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Procedia Engineering 172 (2017) 1286 – 1290

Procedia Engineering

www.elsevier.com/locate/procedia

Modern Building Materials, Structures and Techniques, MBMST 2016

Improvement of Viscoelastic Damping for the Hertz Contact of Particles due to impact velocity

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Abstract

One of the most important task in application of the Discrete Element Method (DEM) to simulation of geomaterials is the adequacy of description of contact between interacting mineral particles. In most of the cases the elastic force is expressed by using Hertz model, while description of viscoelastic damping force is still under discussion. Recently various damping models described by different values of overlap power factor are used in DEM calculations. The general form of viscoelastic damping constant was presented, where well-known damping models present particular cases of general relationships. It known no one already existing models can precisely represent experimental results in most of the cases. In this paper general damping model comprising several existing approaches is developed. Here displacement power factor is evaluated on the basis of experiment results. It was found that the proposed approach gives more accurate simulation results of coefficient of restitution in comparison to already existing damping models.

Peer-review under responsibility of the organizing committee of MBMST 2016

Keywords: DEM, Viscoelastic, Damping, Particles; Model, Coefficient of restitution.

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1. Introduction

Numerical simulation of soils and various geomaterials is an important investigative tool, successfully explored in many areas of civil and geotechnical engineering. On the microscale, the soil subjected to external forces is much discrete and discontinuous, therefore, the particle-based DEM is an ideal method for analyzing mechanical behaviors

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Peer-review under responsibility of the organizing committee of MBMST 2016
doi:10.1016/j.proeng.2017.02.156

[1]. Mechanical state of the solid is defined by dynamic motion and contact behavior of individual particles [2,3,4,5]. Among several effects, the viscoelastic damping is important factor adjusted to reflect the dissipations of energy during collision between soil particles [6,7].

Typically, the elastic properties of contacting particles are well studied and can be justified experimentally. However, the damping properties are more complicated. Traditionally, the viscoelastic damping force is proportional to displacement rate and is characterized by a nonlinear damping parameter C. It is commonly agreed that this integral constant depends on various factors, where the particles overlap $h(t)$ is of major importance. Recently the mostly applied viscoelastic DEM models are classified with respect to a factor β of phenomenological origin, which is the power factor of the particles overlaps $h(t)$. Various authors suggested different damping models having different values of power factor β. Most popular viscoelastic damping models like Kuwabara and Kono [8] (Brilliantov et al. [9]), Tsuji [10], Lee and Hermann [11] and Hu [12] have following power factor β: 1/2, 1/4, 0 and 3/2.

On the other hand, it is well known and experimentally confirmed that for the many materials the coefficient of restitution (COR) and the impact velocity are coherent with each other [13,14,15,16]. It leads to indirect relationship between the impact velocity and the damping parameter C. The most appropriate way to simulate contact energy dissipation is to make experiments to describe contact of the single particle and to determinate the values of COR at the different impact velocities of a particle. A comparative evaluation of normal viscoelastic contact forces of some models, Kuwabara and Kono [8] (Brilliantov et al. [9]), Tsuji [10], Lee and Hermann [11] and Hu [12] is already presented in [17,18].

The further investigation of the existing damping models used in DEM has shown that these models can be used only in the case when during a contact under consideration two interacting bodies interact only with each other. That is, no interaction exists with other bodies during the contact under investigation. In the case when during a contact under consideration two interacting bodies interact not only with each other, but with other bodies, then the overlap $h(t)$ and its velocity $dh(t)/dt$ is determined not only by the damping properties of the bodies of the contact under consideration but also by the other bodies that interact with that two bodies that contact is considered. In this case, the initial contact velocity V_0 is just fictitious parameter insufficient for determining the contact of a multiparticle system. Therefore, for multiparticle systems it is reasonable to take the initial contact velocity V_0 as a constant for all particles and for all contacts. And to make the nonlinear damping parameter C to be dependent only on overlap $h(t)$ raised by the power factor β that is a priori unknown during the simulation of a mutiparticle system. In this case the new damping parameter C can be treated as a function of one variable in contrast to the existing damping parameters C that depends not only on $h(t)$ but also on the initial contact velocity V_0.

In this paper an idea of the new damping parameter C depending only on the overlap $h(t)$ is demonstrated by comparing calculated COR by using new C with experimental results of impact of marble particles.

