified prismatic samples [13,26].

3.3. Test results of full-scale beams [29]

This study includes the test results of four beam samples from the previous investigation to illustrate the applicability of the proposed modeling methodology. Therefore, this article reports only the minimum information necessary for this analysis; the interested reader can find the complete description of the tests in the references [29] and [33].

The beams having a 3000 mm distance between the supports were tested under a four-point bending scheme with equal loading spans. All specimens had nominally identical cross-section dimensions (width $b \approx 280$ mm; height $h \approx 300$ mm; effective depth $d \approx 275$ mm) and three 10 mm S500 steel bars of tensile reinforcement. The reinforcement had 560 MPa yielding strength and 203 GPa modulus of elasticity. Two reference beams were produced from ordinary concrete; the alternative two SFRC beams had 1.0 % (by volume) of hooked-end steel fibers with 1 mm diameter, 53 mm length, and 1020 MPa tensile strength. The average strength of $\varnothing 150 \times 300$ mm cylinders was 50.9 MPa (plain concrete) and 48.0 MPa (SFRC) that well corresponds to the material properties of the beam samples in Section 3.1.

The test program [29] utilized two alternative measurement systems to estimate the average curvature in the pure bending zone, i.e., the surface strains and vertical displacements. The study [29] considered the curvature diagrams constructed using the surface strains as a reliable estimation of the deformation behavior. Thus, this study preserves the choice, and Fig. 8 shows the graphs used for the analysis. Remarkably, this figure demonstrates the same tendencies as shown in Fig. 6b:

- Identical beam samples demonstrate very similar deformation responses. This similarity results from the bar reinforcement effect and well agree with the experimental deformation analysis principles formulated in the dissertation [55].
- The fibers improve the flexural resistance of SFRC elements with bar reinforcement in the post-cracking stage, ensuring the development of efficient composite systems, e.g., [56,57].

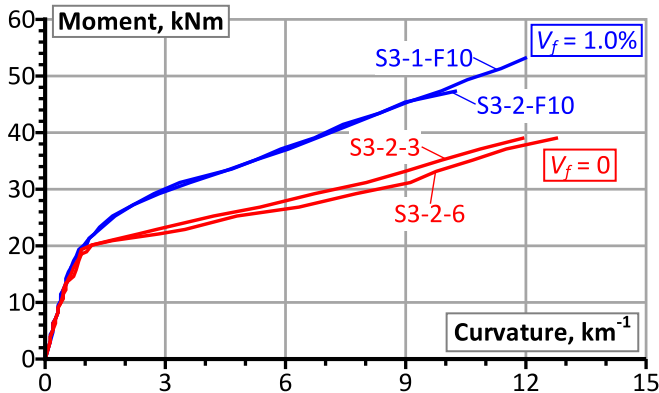


Fig. 8. Moment-curvature diagrams of the full-scale beam samples reported in [29].

4. The residual strength analysis

4.1. The ‘exact’ inverse analysis procedure [37]

Kaklauskas & Ghaboussi [58] described the inverse analysis principles: the equilibrium conditions of axial forces and bending moments define the stress–strain diagram of the tensile concrete in the incremental form for a given moment–curvature relationship. The equilibrium equations determine the stress of the outermost tension layer of the concrete for a given load and deformation increment. This study employs the modified inverse analysis procedure [37] as the ‘exact’ solution reference; Fig. 1 exemplifies the solution results from the literature [29]. The terminology ‘exact’ is placed in quotations because of the iterative nature of the calculation procedure [37], which includes approximations. However, the remaining text uses this word without quotes to simplify the descriptions. The inverse analysis employs the layered cross-section model and the following assumptions:

- The smeared crack model.
- The Euler–Bernoulli plane sections hypothesis.
- All concrete layers in the tension zone follow the same stress–strain law iteratively determined for incrementally increased load. The latter simplification reduces the inverse problem dimension to a single non-linear equation.

However, notwithstanding the numerical enhancements, the solution [37] still requires a refined description of the loading history because of the iterative process requirement to specify the stress–strain model to every layer belonging to the tensile concrete zone—the interested reader finds the detailed description of the limitations of this solution in the article [59]. Fig. 9 exemplifies the direct analysis procedure when the cross-section in Fig. 2 defines the simulation object. This layered cross-section model calculates the bending deformations for specified geometry (Fig. 9a) and material laws (Fig. 9b); the n layers form the cross-section model in Fig. 9c, corresponding to concrete or reinforcement. The equivalent reinforcement area (A_s) defines the corresponding layer thickness, $t_s (= A_s / b)$. Under the plane sections assumption, the linear strain distribution defines the curvature induced by the external bending moment M_{ext} (Fig. 9d), and the internal forces’ (generated in each layer) equilibrium corresponds to the assumed material laws (Fig. 9b). The following equations define the calculation routine [37]:

