

Experimental investigation of dynamic impact of firearm with suppressor

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Abstract: The internal ballistics processes occur in the tube during firearm firing. They cause tremendous vibratory shock forces and robust sounds. The determination of these dynamic parameters is relevant in order to reasonably estimate the firearm ergonomic and noise reduction features. The objective of this study is to improve the reliability of the results of measuring a firearm suppressor's dynamic parameters. The analysis of indicator stability is based on an assessment of dynamic parameters and setting the correlation during experimental research. An examination of the spread of intensity of firearm with suppressor dynamic vibration and an analysis of its signals upon applying the theory of covariance functions are carried out in this paper. The results of measuring the intensity of vibrations in fixed points of a firearm and a shooter have been recorded on a time scale in the form of data arrays (matrices). The estimates of covariance functions between the arrays of digital results in measuring the intensity of firearm vibrations and the estimates of covariance functions of single arrays have been calculated upon changing the quantization interval on the time scale. Software Matlab 7 has been applied in the calculation. Finally, basic conclusions are given.

Keywords: Ergonomics; Vibration; Covariance function; Cross-covariance function; Quantization interval

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1. Introduction

The assessment of firearms' ergonomic safety features is relevant to the production, estimation, usage and periodic control. The internal ballistics processes occur in the tube during firing. They cause tremendous vibratory explosive forces and sharp sounds [1]. These dynamic forces hit on the shooter's body (usually the shoulder), making a recoil from the butt, and the deafening sound harmfully affects the human sensory organs. Different firearm suppressors can be applied to reduce the hazardous impact on human health caused by firearm shooting processes. Parameters of firearms and their accessories (sights, suppressors, etc.) are

research objects of many studies worldwide, as an evaluation of their precision and reliability is required by state authorities of weapon supervision. Modern suppressors of firearms are vastly superior to ear-level protection and the only available form of suppression capable of making certain sporting arms safe for hearing [2]. All firearm suppressors offered significantly greater noise reduction than ear-level protection, usually greater than 50%. Noise reduction of all ear-level protectors is unable to reduce the impulse pressure below 140 dB for certain common firearms.

According to scientists' [3] knowledge, an individual's ability to inhibit their motor response of shooting when a non-target is presented has not been investigated. This phenomenon has been modelled empirically using the sustained attention to response task (SART) computer task.

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In the current investigations, the authors further investigate the SART using a simulated small arms scenario to test whether a lack of motor response inhibition can be modelled in a more ecologically valid environment.

Surface metrology is commonly used to characterise functional engineering, for example, firearm barrels and surfaces [4]. The technologies developed offer opportunities to improve forensic tool-mark identification. Tool-marks are commonly found on fired bullets and cartridge cases. Objectivity was improved through measurement of surface topography and an application of unambiguous surface similarity metrics, such as the maximum value of the areal cross-correlation function.

Six pre-processing algorithms for the detection of firearm gunshots were statistically evaluated [5], using the receiver-operating characteristic method as a previous feasibility metric for their implementation on a low-power very large scale integration (VLSI) circuit. Circuits were intended to serve as the input detection sensors of a low-power environmental surveillance network. Research results indicated that the use of wavelet bank filters, either discrete or continuous, might be the best choice in terms of the compromise between detection efficiency and the power requirements of the intended application.

Information obtained from recorded shooting signals can be useful for law enforcement agencies and defence forces [6]. Direction of arrival estimation of a gunshot is an important issue in shooter localisation. As the distance between the firing position and the sensor increases, the signal to noise ratio decreases and this estimation degrades. In that case, some sort of pre-processing of the firing acoustic signal before the application of the direction of arrival estimation algorithms becomes necessary. The application of spectral subtraction to enhance the signal improves the results. The tests were carried out with simulated directions by delaying copies of a real shooting signal, also with real gunfire signals recorded by a microphone array.

Estimates of the covariance functions of digital arrays of two vibration signals or the covariance functions of a single array have been calculated upon transformation of the digital arrays into vectors. At the end of the 20th century, scientists described discrete Fourier transformation for processing the digital signals [7, 8]. Several researchers [9–12] applied wavelet or small wave theory of wavelet functions for the processing of digital images. The geometric corrections with first-degree polynomials, using either GPS-derived points or topographic map-derived points, yielded RMS error values on the order of +35 m for all three types of satellite image regardless of pixel size [13, 14].

Lithuanian researchers carried out an analysis of the spread of vibration intensity of mechanical objects and its

parameters upon application of the theory of covariance functions [15–17]. American scientists Dutkay and Jorgensen developed a Hilbert-space framework for a number of general multiscale problems from dynamics [18]. They identified a spectral theory for a class of systems based on iterations of a non-invertible endomorphism. They were motivated by the more familiar approach to wavelet theory, which starts with the two-to-one endomorphism. Italian scientist Cattani studied the Shannon wavelets together with their differential properties as connection coefficients [19]. Cattani revealed that the Shannon sampling theorem could be considered in a more general approach suitable for analysing functions ranging in multi-frequency bands. This generalisation coincides with the Shannon wavelet reconstruction of functions. The Shannon wavelets are—functions, and any of their order derivatives could be analytically defined by some kind of a finite hyper-geometric series. These coefficients make it possible to define the wavelet reconstruction of the derivatives of the functions. The assessment of firearms suppression equipment efficiency was measured in three dynamic parameters: acceleration of butt, a force against the shooter's shoulder and the sound pressure of the shot. The scientific paper analyses the correlation between them. It is most convenient to measure the sound pressure of the shot and vibrating acceleration of the firearm butt.

The research group of VGTU carried out an analysis of the estimates of digital signals of a firearm with suppressor vibration parameters upon application of the theory of random functions. This theoretical model is based on the conception of a stationary random function taking into account that errors of measuring the vibration parameters are random and of the same accuracy, i.e. the average error $M\Delta = \text{const} \rightarrow 0$, their dispersion $D\Delta = \text{const}$. Covariance function of digital signals depends only on the difference between the arguments, i.e. on the quantization interval on the time scale.