2. Discrete element approach

The problem of viscoelastic damping is considered in a particle scale. In the present article, for the sake of simplicity, only a one-dimensional normal interaction is considered. The normal contact between two spherical particles is characterized by the time-dependent normal inter-particle displacement $h(t)$, which is called hereafter as overlap, and the total normal contact force $F(h(t)) = F_{el}(h(t)) + F_d(h(t))$, where the forces $F_{el}(h(t))$ and $F_d(h(t))$ can be attributed to the elastic and damping interactions, respectively. In DEM, the Hertz contact solution is widely applied for the $F_{el}(t)$, i.e. $F_{el}(t) = k_n h(t)^{3/2}$, while $F_d(t) = C(h(t))dh(t)/dt$. Here k_n is stiffness coefficient of the Hertz law (given in) and $C(h(t))$ is so called damping parameter. Combining the total normal contact force $F(h(t))$ with the Newton second law $F(t)/m = d^2h(t)/dt^2$, where m is mass of a particle, we obtain the following equation for the motion of a particle during a contact:

$$\frac{d^2h(t)}{dt^2}+\frac{k_n}{m_{eff}}\cdot\left(h(t)\right)^{3/2}+\frac{C\left(h(t)\right)}{m_{eff}}\cdot\frac{dh(t)}{dt}=0\,, \qquad (1)$$

where $k_n = 4/3E_{eff}\sqrt{R_{eff}}$, $R_{eff} = R_iR_j / (R_i + R_j)$ and $E_{eff} = E_iE_j / (E_i(1-\mu_j^2) + E_j(1-\mu_i^2))$, $m_{eff} = (m_im_j)/(m_i + m_j)$ are effective radius, modulus of elasticity and mass, respectively; in these formulae $R_i, R_j, E_i, E_j, \mu_i, \mu_j, m_i$ and m_j are radii, modulus of elasticity, Poison’s ratios and masses of particles i and j that are in contact, respectively. It should be noted, that in eq. (1) and hereafter in the article the indices i, j and ij are omitted.

3. Improvement of damping model by experiment data

By applying dimensional analysis, it was shown in [7] that the damping parameter C for all damping models can be expressed by one common equation:

$$C\left(h(t)\right) = c m_{eff}^{(3-2\beta)/5} V_0^{(1-4\beta)/5} k_n^{(2+2\beta)/5} h^{\beta}(t)\,, \tag{2}$$

where β is the power factor of the overlap $h(t)$ that depends on selected damping model. In eq. (2), c is a nondimensional coefficient that can be treated as a function depending on COR and is unique for all damping models containing different β. In eq. (2) V_0 is the initial contact velocity used only for eq. (2) and this V_0 not always coincides with the real initial contact velocity $V_{0,real}$.

For easier comparison of calculated and experimental COR experimental COR is approximated by a function $e_{exp}(V_{0,real})$ that depends on the real initial contact velocity $V_{0,real}$. The algorithm is based on determination of the calculated COR function $e_{calc}(V_{0,real})$ that is obtained by using eq. (2) and by modeling a collision of single particle whose experimental COR on $V_{0,real}$ is determined experimentally. The aim of the algorithm is to find such value of the power factor β of eq. (2) that the calculated COR $e_{calc}(V_{0,real})$ would be close enough to the experimental COR $e_{exp}(V_{0,real})$.

If function $c(e(V_0))$ is known, then the new damping parameter at another real value of the initial contact velocity $V_{0,real}$ can be obtained by the following function:

$$C\left(h^{\beta}\right) = c \cdot \left(\frac{V_{0,real}}{V_0}\right)^{(1-4\beta)/5} m_{eff}^{(3-2\beta)/5} V_0^{(1-4\beta)/5} k_n^{(2+2\beta)/5} \bar{h}^{\beta}\,. \tag{3}$$

In this case, it is possible to state that the real non dimensional damping coefficient at $V_{0,real}$ is:

$$c_{real}\left(V_{0,real}\right) = c\left(e\left(V_0\right)\right) \cdot \left(\frac{V_{0,real}}{V_0}\right)^{(1-4\beta)/5}\,. \tag{4}$$

Taking into account all mentioned features, the most appropriate value of the power factor β can be found by least squares

$$\min \rightarrow \left[\frac{1}{V_{0,real,2} - V_{0,real,1}} \int_{V_{0,real,1}}^{V_{0,real,2}} \left[e_{exp}(V_{0,real}) - e_{calc}(V_{0,real})\right]^2 dV_{0,real}\right]^{1/2} \tag{5}$$

where $V_{0,real,1}$ and $V_{0,real,2}$ are minimum and maximum of the initial contact velocities that can occur during the simulation. In order to simplified the algorithm and to reduce computational time, in eqs. (2), (3) and (4) $V_0 = (V_{0,real,1} + V_{0,real,2})/2$.