$$\kappa = M_{ext} / IE; \quad (3a)$$

$$IE = b \sum_{i=1}^n \left[\frac{t_i^3}{12} + (y_i - y_c)^2 \right] t_i E_i; \quad (3b)$$

$$y_c = SE / AE; \quad (3c)$$

$$SE = b \sum_{i=1}^n y_i t_i E_i; \quad (3d)$$

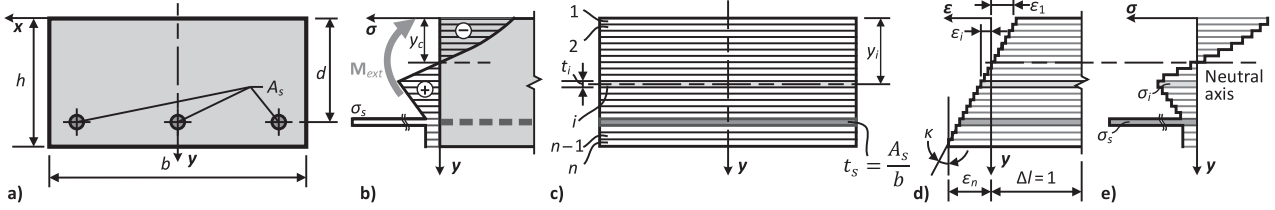


Fig. 9. Layered model of the cross-section discretization (the drawing concept was taken from [59]): a) RC cross-section; b) assumed materials models; c) layered approximation; d) and e) approximated strain and stress distributions.

$$AE = b \sum_{i=1}^n t_i E_i. \quad (3e)$$

Here AE , SE , and IE define the cross-section area and the first and second moments of inertia of the cross-section multiplied by the deformation modulus E_i :

$$E_i = \sigma_i / \varepsilon_i; \quad (4a)$$

$$\varepsilon_i = \kappa(y_i - y_c). \quad (4b)$$

Thus, the assumed material laws (Fig. 9b) define the stresses σ_i corresponding to the estimated strain ε_i ; Eqs. (3a) and (3c) determine the curvature and the neutral axis coordinate. Fig. 10 shows the flowchart of the inverse analysis procedure, employing the layered cross-section model (Fig. 9). The assumed geometry parameters define the model (Fig. 9c); the calculations use the pre-defined materials models for compressive concrete and bar reinforcement; the stress-strain model of the tensile concrete describes the analysis result. The analysis makes no difference between ordinary concrete and SFRC, assuming they are homogeneous materials with specific physical properties.

The load increase from M_1 to M_{max} defines the iterative solution steps. The first iteration assumes the initial deformation modulus ($E_{m,0}$) and determines the curvature using Eq. (3a). Condition 1 in Fig. 10 defines further calculation flow: the analysis repeats while the relative error (i.e., the calculated $\kappa_{calc,m}$ and experimental $\kappa_{exp,m}$ curvature ratio) exceeds the assumed tolerance Δ , and Condition 2 is fulfilled. Satisfying Condition (1) fixes the obtained strain (ε_m) and stress (σ_m) values, and the derived stress-strain model of the tensile concrete further extends by adding this defined pair. Otherwise, the condition $k > N$ skips the iteration results, and the analysis moves to the next load step. The latter reduces the oscillations in the derived stress-strain relationship [59]. Finally, Condition 3 terminates the inverse analysis.

The composition of the initial data set firmly affects the inverse analysis outcomes, i.e., even small curvature increments induce sudden oscillations in the derived constitutive model, sometimes destabilizing the convergence process. Fig. 1b shows an example; the reference [37] presents a more unstable result. Investigating the convergence problems, Gribniak et al. [59] identified the aspects that stabilized the solution. Thus, the zero value determines the initial deformation modulus at every load step in this study, i.e., $E_{m,0} = 0$; the layered section model includes 125 layers, and the tolerance $\Delta = 10^{-5}$. For simplicity, the elastic material laws are used for compressive concrete and bar reinforcement.

Fig. 11 shows the inverse analysis results of the reference B1 beam (Fig. 6b); the calculations employ the modulus of elasticity of concrete and steel bars equal to 32.9 GPa (calculated using Eurocode 2 [52] as the compressive strength function) and 196 GPa (defined experimentally). Fig. 11a shows the obtained stress-strain diagram of the tensile concrete, tracing the solution iterations. Unfortunately, this highly oscillated line is barely acceptable to represent the materials model. Therefore, Gribniak [55] developed the smoothing procedure, which combines stochastic modeling with a modified moving average approximation. Fig. 11b presents the smoothed stress-strain diagram averaging five inverse simulations of stochastically selected moment-curvature data points, generating individual scatterplots. The moving average of all five stress-strain sets described the averaged constitutive model in the studies [29,37,54–56,59]. The thesis [55] describes the inverse analysis procedure in detail and presents the corresponding MATLAB code.