2. A covariance model of firearm with suppressor vibration signal parameters

The object of the study is the firearm with a suppressor (see Fig. 1). For a theoretical model, we assumed that errors in measuring the intensity of digital signals of a firearm vibration field are random. In each column of the array of measuring the intensity of the vibration field, the trend of the measuring data of the column is eliminated.

The research group of VGTU considered the random function (formed according to the arrays of measuring the intensity of the vibration field) a stationary function (in a broad sense), i.e. its average value $M\{\varphi(t)\} \rightarrow \text{const}$, and covariance function depends only on the difference of



Fig. 1 A firearm with a suppressor system

arguments— $K_\varphi(\tau)$. The auto-covariance function of a single data array or the cross-covariance function of two data arrays $K_\varphi(\tau)$ shall be written as follows [14]:

$$K_\varphi(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} \delta\varphi_1(u)\delta\varphi_2(u+\tau)du, \quad (1)$$

where $\delta\varphi_1(u), \delta\varphi_1(u+\tau)$ are the centred values of vibration intensity measurements, u is the vibration parameter, $\tau = k \cdot \Delta$ the variable quantization interval, k the number of units of measurement, Δ the value of a unit of measurement, and T time.

According to the available data on measurements of vibration parameters, the estimate $K'_\varphi(\tau)$ of covariance function is calculated as follows:

$$K'_\varphi(\tau) = K'_\varphi(k) = \frac{1}{n-k} \sum_{i=1}^{n-k} \delta\varphi_1(u_i)\delta\varphi_2(u_{i+k}), \quad (2)$$

Where n is the total number of discrete intervals.

Formula (2) may be applied in a form of an auto-covariance function or a cross-covariance function. When the function is an auto-covariance function, the arrays $\varphi_1(u)$ and $\varphi_2(u+\tau)$ are parts of single arrays, and when the function is a cross-covariance function, they are two different arrays.

The estimate of a normed covariance function is

$$R'_\varphi(k) = \frac{K'_\varphi(k)}{K'_\varphi(0)} = \frac{K'_\varphi(k)}{\sigma_\varphi^2}, \quad (3)$$

where σ'_φ is the estimate of the standard deviation of a random function.

For an elimination of the trends of columns in the i -th digital array of measurements, the following formulas are applied:

$$\delta\varphi_i = \varphi_i - e \cdot \varphi_i^{-T} = (\delta\varphi_{i1}, \dots, \delta\varphi_{im}), \quad (4)$$

where $\delta\varphi_i$ is the i -th digital array of reduced values where a trend of column is eliminated; φ_i is the i -th array of the vibration intensity, e a unit vector with the sizes $(n \times 1)$; n the number of lines in the i -th array, $\bar{\varphi}_i$ the vector of average values of columns in the i -th array, $\delta\varphi_{ij}$ the j -th column (vector) of the reduced values in the i -th array.

The vector of average values of columns in the i -th array is calculated according to the following formula:

$$\bar{\varphi}_i^T = \frac{1}{n} e^T \cdot \varphi_i = \frac{1}{n} \varphi_i^T \cdot e. \quad (5)$$

A realisation of the random function of the j -th column of the i -th array of vibration intensity in the form of vectors shall be expressed as follows:

$$\delta\varphi_{ij} = (\delta\varphi_{ij,1}, \dots, \delta\varphi_{ij,m}). \quad (6)$$

The estimate of the covariance matrix of the i -th array of wavelet intensity is expressed as follows:

$$K'(\delta\varphi_i) = \frac{1}{n-1} \delta\varphi_i^T \delta\varphi_i. \quad (7)$$

The estimate of covariance matrix of two arrays of vibration intensity is written as follows:

$$K'(\delta\varphi_i, \delta\varphi_j) = \frac{1}{n-1} \delta\varphi_i^T \delta\varphi_j, \quad (8)$$

where the sizes of $\delta\varphi_i$, $\delta\varphi_j$ arrays should be the same.

The estimates $K'(\delta\varphi_i)$ and $K'(\delta\varphi_i, \delta\varphi_j)$ of covariance matrices are reduced into estimates of matrices of correlation coefficients $R'(\delta\varphi_i)$ and $R'(\delta\varphi_i, \delta\varphi_j)$ [13, 14]:

$$R'(\delta\varphi_i) = D_i^{-1/2} K'(\delta\varphi_i) D_i^{-1/2}, \quad (9)$$

$$R'(\delta\varphi_i, \delta\varphi_j) = D_{ij}^{-1/2} K'(\delta\varphi_i, \delta\varphi_j) D_{ij}^{-1/2}, \quad (10)$$

where D_i , D_{ij} are the diagonal matrices of members of principal diagonals in the estimates of covariance matrices $K'(\delta\varphi_i)$ and $K'(\delta\varphi_i, \delta\varphi_j)$, respectively.

The accuracy of the calculated coefficients of correlation is defined by the standard deviation σ_r , and the value of the latter is assessed according to the following formula:

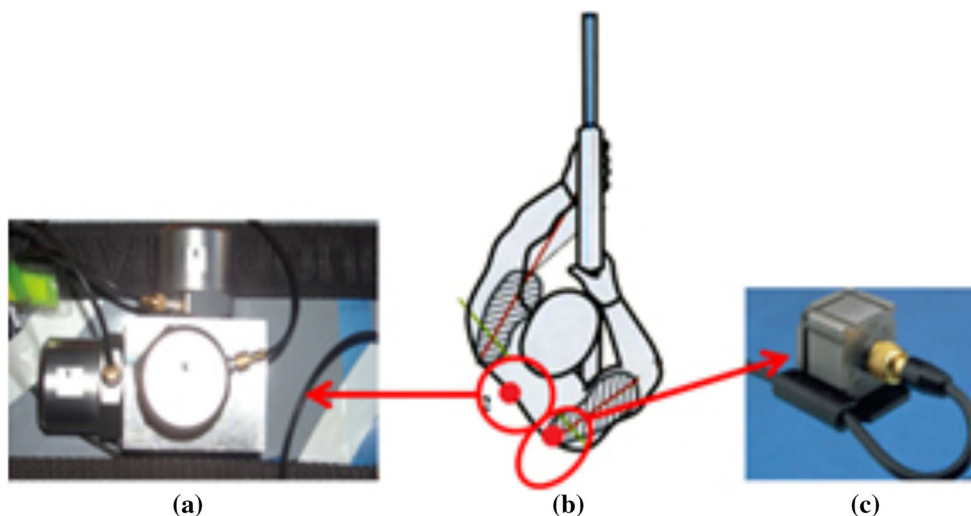
$$\sigma_r = \frac{1}{\sqrt{n}} (1 - r^2), \quad (11)$$

where $n = 8,000$, r is the coefficient of correlation. The maximum value of the standard deviation is obtained when the value of r is close to zero, and in this case, $\sigma'_r \approx 0.01$, when $r \approx 0.5$, we obtain $\sigma'_r \approx 0.008$.

