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ORIGINAL PAPER

Analysis of identification of digital images from a map of cosmic microwaves

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Abstract: This paper discusses identification of digital images from the cosmic microwave background radiation map formed according to the data of the European Space Agency "Planck" telescope by applying covariance functions and wavelet theory. The estimates of covariance functions of two digital images or single images are calculated according to the random functions formed of the digital images in the form of pixel vectors. The estimates of pixel vectors are formed on expansion of the pixel arrays of the digital images by a single vector. When the scale of a digital image is varied, the frequencies of single-pixel color waves remain constant and the procedure for calculation of covariance functions is not affected. For identification of the images, the RGB format spectrum has been applied. The impact of RGB spectrum components and the color tensor on the estimates of covariance functions was analyzed. The identity of digital images is assessed according to the changes in the values of the correlation coefficients in a certain range of values by applying the developed computer program.

Keywords: Planck microwave map; Digital images; Identification; Covariance function

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1. Introduction

In this paper, the cosmic microwave background (CMB) radiation map formed according to the data from the European Space Agency (ESA) "Planck" satellite telescope was used. The Planck map is a 50-million-pixel image of the whole celestial sphere of the universe. The top frequency of the spectrum of microwave radiation is about $f \rightarrow 160{,}23$ GHz ($\lambda \rightarrow 1063$ mm). The CMB is the thermal radiation left over from the time of recombination in Big Bang cosmology. The CMB has a thermal black body spectrum at a temperature of 2.72548 ± 0.00057 K [1–6].

The map of the celestial sphere (Fig. 1) is divided into two parts (upper and lower) by the so-called axis of evil. In the lower part (below the axis of evil), the biggest structure of the universe a huge hollow superemptiness with a diameter of 1.8 billion light years and the "cold spot" of the universe was discovered.

Here, the identification of digital images is carried out with the use of numerical photogrammetric methods and the theory of random functions. The spatial position of pixels of a digital image is defined by the spatial range of frequencies of the color waves, that is, on the radiometric level using the RGB format spectrum. The theoretical model is based on the concept of a stationary random function taking into account that the errors of frequencies of the color waves will be random and of the same accuracy; that is, the average error $M\Delta = \text{const} = 0$, the errors' dispersion $D\Delta = \text{const}$, and the covariance function of the digital images depends only on the difference between the arguments, that is, on the pixel quantization interval. The estimates of the covariance functions of two digital images or autocovariance functions of single images are calculated according to the random functions formed by expansion of pixel arrays of the digital images in the form of pixel vectors.

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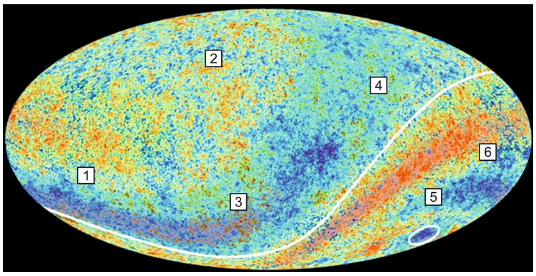

Fig. 1 The "Planck" map of the cosmic microwave background (CMB)

To process the digital signals, discrete Fourier transformation [7, 8] is usually applied. One of the newest digital image processing theories is wavelet theory (the theory of wavelet functions) [9–12].

2. The model of covariance functions for digital images

For the theoretical model, we will assume that errors of pixel parameters of digital images are random. A random function is formed by expansion of the pixel array of a digital image into one-dimensional space on one axis of coordinates column by column. In each column of a pixel array, the trend of the parameters of the column is eliminated. The parameters include pixel color intensity indicators in the RGB format spectrum. We will consider the random function formed in this way to be stationary (in a broad sense), because its average value $M\{h(t)\} \to const$, and the covariance function depends only on the difference between the arguments $K_h(\tau)$. The continuous covariance function $K_h(\tau)$ of two fragments of one digital image or pixel arrays of fragments of two digital images $h_l(u)$ and $h_j(u+\tau)$ (which are considered as realizations of random functions) shall be written as follows [13, 14]:

$$K_h(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} \delta h_l(u) \delta h_j(u+\tau) du, \tag{1}$$

where $\delta h_l(u)$, $\delta h_j(u+\tau)$ are centered fragments of pixel parameters, u is a pixel parameter of a fragment, T is the length of a fragment in conditional units, $\tau = k \cdot \Delta$ is the variable quantization interval, Δ is the value of a pixel parameter, and k is the number of pixels in the quantization interval.