4.2. Simplified analytical model

The sensitivity of the exact inverse analysis procedure to the deformation gradient in the dataset (Fig. 11a) motivated the development of

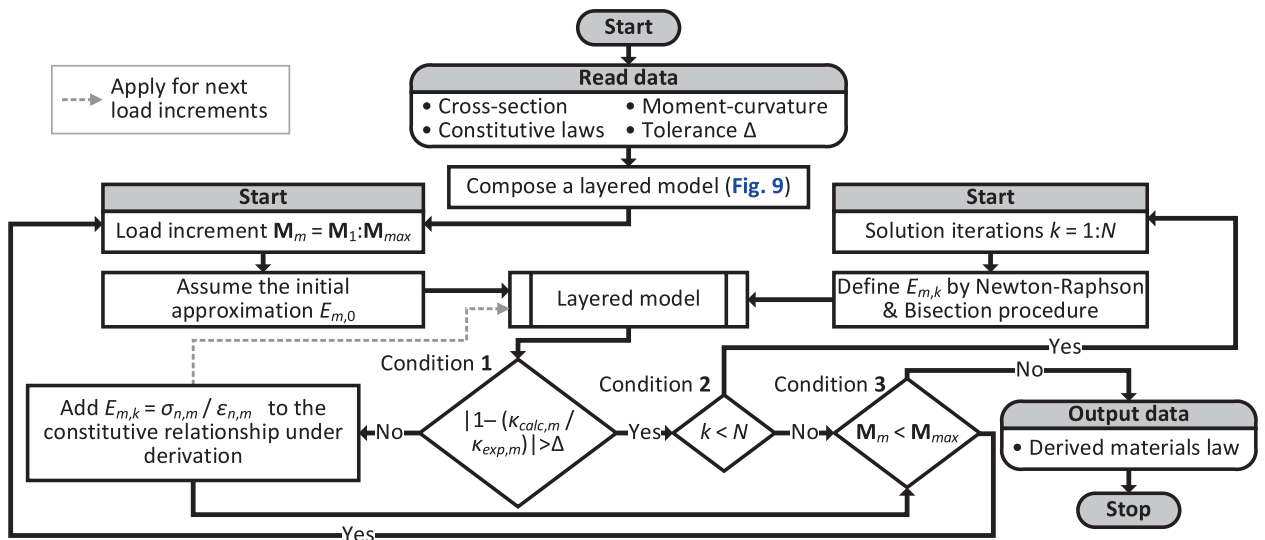


Fig. 10. The flowchart of the inverse analysis procedure (the schematic concept taken from [59]).

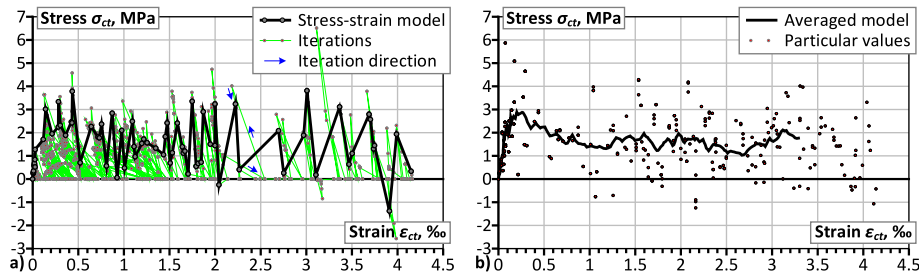


Fig. 11. Inverse analysis results for the B1 beam sample: a) the exact stress–strain diagram with iterations; b) smoothed (averaged) stress–strain model of the tensile concrete.

the simplified analytical model in this study. Similarly to the method described in Section 4.1, the proposed model assumes the concepts of smeared cracking and plane sections. The analytical model also uses elastic laws to describe the material behavior of compressive concrete and bar reinforcement. However, the theoretical concept employs a rectangular model to describe the distribution of stresses in tensile concrete. This simplified assumption ensured a closed-form analytical solution to the inverse problem. A series of recent works [42,50,51] developed the specimen geometry (Fig. 9a), which provides the shape of the tensile stresses' distribution in concrete close to rectangular, which represents a typical approximation of tensile stresses in SFRC, e.g., [12]. Thus, rectangular stress distribution becomes applicable for SFRC and ordinary concrete with bar reinforcement and ensures the implementation of the previous idea [29] of the subtraction of the average stress-average strain diagrams calculated for SFRC RC and RC beam samples for quantifying the fiber effect.