3. The results of the experiment and the analysis

The firearm with suppressor used in the study is shown in Fig. 1(a); the suppressor in Fig. 1(b); and the magnified image of the suppressor in Fig. 1(c).

Fig. 2 The measurement means used for measuring the shooter's vibrations



To measure the parameters of vibrations, force and acoustic pressure, the following devices from 'Brüel and Kjer' were used: three accelerometers 8344 (Fig. 2(a)) and a triaxial accelerometer 4506 (Fig. 2(c)). The accelerometers 8344 were fixed to the sensor-fixing block. The block, in its turn, was fixed to the plate put on the back of the person involved in shooting (Fig. 2(b); the point 2). The accelerometer 4506 was fixed onto the shoulder of the shooter that the firearm was abutted on (Fig. 2(b), the point 1).

The accelerometer 8341 (Fig. 3(a)) and the force sensor 8320-002 (Fig. 3(b)) were fixed to the end part of the firearm barrel. The equipment for measuring the acoustic pressure is shown in Fig. 3(f); it includes an audio analyser 2250 and a hydrophone 8103. In Fig. 3, the mobile equipment for measurement of the results processing, collection and control '3660-D' with computer DELL is presented.

Figure 4 shows the time graphs of changes in the measured force of the end part of the firearm barrel (Fig. 3(b) and (c)) directed at the shooting person during the gunshot for the cases with and without a suppressor system.

Figure 5 shows the time graphs of changes in the measured acoustic pressure (Fig. 3(f)) during the gunshot and the spectral density for the cases (a) with and (b) without a suppressor system.

Figure 6 shows the time graphs of changes in the measured acceleration of the shooter's shoulder (Fig. 2(b); the point 1) during the gunshot and the spectral density (in three directions: the red—across the back; the blue—along the back; and the green—perpendicular to the back) for the cases with and without a suppressor system.

To assess the impact of the suppressor system on the parameters under measurement (force, acoustic pressure and acceleration), the degree of silencing was calculated:

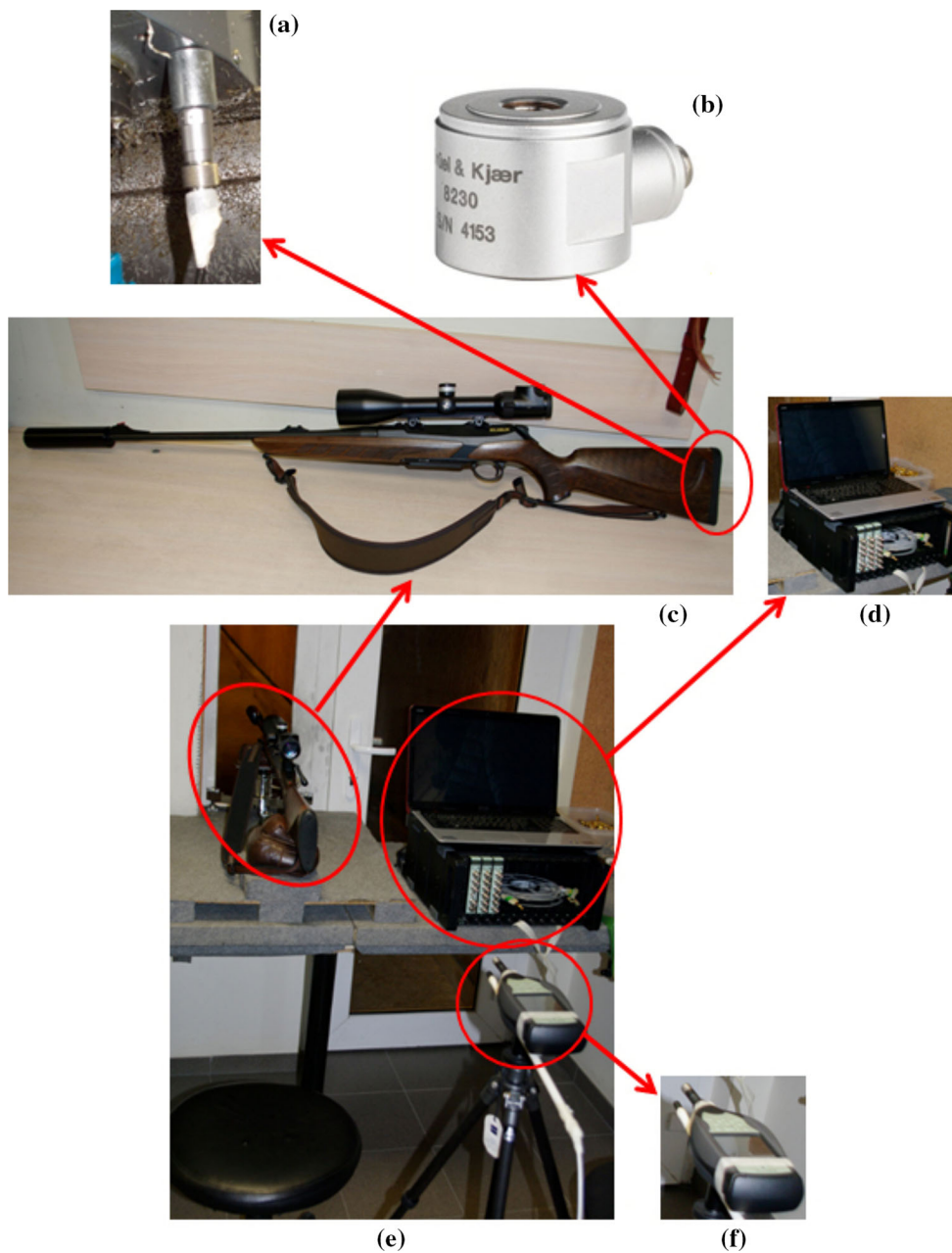


Fig. 3 The means for measuring and analysing the parameters of the force, acoustic pressure and acceleration caused by a firearm gunshot