The estimate $K'_h(\tau)$ of the covariance function according to the available measuring data shall be calculated as follows:

$$K'_h(\tau) = K'_h(k) = \frac{1}{n-k} \sum_{i=1}^{n-k} \delta h_l(u_i) \delta h_j(u_{i+k}), \tag{2}$$

where n is the total number of discrete intervals.

Formula (2) may be applied as an autocovariance function or a crosscovariance function. If the function is an autocovariance function, the fragments $h_l(u)$ and $h_j(u+\tau)$ are fragments of a single digital image, and if the function is a crosscovariance function, they are fragments of two different images.

The estimate of the normalized covariance function is

$$R'_h(k) = \frac{K'_h(k)}{K'_h(0)} = \frac{K'_h(k)}{\sigma'^2_h}, \tag{3}$$

where σ'_h is the estimate of the standard deviation of a random function.

For elimination of the trend of columns in the pixel array of the ith digital image, the following formulas shall be used:

$$\delta H_i = H_i - e \cdot \bar{h}_i^T = (\delta h_{i1}, \delta h_{i2}, .., \delta h_{im}), \tag{4}$$

where δH_i is the reduced pixel array of the ith digital image where the trend of columns is eliminated, H_i is the pixel array of the ith image, e is the unit vector with sizes $(n \times 1)$, n is the number of rows in the ith array, $\bar{h}_i$ is the vector of average values of columns in the ith pixel array, and δh_{ij} is the jth column (vector) of the ith reduced pixel array.

The vector of average values of columns of the ith array shall be calculated according to the following formulas:

$$\bar{h}_i^T = \frac{1}{n} e^T \cdot H_i \tag{5}$$

or

$$\bar{h}_i = \frac{1}{n} H_i^T \cdot e.$$

The realization of the random function of the ith pixel array of the digital image in the form of a vector shall be expressed as follows:

$$\delta h_i = \begin{pmatrix} \delta h_{i1} \\ \delta h_{i2} \\ \cdots \\ \delta h_{im} \end{pmatrix} = \left(\delta h_{i1}^T \delta h_{i2}^T \ldots \delta h_{im}^T \right)^T. \tag{6}$$

The estimate of the autocovariance matrix of the ith pixel array of the digital image is expressed as follows:

$$K'(\delta H_i) = \frac{1}{n-1} \delta H_i^T \delta H_i. \tag{7}$$

The estimate of a covariance matrix of two pixel arrays of a single digital image or pixel arrays of two digital images is expressed as follows:

$$K'\left(\delta H_i, \delta H_j\right) = \frac{1}{n-1} \delta H_i^T \delta H_j, \tag{8}$$

where the sizes of arrays δH_i and δH_j should be the same.

By applying the theory of covariance functions, the influence of the components of the RGB format spectrum on the expressions of covariance functions has been examined. In addition, the expressions of the covariance functions of digital images by the continuous RGB spectrum in the sense of a color tensor were assessed. The variation of the values of correlation coefficients in the matrices of correlation coefficients is shown in a relation diagram.

The estimates of covariance matrices $K'(\delta H_i)$ and $K'\left(\delta H_i, \delta H_j\right)$ are reduced to estimates of the matrices of correlation coefficients $R'(\delta H_i)$ and $R'\left(\delta H_i, \delta H_j\right)$ [13–15]:

$$R'(\delta H_i) = D_i^{-1/2} K'(\delta H_i) D_i^{-1/2}, \tag{9}$$

$$R'\left(\delta H_i, \delta H_j\right) = D_{ij}^{-1/2} K'\left(\delta H_i, \delta H_j\right) D_{ij}^{-1/2}, \tag{10}$$

where D_i and D_{ij} are the diagonal matrices of the basic diagonal terms of the estimates of covariance matrices $K'(\delta H_i)$ and $K'\left(\delta H_i, \delta H_j\right)$, respectively.

The accuracy of the calculated coefficients of correlation is defined by the standard deviation σ_r, whose value is found from the following formula:

$$\sigma_r = \frac{1}{\sqrt{k}}\left(1 - r^2\right), \tag{11}$$

$k \to 30{,}000$, and r is the correlation coefficient. The maximum value of the estimate of the standard deviation is obtained when the value of r is close to zero, and in this case $\sigma'_r \approx 0.004$; when $r \approx 0.5$, the deviation $\sigma'_r \approx 0.003$.

3. Results of experiment and analysis

In the processing of the digital images of the Planck map of the universe, a total of six fragments of the map were used, namely four fragments (1, 2, 3, and 4) from the upper part of the map and two (5 and 6) from the lower part under the axis of evil (Fig. 1). The arrays of the digital fragments of the map were numbered in sequence. The data arrays of the digital images were processed according to the developed digital programs by applying the operators of the Matlab 7 program package.