Fig. 12 compares the averaged stress–strain model of the B1 beam sample (Fig. 11b) obtained using the exact inverse analysis procedure and the proposed theoretical simplification to illustrate this approximation feasibility. The relative difference in deformation energy (the area limited by the stress–strain diagram) estimated until the 2‰ strain of the red and black graphs in Fig. 12 is 11.8 %. The dashed line and colored area define the rectangular approximation of the black curve in Fig. 12. The obtained adequacy of the theoretical simplification seems reasonable because both red and black diagrams approximate the oscillated line shown in Fig. 11a. Moreover, the proposed analytical approach substantially reduces the oscillation of the stress–strain graph without any additional approximations.

Fig. 13 schematizes the simplified analytical solution—Fig. 13a shows the cross-section reinforced with three 8 mm steel bars identical to Figs. 2 and 9a; Fig. 13b and 13c illustrate the strain and stress distributions corresponding to the bending moment M_{ext} . In addition, Fig. 13c shows the assumed rectangular approximation of the stress distribution in the tensile concrete—it describes the deformation energy that is equivalent to the exact stress diagram (e.g., the green line in

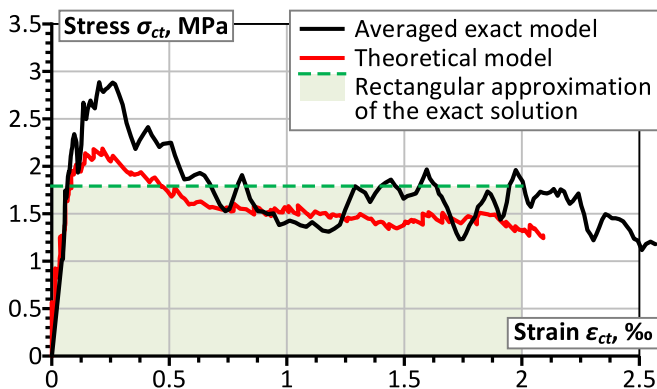


Fig. 12. Comparison of the averaged exact (Fig. 11b) and analytical models.

Fig. 13c). Still, such an approximation simplifies localizing the gravity center of the equivalent rectangular area (Fig. 13d).

According to the proposed theoretical concept, the inverse analysis employs the force equilibrium and strain compatibility conditions. The following equations determine the balance of bending moments (regarding the centroid of the equivalent stress diagram in Fig. 13d) and the internal forces' equilibrium:

$$F_c(d_t^* - d_c) - F_s(d_t^* - d) - M_{ext} = 0, \quad d_t^* = (h + y_c)/2, \quad d_c = y_c/3; \quad (5a)$$

$$F_c - F_s - F_t^* = 0. \quad (5b)$$

Here, F_c and F_s are the forces acting on the compressive concrete and tensile reinforcement; F_t^* is the equivalent force resultant from the tensile concrete; d_t^* and d_c are the distances from the most compressed edge of the cross-section to the gravity centers of the tensile and compressive stress diagrams (Fig. 13d). Fig. 13 defines the remaining notations. The respective stresses express the internal forces as follows

$$\sigma_c \frac{y_c b}{12} (3h + y_c) - \sigma_s A_s \left(\frac{h + y_c}{2} - d \right) - M_{ext} = 0; \quad (6a)$$

$$\sigma_c \frac{y_c b}{2} - \sigma_s A_s - \sigma_t^* (h - y_c) b = 0. \quad (6b)$$

For given curvature, κ , the strain compatibility conditions (Fig. 13b) allow the formulation of the following expressions for the stresses in the most compressed concrete layer and tensile reinforcement (Fig. 13d):

$$\sigma_c = \kappa y_c E_c; \quad (7a)$$

$$\sigma_s = \kappa (d - y_c) E_s. \quad (7b)$$

The following formula results from rearranging Eq. (6a) through accounting to expressions (7):

$$\kappa y_c E_c \frac{y_c b}{12} (3h + y_c) - \kappa (d - y_c) E_s A_s \left(\frac{h + y_c}{2} - d \right) - M_{ext} = 0, \quad (8)$$

representing a third-order polynomial of the neutral axis coordinate (y_c)

$$C_3 y_c^3 + C_2 y_c^2 + C_1 y_c + C_0 = 0, \quad (9)$$

with coefficients

$$C_3 = \kappa E_c b / 12; \quad (10a)$$

$$C_2 = \kappa (2E_s A_s + E_c b h) / 4; \quad (10b)$$

$$C_1 = \kappa E_s A_s (h - 3d) / 4; \quad (10c)$$

$$C_0 = \kappa d^2 E_s A_s \left(1 - \frac{h}{2d} \right) - M_{ext}. \quad (10d)$$