(a) for force:

$$T_F = (1 - F_{i \text{ with suppressor}} / F_{i \text{ without suppressor}}) \times 100\%; \quad (12)$$

(b) for acoustic pressure:

$$T_P = (1 - (P_{i \text{ with suppressor max}} + |P_{i \text{ with suppressor min}}|) / (P_{i \text{ without suppressor max}} + |P_{i \text{ without suppressor min}}|)) \times 100\%; \quad (13)$$

(c) for acceleration of the shooter's shoulder:

$$T_A = (1 - (A_{i \text{ with suppressor max}} + |A_{i \text{ with suppressor min}}|) / (A_{i \text{ without suppressor max}} + |A_{i \text{ without suppressor min}}|)), \quad (14)$$

where T_F , T_P and T_A denote the degree of silencing; F_i with suppressor, P_i with muzzle suppressor and A_i with suppressor denote

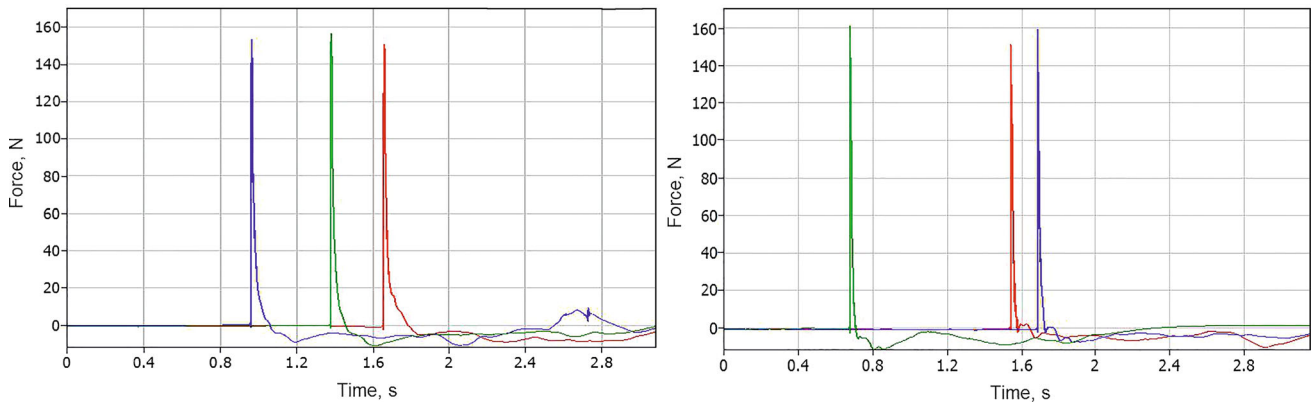


Fig. 4 The time graphs of changes of the force directed towards the shooter by the end part of the firearm barrel (Fig. 3(b) and (c)) during a gunshot for three tests: left with a suppressor system; right without a suppressor system

