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NUMERICAL DEFORMATION ANALYSIS OF BRIDGE CONCRETE GIRDERS

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Abstract. Present research was aiming at deriving tension stiffening relationship based on EC2 provisions for deformation analysis of bending RC structures. According to the algorithm proposed by the first author, a tension stiffening relationships were derived from moment-curvature diagrams of reinforced concrete beams calculated according to EC2 technique. The obtained tension stiffening relationship was applied in the parametric study, using non-linear finite element software ATENA and *layered* model. Theoretical results were compared with experimental data of beams reported in the literature. The defined tension stiffening relationship was also applied for calculation of moment-curvature response of reinforced concrete bridge girder. The analyses have shown that the deformations calculated using the derived tension stiffening relationship and the EC2 technique were in good agreement.

Keywords: reinforced concrete, bridge, cracking, tension stiffening, curvature, numerical modelling.

1. Introduction

In many countries, the design of reinforced concrete bridge structures is based on limit state method. The concrete bridge girder or another bridge element has to be designed to satisfy the requirements of ultimate and serviceability limit states. The longitudinal reinforcement of reinforced concrete flexural member is defined according to the strength, cracking and stiffness requirements. In various design code methods, strength analysis of reinforced concrete flexural member is based on similar approaches.

On the contrary, cracking and stiffness techniques of various design codes are based on different assumptions and approaches (Borosnyói, Balázs 2005; Kaklauskas 2001; CEN 2004). The results obtained using different design code methods can vary several times (Borosnyói, Balázs 2005). Main disadvantage of design code methods is their limited application for simple cases of structural shape and loading.

Another alternative for structural engineers is using finite element methods. This method can evaluate irregular geometrical shape of the structure, specific loading conditions and non-linear properties of the materials (Zergua, Naimi 2006). Results of analysis significantly depend on constitutive models of concrete and reinforcing steel. Modelling the reinforcing steel behaviour is simple. For concrete in compression, many models were proposed (Popovics 1970). However, modelling of behaviour of cracked tensile concrete is a complicated issue. Due to bond with reinforcement, the cracked concrete between cracks carries a certain amount of tensile force normal to the cracked plane. The concrete adheres to reinforcement bars and contributes to overall stiffness of the structure. The phenomenon, called *tension stiffening,* has significant influence on the results of short-term deformational analysis. An accurate assessment of the tension stiffening effect is of primary importance in the cases of lightly reinforced concrete members (Gilbert 2007), concrete members internally reinforced by FRP bars (Mota *et al.* 2006) or externally reinforced by FRP laminates (Valivonis, Skuturna 2007).

Based on a variety of assumptions, many constitutive models for cracked concrete in tension have been proposed for case of short-term loading (Kaklauskas, Ghaboussi 2001; Torres *et al.* 2004; Nayal, Rasheed 2006; Kaklauskas *et al.* 2007). Model proposed by Kaklauskas and Ghaboussi (2001) was applied in the layered deformational approach called the *flexural* model (Kaklauskas 2004). Subsequently, this model was used by Juozapaitis *et al.* (2006) for the analysis of behaviour of suspension pedestrian bridge structure. Interaction of concrete and reinforcement in RC bridge decks subjected to monotonic loading history was also studied by Muttoni and Ruiz (2007).

It should be noted that shrinkage has a significant effect on cracking of reinforced concrete members (Bischoff and Johnson 2007). State-of-art review of shrinkage effect on deformations of reinforced concrete bridge elements is present-

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ed by Gribniak *et al.* (2007). The shrinkage effect on cracking moment is given in reference (Gribniak *et al.* 2008).

The proposed tension stiffening relationships were derived from tension, shear or bending tests of reinforced concrete members. However, there was no tension stiffening relationship proposed which would conform to the deformation analysis by EC2 code technique. Therefore, structural engineers cannot perform the FE analysis based on design code principles.

In this research, stress-strain relationships for cracked tensile concrete which satisfies EC2 provisions (CEN 2004) have been derived. According to the algorithm proposed by Kaklauskas and Ghaboussi (2001), a tension stiffening relationship has been derived from moment-curvature diagrams of reinforced concrete beams, calculated according to EC2 technique. The obtained tension stiffening relationship was applied in the numerical experiment, using nonlinear finite element software ATENA and *layered* model (Kaklauskas 2004). Theoretical results were compared with experimental data of beams reported in the literature (Clark, Speirs 1978). The defined tension stiffening relationship was also applied for calculating the moment-curvature response of reinforced concrete bridge girder.