To find $c(e(V_0))$ is a quite complicated task since there is not any analytical way to solve eq. (1) when $\beta \neq 1/4$, therefore the most accurate, but also the most time demanding approach is to simulate particles motion and to find a dependency between c and e for every β.

4. Algorithm application

The developed model was tested with the experiment results provided by Goldsmith [5] to illustrate its efficiency. Properties of marble spheres are following: diameter $d = 0.04$ m, density $\rho = 2900$ kg/m^3, Young's modulus $E = 71$ GPa and Poisson's ratio $\mu = 0.3$. Experimentally obtained COR, [5], were approximated by following function: $e_{exp}(V_{0,real}) = 0.7226 - 0.031 ln(V_{0,real})$. These experimental results and their approximation $e_{exp}(V_{0,real})$ are shown in Fig. 1.

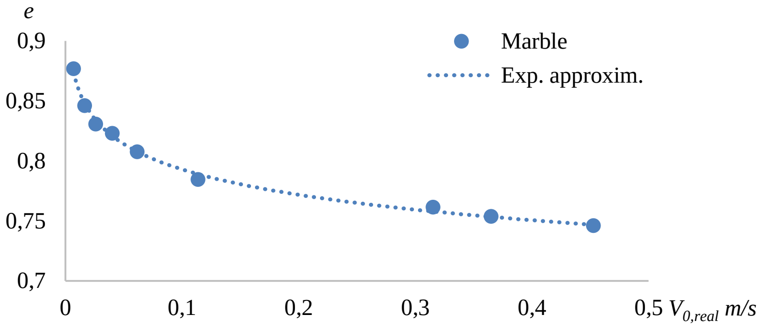

Fig. 1. Coefficient of restitution versus initial contact velocity $V_{0,real}$.

After algorithm adaptation, it was found that eq. (3) with $\beta = 2/5$ gives more accurate simulation results than well-known already existing damping models, see Fig. 2. In this case $V_0 = 0.25$ m/s.

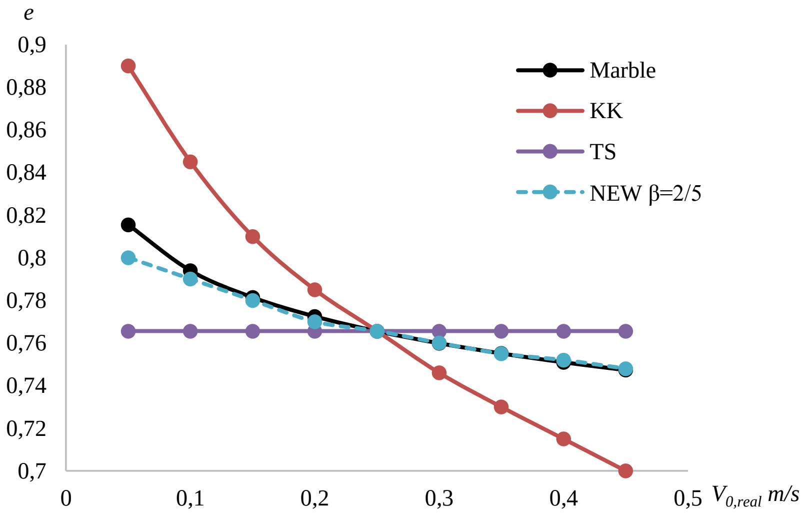

Fig. 2. Dependencies of coefficients of restitutions on initial contact velocity $V_{0,real}$.

Most appropriate known damping models like Tsuji (TS) and Kuwabara and Kono (KK) also were applied for comparison purpose with $V_0 = 0.25$ m/s and $e(V_0) = 0.766$ for all the models.

5. Conclusions

The proposed approach enables us to obtain only overlap depending damping parameter *C* that gives more accurate calculated coefficient of restitution with respect to the initial contact velocity in comparison to the already existing damping models.

Validity of the modification experiment-based of viscoelastic damping model for the Hertz Contact of spherical particles was proved by considering results of available experiment with marble particles. It was found that the power factor’s value of overlap equal to 2/5 yields much smaller error compared to widely used models.

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