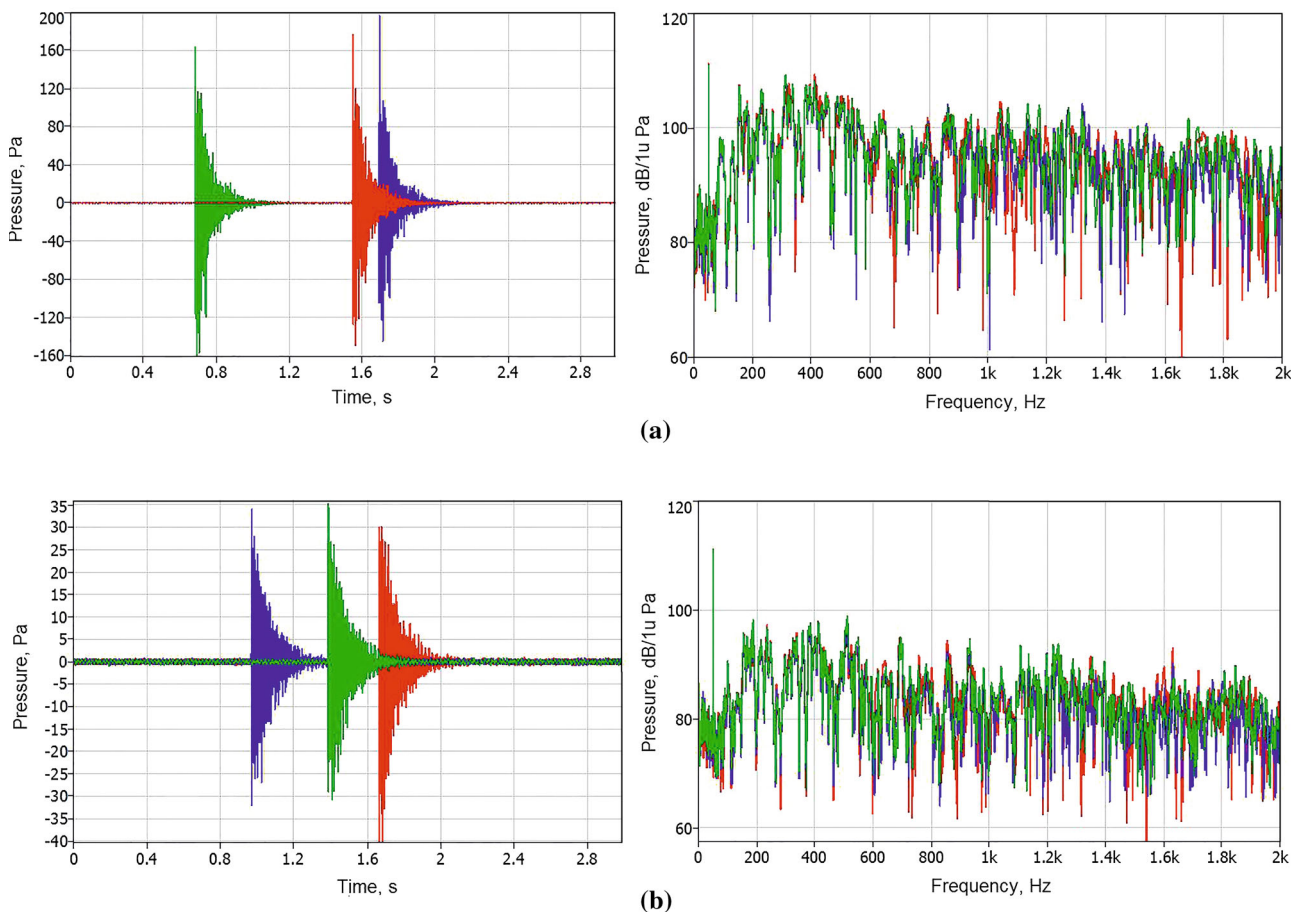


Fig. 5 The time graphs of changes in acoustic pressure (Fig. 3(f)) during a gunshot for three tests and the spectral density: (a) without a suppressor system; (b) with a suppressor system

the average values of force, acoustic pressure and acceleration of the shooter’s shoulder, respectively, obtained for three measurements, when a suppressor system was used during the gunshot; F_i without suppressor, P_i without suppressor and A_i without suppressor denote the average values of force, acoustic pressure and acceleration of the

shooter’s shoulder, respectively, obtained for three measurements, when a suppressor system was not used during the gunshot.

On assessing the value of the force, the degree of silencing is equal to:

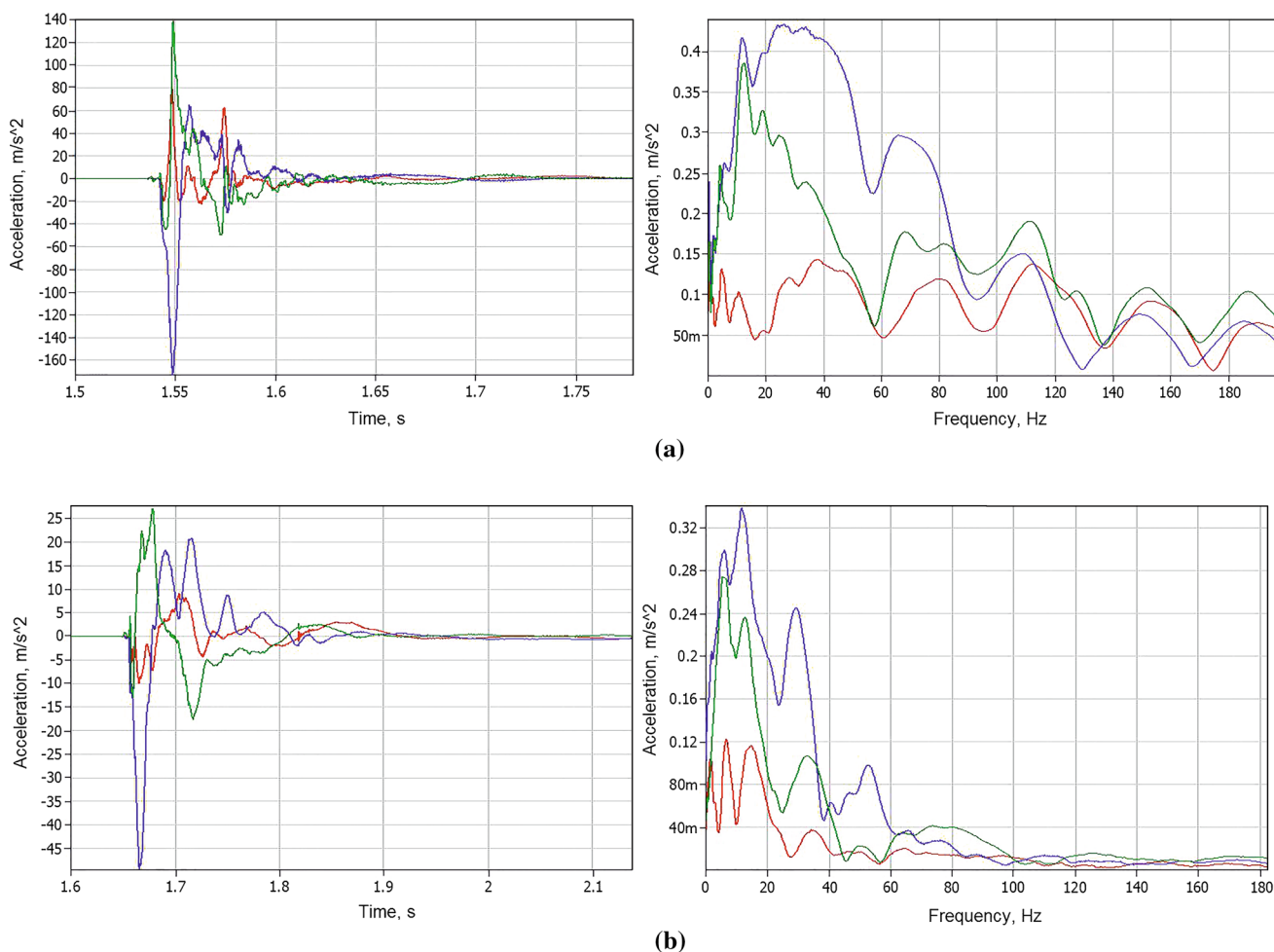


Fig. 6 The time graphs of changes in the measured acceleration of the shooter’s shoulder (Fig. 2(b); the point 1 during the gunshot and the spectral density (in three directions: the *red*—across the back; the

blue—along the back; and *green*—perpendicular to the back) for the cases without and with a suppressor system

$$T_F = (1 - F_{\text{with suppressor}}/F_{\text{without suppressor}}) \times 100\% = 1.1\%.$$