In the above equations, E_c and E_s are the elasticity moduli of concrete and steel; A_s is the reinforcement area (in the considered case, $A_s =$

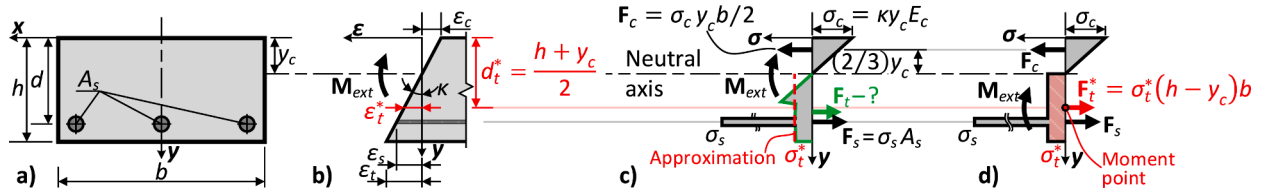


Fig. 13. Analytical simplified model of the pure bending zone: a) cross-section with bar reinforcement; b) and c) strain and stress distribution under bending moment M_{ext} ; d) rectangular stress approximation in tensile concrete.

150.8 mm²). The polynomial (9) has three roots. The following solution comes from the condition $0 < y_c \leq h$:

$$y_c = \frac{1}{3C_3} \left\{ 2\sqrt{C_2^2 - 3C_3C_1} \cos\left(\frac{1}{3}\cos^{-1}\left\{ \frac{-27C_3^2C_0 - 9C_3C_2C_1 + 2C_2^3}{2[C_2^2 - 3C_3C_1]^{1.5}} \right\}\right) - C_2 \right\}. \quad (11)$$

Eq. (6b) defines the equivalent stress in the tensile concrete (σ_t^*) using expressions (7) as follows

$$\sigma_t^* = \kappa \frac{y_c^2 b E_c - 2(d - y_c) E_s A_s}{2(h - y_c)b}. \quad (12)$$

Fig. 13b determines the corresponding strain as

$$\varepsilon_t^* = \kappa(h - y_c)/2. \quad (13)$$

As Fig. 1 schematizes, the equivalent stress σ_t^* of the SFRC RC beam (Fig. 2) combines the tension softening, tension stiffening, and fiber bridging effects. The following section describes the procedure to estimate the fibers' contribution to the residual stiffness of the cracked concrete.

4.3. Estimating the fiber bridging effect

This section implements the previous idea [29] of the subtraction of the average stress-average strain diagrams calculated for SFRC RC and RC beam samples (Fig. 1b) for quantifying the fiber bridging effect using the moment-curvature relationships in Fig. 6b and the simplified theoretical model (Section 4.2). At the first stage, the theoretical model transforms the diagrams in Fig. 6b into the equivalent stress-strain relationships, as Fig. 14a shows. This figure demonstrates the normalized graphs regarding the tensile strength and modulus of elasticity estimated using functional expressions of the compressive strength from Eurocode 2 [52]; Table 1 specifies the estimated materials parameters of the beams.

In the second stage, subtracting the stress-strain diagrams of SFRC and plain concrete beams estimates the fiber bridging effect. The grey lines in Fig. 14a define the averaged response of two nominally identical

Table 1

Estimated materials characteristics of the beam samples.

Beam	B1	B2	B1F	B2F
Tensile strength (f_{ct}), MPa	2.79	2.79	3.21	3.21
Modulus of elasticity (E_{cm}), GPa	32.4	32.4	34.1	34.1
Cracking strain (ε_{cr}), ‰	0.086	0.086	0.094	0.094

specimens, representing the boundaries of the fiber effect zone. Fig. 14b shows the corresponding normalized diagram and a logarithmic approximation of the red dots, describing the difference between the stress-strain diagrams of the fiber-reinforced and plain concretes. The determination coefficient (R^2) indicates that the logarithmic model explains more than 95 % residual strength variation.

Fig. 14b estimates the fiber efficiency, expressed in the residual strength of SFRC (σ_f) terms, similar to the graph shown in Fig. 1c. To illustrate this similarity, Fig. 14b also includes the residual strength estimation results from the previous study [29]—green dots correspond to the full-scale beam's test results described in Section 3.3. Excepting relatively small deformations (from two to six cracking strains, ε_{cr}), the residual strength estimation results of the small (standardized) and full-scale beam samples demonstrate remarkable agreement. This resemblance motivates the modeling example in Section 4.4 when the standardized beam samples estimate the fiber bridging effect (i.e., the red line in Fig. 14b) and define the residual strength relationship to model the full-scale beams tested in the previous study [29].