The values of the quantization interval of normalized covariance functions vary from 1 to n/2, where $n = 30{,}000$ is the number of pixels in a single array. For each fragment of the map, a random function in the form of a random vector was formed. For each vector, the estimate $K'_h(\tau)$ of the normalized autocovariance function $K_h(\tau)$ was calculated, and six graphical expressions of the normalized autocovariance functions were obtained. In addition, the estimates of the normalized crosscovariance functions $K'_h(\tau)$ were calculated for six vectors, and 15 graphical expressions for them were obtained.

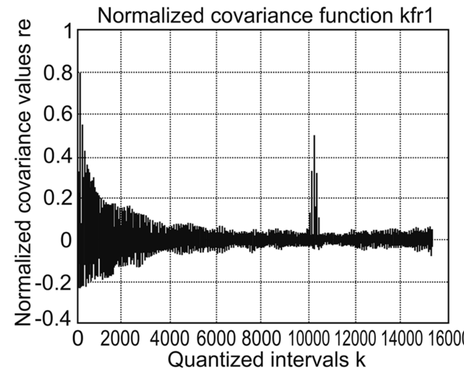

Fig. 2 A normalized autocovariance function of the digital image of point 1

An analysis was done by applying all RGB color tensors and their component colors: red (R), green (G), and blue (B).

The normalized autocovariance functions of the fragments of the map have the maximum value of the correlation coefficient $r \to 1.0$ at the quantization interval values $k \to 0$; this value decreases to $r \approx 0$ at $k \to 1000 : 8000$ pixels. In the course of further growth of the quantization interval, a sudden "peak-shaped" increase in the correlation coefficient r, about 200 pixels wide, takes place, where $r \to 0.3 : 0.6$ at $k \to 10{,}000$ pixels. This shows the appearance of possibly sudden changes (differences) of microwave frequencies in the mentioned quantization interval. The graphical expressions are provided in Figs. 2, 3 and 4 below.

The graphical expressions of the normalized crosscovariance functions of the vectors of universe map fragments are highly incoherent. This may be seen from Figs. 5, 6, 7 and 8. The values of the normalized crosscovariance functions are low and vary in the range between $r \to -0.1 : 0.1$ and $r \to -0.3 : 0.3$ in the whole quantization interval.

In Fig. 9, the graphical expression of the resumptive (spatial) correlation matrix of all six vectors of digital

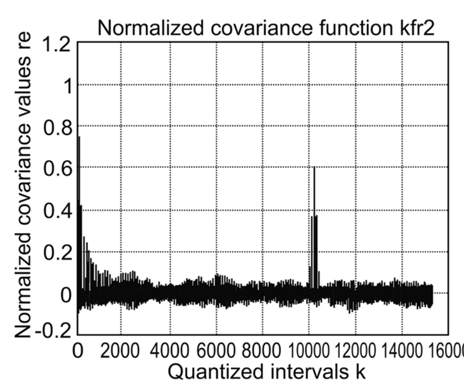

Fig. 3 A normalized autocovariance function of the digital image of point 2

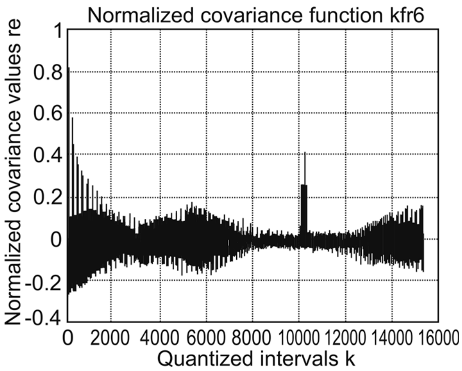

Fig. 4 A normalized autocovariance function of the digital image of point 6

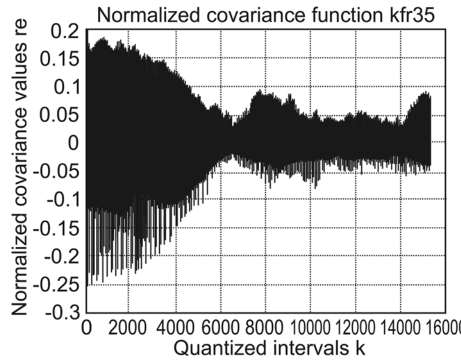

Fig. 7 A normalized crosscovariance function of the digital image of points 3 and 5