On assessing the value of acoustic pressure, the degree of silencing is equal to:

$$T_P = (1 - (P_{\text{with suppressor max}} + |P_{\text{with suppressor min}}|) / (P_{\text{without suppressor max}} + |P_{\text{without suppressor min}}|)) \times 100\% = 79.1\%.$$

On assessing the value of the acceleration, the degree of silencing is equal to:

$$T_A = (1 - (A_{\text{with suppressor max}} + |A_{\text{with suppressor min}}|) / (A_{\text{without suppressor max}} + |A_{\text{without suppressor min}}|)) \times 100\% = 75.0\%.$$

The degree of silencing found for three measured parameters (force, acoustic pressure and acceleration) shows that on the force measurement (when the sensor is between the end of the barrel and a rubber gasket) no general degree of silencing is shown because the said rubber gasket (between the gasket and the person) absorbs a considerable

share of energy, so indications of the sensor do not show the real degree of silencing. So the general degree of silencing of the system may be assessed according to the acoustic pressure and the acceleration of the shooter’s back. It may be seen that in the presence of the suppressor system, the measured acoustic pressure is decreased by 20.9%, as compared with the acoustic pressure in the absence of the system (when measured in the side of the shooter); in addition, the acceleration of the shooter’s back is decreased by 25%, when the suppressor system is used.

To establish whether the earlier measured parameters are interrelated, an analysis of signals of the said parameters was carried out.

In both states (without and with the suppressor system), vectors of dynamic signals were measured, and arrays of the results of measurement of 3 vectors were obtained. The signals were fixed in time intervals $\tau_{\Delta} = 1.95312500 \times 10^{-4}$, 3.2 s. Each vector of the array includes $n = 16,386$ values of dynamic signals.

The data arrays for two conditions obtained in measurement data processing procedures were used in the formation of two groups of data. The arrays of the measurement data were processed by the developed software upon applying operators of the Matlab 7 program package.

The values of the quantization interval of normed covariance functions vary from 1 to $n/2$, where $n = 16,386$ —the number of vector lines (values) in the array of vibration signals. The array of measurements of dynamic signals is formed of 3 vectors (columns) for each condition of the system. For each vector, the estimate $K_{\phi}'(\tau)$ of the normed auto-covariance function $K_{\phi}(\tau)$ was calculated, and 3 graphical expressions of normed auto-covariance functions were obtained for each condition. In addition, the estimates $K_{\phi}'(\tau)$ of the normed cross-covariance functions were calculated for all vectors at 3 points upon each condition; thus a total of 15 graphical expressions were found. Numbering of dynamic vectors is presented in Table 1.

The normed auto-covariance functions achieve the maximum value of the correlation coefficient when $r = 1.0$ is the value of the quantization interval.

In both cases, the expressions of the normed covariance functions of the relevant dynamic characteristics of the firearm are similar enough and distinguish themselves for very rapidly ‘damping’ values of functions. Their values approach zero already when the quantization intervals $k \rightarrow 10 - 100$ ($\tau_k \rightarrow 0.002 - 0.02$ s). So during changes of the quantization interval, the correlation of dynamic characteristics decreases very rapidly both when suppressor systems are not used and in their presence.

The values of the normed cross-covariance functions of dynamic characteristics of 6 vectors of the forearm are close to zero $|r| \rightarrow 0 - 0.25$, except for the data of three functions. They are the cross-covariance function between the vector 1 and the vector 4 when $r \rightarrow 0.55$ ($k \rightarrow 500, \tau_k \rightarrow 0.1$ s), the cross-covariance function between the vector 2 and the vector 5 when $r \rightarrow 0.90$ ($k \rightarrow 700, \tau_k \rightarrow 0.14$ s), the cross-covariance function

Table 1 Numbering of dynamic vectors

Vector no.	Vector description
<i>Without the suppressor system</i>	
1.	Acoustic pressure (Pa)
2.	Force (N)
3.	Acceleration (m/s^2)
<i>With the suppressor system</i>	
4.	Acoustic pressure (Pa)
5.	Force (N)
6.	Acceleration (m/s^2)

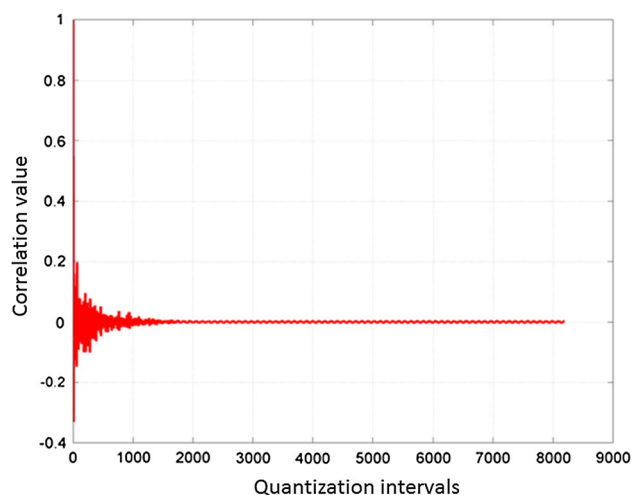


Fig. 7 The normed auto-covariance function of dynamic signals of vector 1

between the vector 3 and the vector 6 when $r \rightarrow 0.65$ ($k \rightarrow 600, \tau_k \rightarrow 0.12$ s). The values of the normed cross-covariance functions of the mentioned vectors are close to zero in any other ranges of the quantization interval

Thus, the maximum correlation is found between the same dynamic characteristics in the presence and absence of the suppressor system, i.e. between acoustic pressure dynamic vectors, between force dynamic vectors and between acceleration dynamic vectors. The correlation between vectors of different dynamic characteristics is low, i.e. $|r| \rightarrow 0 - 0.2$.

The most important graphical expressions of the normed auto-covariance and cross-covariance functions are presented in Figs. 7, 8, 9, 10, 11, 12, 13, 14 and 15.

The graphical expressions of the generalised (spatial) correlation matrices of the arrays of 6 vectors of dynamic signals are provided for two states (Fig. 16).

The expression of the correlation matrix turns into a block of six pyramids (Fig. 16), where the values of correlation coefficients are shown by tints of the colour spectrum. The chromatic projection of the pyramids is shown in the horizontal plane of the chart in Fig. 16.