4.4. Modeling example

This section considers the test outcomes discussed in Section 3.3. Fig. 15 shows the normalized stress-strain diagrams obtained from the results of Fig. 8 using the exact inverse analysis procedure described in Section 4.1. These stress-strain relationships were derived by eliminating the shrinkage effect realized in RC elements before applying the mechanical load. However, the description of the shrinkage elimination procedure is irrelevant to the topic considered in this study because of keeping the small-size beam samples (Fig. 2) in water tanks. Therefore, this article does not include a description of this elaborate analysis procedure—an interested expert can find the details in the literature

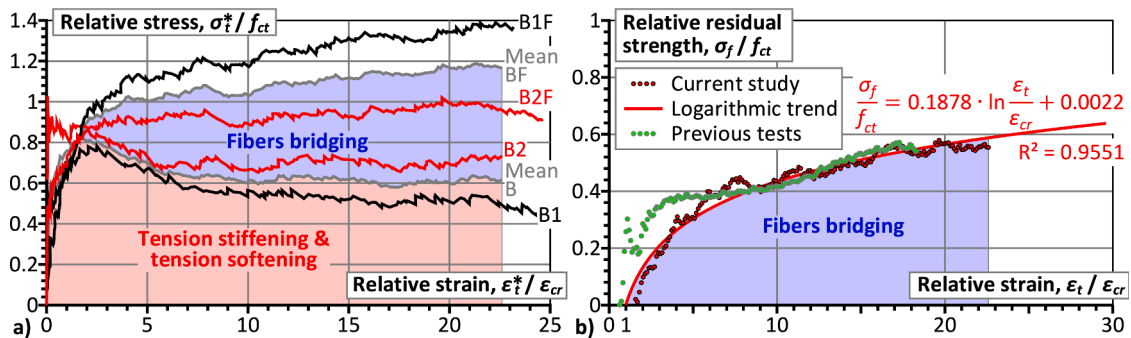


Fig. 14. Residual strength analysis: a) normalized equivalent stress diagrams of the standardized beam samples (Fig. 6b); b) logarithmic model of the fiber bridging effect.

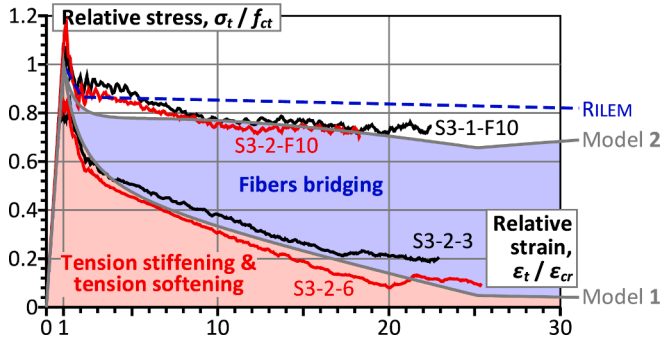


Fig. 15. Stress-strain relationships of the full-scale beams from Fig. 8.

[29,37,54,55,60].

Fig. 15 also includes three analytical models of tensile concrete. The first two simplifications correspond to the proposed modeling approach depicted in Figs. 1 and 14. Model 1 combines the tension stiffening and tension softening responses characteristic of RC beams without fibers. The following analytical expressions define Model 1 [29]:

$$\frac{\sigma_t}{f_{ct}} = \begin{cases} \varepsilon_t / \varepsilon_{cr}, & \varepsilon_t / \varepsilon_{cr} \leq 1; \\ \mathcal{B} + (1 - \mathcal{B}) \frac{\varepsilon_{cr}}{\varepsilon_t}, & \varepsilon_t / \varepsilon_{cr} > 1, \end{cases} \quad (14)$$

with parameter $\mathcal{B}$, expressed as

$$\mathcal{B} = \frac{1}{2(1 + \exp 4)} + 0.4 \cdot \max\left(0; \frac{\beta - [\varepsilon_t / \varepsilon_{cr}]}{\beta - 1}\right). \quad (15)$$

In the above equation, the factor β describes the ultimate relative strains ($\varepsilon_{t,u}$) at disappearing the tension-stiffening effect. According to [58], the factor β depends on the reinforcement percentage (ρ) as

$$\beta = \frac{\varepsilon_{t,u}}{\varepsilon_{cr}} = \max(5; 7.12 \cdot \rho^2 - 27.6 \cdot \rho + 32.8). \quad (16)$$

For rectangular cross-section (e.g., Fig. 9a), the following formula defines the ratio ρ :

$$\rho = A_s / (b \cdot d) \times 100\%. \quad (17)$$

In Fig. 15, Model 2 represents a conjunction of Model 1 (Eq. (14)) and the proposed logarithmic expression describing the fiber bridging effect (Fig. 14b). This figure demonstrates a good agreement between the stress-strain diagrams of the SFRC beams S3-1-F10 and S3-2-F10 and Model 2. Fig. 15 shows that the proposed modeling approach can adequately account for the tension stiffening effect (combined with tension softening) and fiber bridging contribution, determining the material model of SFRC elements with bar reinforcement.