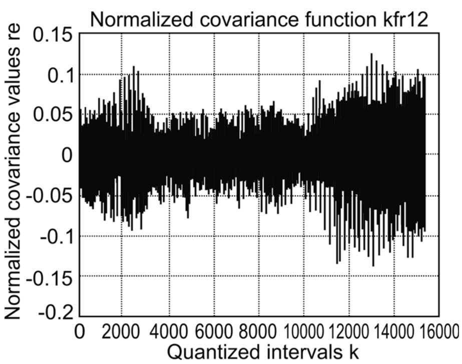

Fig. 5 A normalized interrelation covariance function of the digital image of points 1 and 2

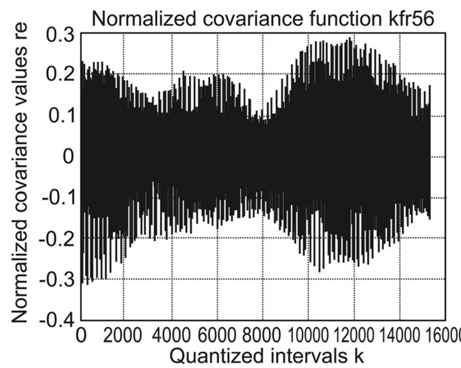

Fig. 8 A normalized crosscovariance function of the digital image of points 5 and 6

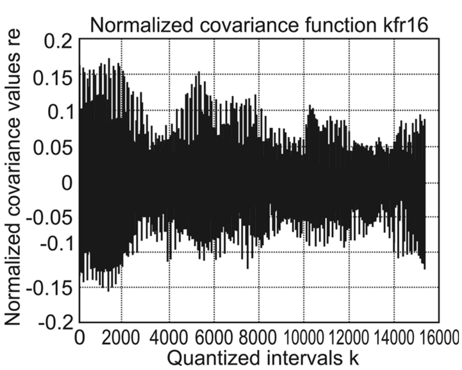

Fig. 6 A normalized interrelation covariance function of the digital image of points 1 and 6

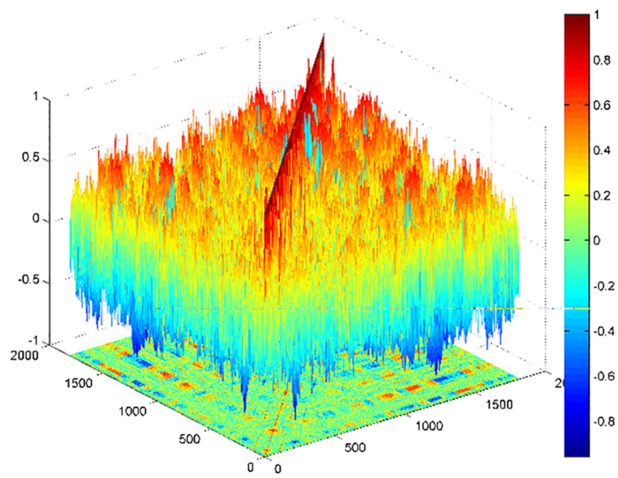

Fig. 9 The correlation of spatial matrix of all six digital images

images from the Planck microwave map of the universe is provided. The graphical expression of the correlation matrix is a block of six pyramids, where the values of the correlation coefficients are shown by different colors of the spectrum. In the horizontal plane, the chromatic projection of the pyramids is shown.

4. Conclusions

The normalized autocovariance and crosscovariance functions of the vectors of digital images from the Planck microwave map of the universe make it possible to establish the changes in correlation between digital images according to the quantization interval of pixels of their respective vectors.

The maximum values $r \to 1.0$ of the autocovariance functions of the vectors of digital images from the Planck microwave map are found in the quantization interval $k \to 0$. Then, as the quantization interval grows, the rates of decrease in the probabilistic dependence between pixels in a digital image become different and depend on the position of the digital image in the celestial sphere. In the lower part of the sphere, the rate of decrease in the correlation between pixels in a digital image is slower and $r \to 0$ at the quantization interval $k \to 8000$ pixels. Thus, the frequency of radiation of the microwave spectrum and the intensity (energy) of microwaves at different points of the celestial sphere are possibly different, and the structure of the universe is possibly not homogeneous.

The normalized crosscovariance functions of digital images are of similar irregular character, because the correlation coefficients have small positive and negative values in the range between -0.3 and $+0.3$. Hence, the interdependence between digital images from different points of the universe is low.

The parameters (such as the covariation and quantization interval) of the covariance functions of the digital images from the upper part of the universe formed by using all RGB color tensors are close to those of the covariance functions of the digital images formed for vectors of red color (R).

The parameters of the covariance functions of the digital images from the lower part of the universe (below the axis of evil) formed by using all RGB color tensors are close to those of the covariance functions of the digital images formed for vectors of blue color (B).

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