4. Conclusions

Application of the covariance method proposed by the research group of VGTU for investigation and analysis of shooting dynamic processes signals is more accurate and reliable. This method is quite relevant to identify the ergonomic features of a firearm with a suppressor: the efficiency of the reduction of the firearm butt recoil force and the suppression of the sharp shooting sound.

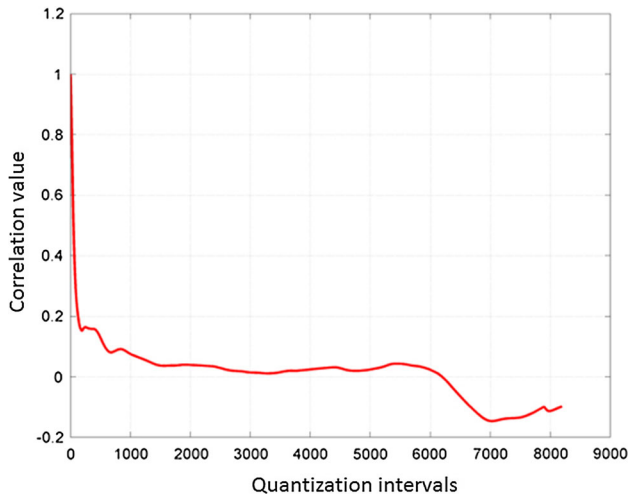


Fig. 8 The normed auto-covariance function of dynamic signals of vector 2

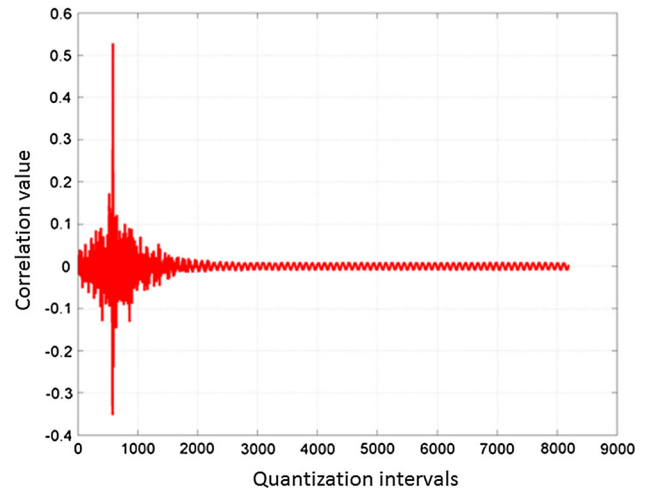


Fig. 11 The normed cross-covariance functions of dynamic signals between vector 1 and vector 4

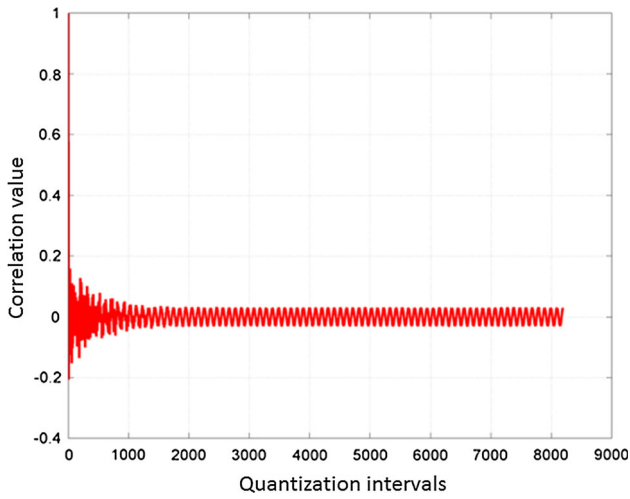


Fig. 9 The normed auto-covariance function of dynamic signals of vector 4

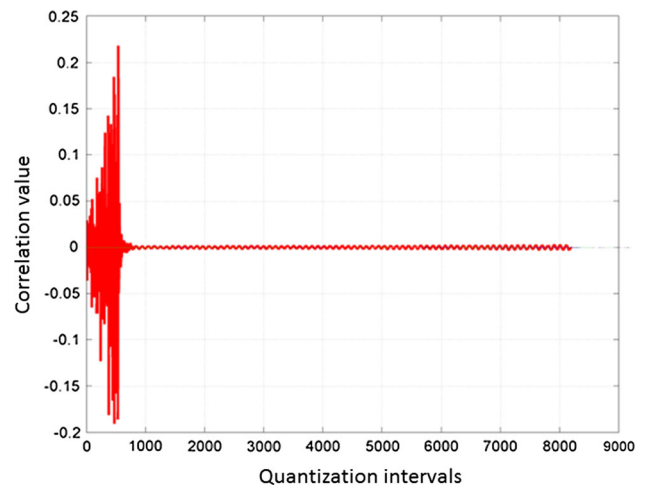


Fig. 12 The normed cross-covariance functions of dynamic signals between vector 1 and vector 6