To illustrate the structural analysis adequacy, the stress-strain models shown in Fig. 15 were used as constitutive laws for tensile

concrete (Model 1) and SFRC (Model 2) to simulate the moment-curvature response of the full-scale beams (Fig. 8). This analysis employs the layered cross-section model (Fig. 9) implemented in the thesis [55] using a MATLAB code because of its flexibility in specifying constitutive models of materials. Fig. 16a shows the calculation results, revealing the remarkable adequacy of the predictions. In this context, the accurate predictions of the plain concrete beams ($V_f = 0$) are not surprising because of the tailoring tension stiffening model in the reference [29]. However, the perfect capturing of the SFRC beams' deformations proves the correctness of the proposed SFRC modeling concept.

Remarkably, as mentioned at the beginning of this section, the stress-strain diagrams in Fig. 15 were determined by eliminating the shrinkage effect. This elimination improves the constitutive modeling adequacy [55,61] but requires assessing the shrinkage impact in the structural analysis. Equivalent internal force and bending moment can simulate the concrete shrinkage effect restrained by the bar reinforcement [61]. Since the shrinkage affects only the tension-stiffening model and does not alter the fiber bridging model (Fig. 14b), this manuscript skips the shrinkage simulation explanations—the interested reader can find all the necessary information in the literature references [37,50,60,61].

As an alternative to the proposed modeling concept, Fig. 15 also includes the SFRC model developed using the results of Fig. 7b and RILEM methodology [8]—the blue dashed line determines the model constructed for the full-scale beams (using the size factor $k_h = 0.779$). The agreement between these two models of SFRC RC (i.e., RILEM and Model 2) is noticeable. The essential difference between the modeling concepts is that the RILEM concept does not account for the reinforcement ratio effect on the tension stiffening behavior. In other words, the RILEM offers the same model (i.e., Fig. 15) for all beams tested within the framework of the study [29]. However, as the reference [29] revealed, neglecting the reinforcement ratio effect contradicts the experimental observations. Numerous literature sources also proved this dependency (e.g., [25,33,49,54,55,58,61]), which does not raise any doubts. Thus, neglecting the bar reinforcement effect defines a fundamental drawback of the RILEM model [8].

Furthermore, the stress-strain diagrams in Fig. 15 were defined as eliminating the shrinkage effect that requires accounting for the shrinkage in the structural analysis. Fig. 16b illustrates this situation when the solid lines describe the moment-curvature diagrams calculated using the RILEM model (Fig. 15) and neglecting the shrinkage effect. Such modeling is typical for engineering analysis of structural elements subjected to short-term loading and represents the practical simulation results using the RILEM model. The dashed lines in Fig. 16b situate the hypothetical case, which accounts for the shrinkage deformations realized before the mechanical load. The modeling results' improvement is apparent, confirming the shrinkage modeling results reported in the literature [54,55,60].

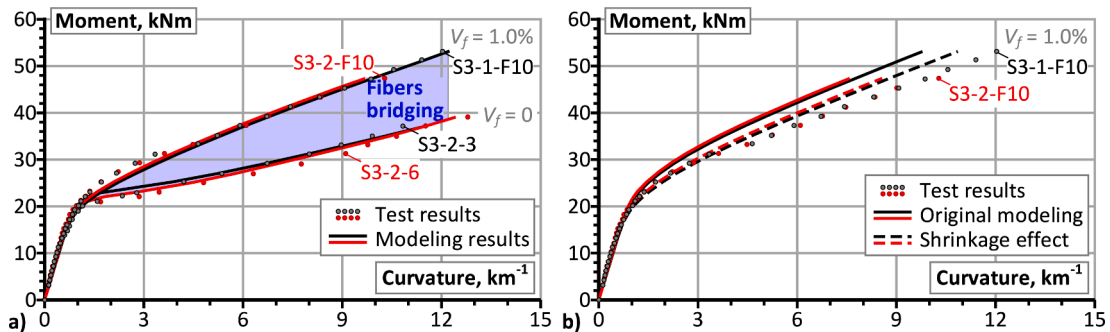


Fig. 16. Moment-curvature modeling results of the full-scale beams reported in [29] using different models of the tensile concrete: a) Model 1 (fibers volume fraction, $V_f = 0$) and Model 2 ($V_f = 1.0\%$) predictions; b) RILEM [8] simulations accounting for the shrinkage effect.