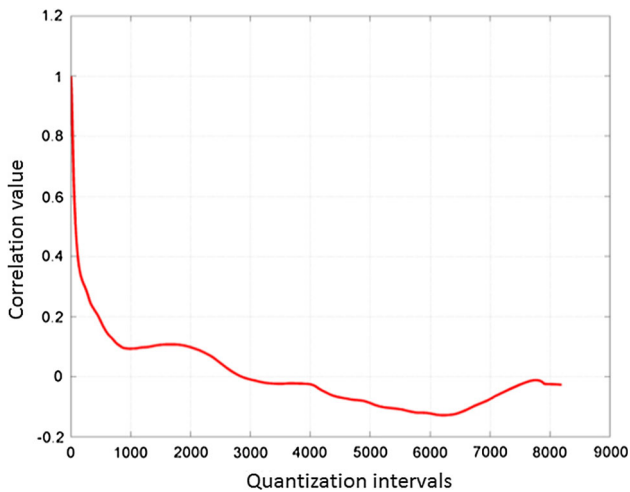


Fig. 10 The normed auto-covariance function of dynamic signals of vector 5

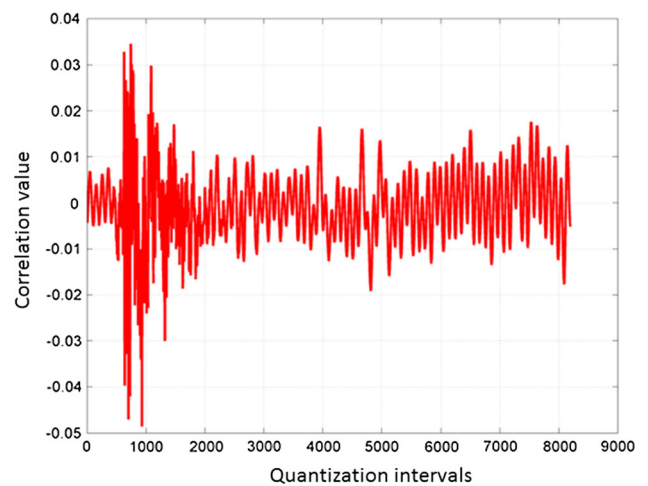


Fig. 13 The normed cross-covariance functions of dynamic signals between vector 2 and vector 4

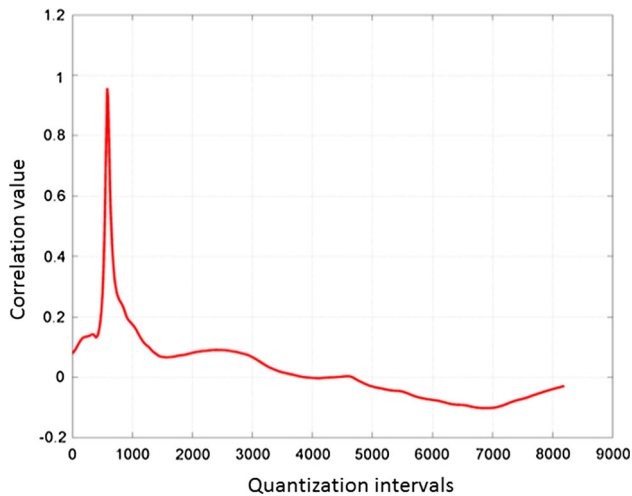


Fig. 14 The normed cross-covariance functions of dynamic signals between vector 2 and vector 5

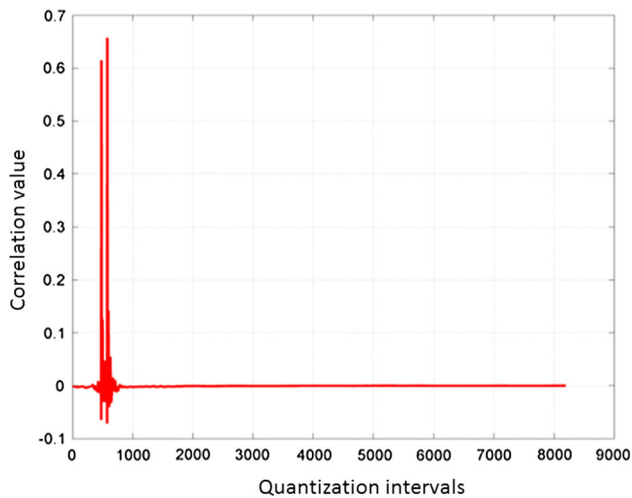


Fig. 15 The normed cross-covariance functions of dynamic signals between vector 3 and vector 6

Different physical quantities were measured: the acceleration of the firearm butt, the firearm recoil force against the shooter's shoulder and the shooting sound pressure. The correlation between the measured dynamic parameters was settled after the analysis in accordance with the methodology presented in the paper. On the basis of this correlation it is possible to predict the trends of other parameters according to one of the measured parameters.

The normed auto-covariance and cross-covariance functions of dynamic characteristics of a firearm make it possible to establish the changes of the correlation, i.e. statistical interdependence between the data vectors according to the signal quantization interval on the time scale. The values of the normed auto-covariance functions of dynamic characteristics approach zero very rapidly (when $k \rightarrow 10 - 100$, $\tau_k \rightarrow 0.002 - 0.02$ s in both cases)

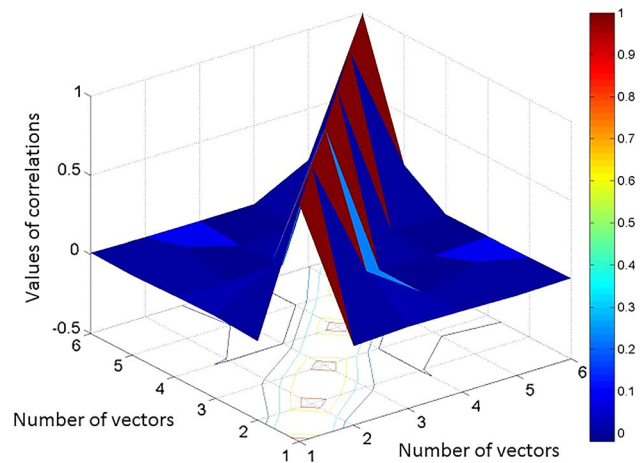


Fig. 16 The graphical expression of the generalised (spatial) correlation matrix of the array of vectors of dynamic characteristics of the firearm

(with and without the suppressor system). These changes in the correlation between the vectors of the relevant dynamic signals on changes of the quantization intervals are shown in graphical expressions.

The maximum correlation takes place, when $r \rightarrow 0.55 - 0.90$ at $k \rightarrow 500 - 700$ ($\tau_k \rightarrow 0.1 - 0.14$ s) at $k \rightarrow 500 - 700$ ($\tau_k \rightarrow 0.1 - 0.14$ s) and changes in a narrow range of the quantization interval $\Delta k \rightarrow 50$ ($\tau_k \rightarrow 0.01$ s) are found between the same dynamic characteristics both in the presence and in the absence of the suppressor system. This fact demonstrates that the suppressor system does not suppress sound in the narrow time range $\tau_k \rightarrow 0.01$ s, when the quantization interval $\tau_k \rightarrow 0.1$ s i.e. 0.1 s from the moment of appearance of the signal.

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