4.5. Peculiarities of the proposed modeling concept

This study employs standardized beam samples with tailored geometry that ensures multiple cracks form in relatively small laboratory specimens (Fig. 5), reducing the scatter of the residual strength estimation results [42]. The specimen geometry (Fig. 9a) allows reaching the distribution shape of the tensile stresses in concrete close to rectangular, which is a typical approximation for SFRC. This stresses' conformity allows subtracting the stress-strain diagrams determined for SFRC RC and RC beams and, thus, quantifying the fiber bridging effect. The revealed outcomes allow identifying the following essential aspects:

- The proposed simplified model (Section 4.2) does not explicitly define a material model of the concrete. Instead, Eq. (12) establishes a measure of the equivalent residual strength of reinforced material. The present study uses this measure to estimate the fiber bridging effect, and the numerical verification (Fig. 16a) proved the adequacy of the mechanical resistance components' (corresponding to the tension softening, tension stiffening, and fiber bridging effects) superposition hypothesis raised in [29]. Thus, FE simulations of SFRC RC elements could apply fiber bridging models obtained using test results of the standardized laboratory specimens.
- The above simplification considers the averaged behavior of the tensile concrete (Fig. 13d), smoothing the stress diagram at the earlier cracking stage (compare the exact and simplified stress-strain models in Fig. 12). This model could cause some errors (underestimation) in determining the fiber bridging effect as shown in Fig. 14b. However, for the deformations exceeding $6\epsilon_{cr}$, the residual strength assessments are adequate. Moreover, the deformation analysis corresponding to the crack formation stage is not essential in engineering applications (e.g., Fig. 16a), typically focused on the stabilized cracking stage [46].
- The comparative analysis of the proposed and RILEM modeling concepts (Figs. 15 and 16b) reveals the adequacy of the developed simplified model. At the same time, contrary to RILEM, this technique ensures adequately evaluating tension stiffening (RC feature) and fiber bridging (SFRC feature) effects.
- The analytical results, employing the closed-form solution to the residual strength problem (Section 4.2), do not require any additional smoothing (e.g., Fig. 11b) that makes the prediction confidence comparable to the precision of the monitoring devices. Still, the stress-strain diagrams in Fig. 14a demonstrate substantial scatter, varying from 7 % to 35 % for deformations exceeding $6\epsilon_{cr}$. The RILEM tests (Fig. 7a) show a comparable scatter, which ensures associating this spread with the stochastic nature of the concrete cracking [1,62]. A sufficient number of test samples will estimate this systematic uncertainty [63].
- The proposed analytical solution (Section 4.2) requires neither iterative calculations nor the loading history description. Thus, it can quantify the residual stiffness of the samples subjected to cyclic and repeated loads, e.g., [42,50,64]. The testing procedure can also be extended to estimate the effects of unfavorable environmental conditions [65,66] and elevated temperatures [67] by repeatedly testing the beam samples (Fig. 2) before and after exposing them to adverse conditions.

5. Conclusions

This study investigates the fiber bridging effect in SFRC beams with bar reinforcement. It employs standardized beam samples for laboratory testing. The analysis hypothesizes the possibility of separating the mechanical resistance components, corresponding to tension stiffening and fiber bridging effects. The plain concrete and SFRC beams' tests illustrate the proposed analysis concept. The following essential conclusions came from this study:

- The proposed simplified analytical model does not explicitly define a material law and represents an equivalent measure of the residual strength of reinforced material. This straightforward theoretical approach estimates the fiber bridging effect, subtracting the residual strength values of the standardized SFRC and plain concrete beams. The comparison with the exact solution to the inverse problem, RILEM modeling results, and full-scale beams' simulation proved the hypothesized superposition of the tension stiffening and fiber bridging components.
- The proposed modeling concept adequately represents the structural behavior of full-scale SFRC beams with bar reinforcement. This study developed the SFRC model suitable for hooked-end steel fibers with 1% reinforcement volume. Further model development requires an extended testing program. Still, standardized geometry, testing procedure, and simplified analytical approach are adequate for modeling the fiber effect in full-scale RC structural elements.
- The RILEM model of SFRC beams ignores the bar reinforcement effect on tension stiffening, contradicting experimental results reported in the literature and limiting the model's versatility.
- The proposed analytical model determines the closed-form solution to the residual strength problem, which ensures the analysis of the elements subjected to cyclic and repeated loads with prediction confidence comparable to the precision of the monitoring devices.

CRedit authorship contribution statement

Viktor Gribniak: Methodology, Conceptualization, Software, Supervision, Resources, Validation, Formal analysis, Visualization, Writing - original draft, Writing - review & editing. **Aleksandr Sokolov:** Investigation, Visualization, Formal analysis, Data curation, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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