2. Moment-curvature behaviour according to *Eurocode 2*

Based on *Eurocode 2* method, a reinforced concrete bending member is divided into two regions: region *I*, uncracked, and region *II*, fully cracked (Fig. 1). In region *I*, both the concrete and steel behave elastically, whereas in region *II* the reinforcing steel carries all the tensile force on the member after cracking.

The average strain of cracked reinforced concrete member can be expressed as:

$$\varepsilon_m = (1-\zeta)\varepsilon_{s1} + \zeta\varepsilon_{s2}, \tag{1}$$

where ε_{s1} and ε_{s2} are the strains in region *I*, uncracked state, and region *II*, fully cracked state, respectively; ζ is distribution coefficient, calculated using the equation:

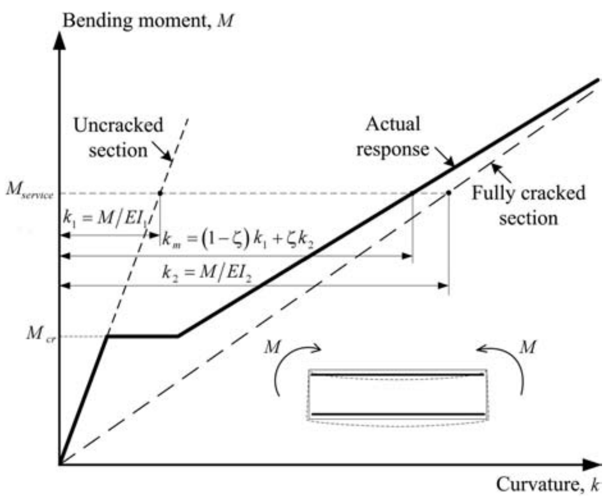

Fig. 1. Moment-curvature relationship according to the *Eurocode 2*

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 = 1 - \beta\left(\frac{M_{cr}}{M}\right)^2, \tag{2}$$

where σ_{sr} and σ_{s2} are the stresses in the tensile steel, calculated on the basis of a fully cracked section under the cracking load M_{cr} and the load considered M, respectively.

Curvature of uncracked and cracked cross-section of RC member is calculated using classical formulae of strength of materials, taking into account stiffness of uncracked and fully cracked cross-section respectively:

$$k_{1(2)} = \frac{M}{E_c I_{1(2)}}, \tag{3}$$

where k_1 and k_2 are the curvatures in uncracked and fully cracked state, respectively; I_1 and I_2 are the moment of inertia of uncracked and fully cracked states respectively; E_c is modulus of elasticity of concrete.

Average curvature of reinforced concrete member (Fig. 1) can be expressed as:

$$k_m = (1-\zeta)k_1 + \zeta k_2. \tag{4}$$

Cracking moment of reinforced concrete flexural member is written as:

$$M_{cr} = f_{ct} W_1, \tag{5}$$

where f_{ct} is concrete tensile strength; W_1 is section modulus in region *I*.

3. Method for deriving $\sigma_t - \varepsilon_t$ relationships

Tension stiffening relationships most often are defined from tension or shear tests. Kaklauskas and Ghaboussi (2001) have proposed a method for determining average stress-average strain relations for concrete in tension from experimental moment-strain (curvature) diagrams of RC beams.

Curvature of reinforced concrete bending member can be determined from strains for two different fibres:

$$k = \frac{(\varepsilon_t - \varepsilon_c)}{h} = \frac{(\varepsilon_s - \varepsilon_c)}{d}, \tag{6}$$

where ε_c and ε_t are average strains at extreme concrete fibre in compression and tension respectively; ε_s is average tensile reinforcement strain; h and d are the overall depth and effective depth of the cross-section, respectively.

The location of the neutral axis can be defined by

$$y_c = \frac{\varepsilon_c}{\varepsilon_t - \varepsilon_c} h = \frac{\varepsilon_c}{\varepsilon_s - \varepsilon_c} d, \tag{7}$$

and from strain compatibility considerations, the strain at any fibre can be expressed as:

$$\varepsilon_i = k y_i, \tag{8}$$

where y_i is the distance of the fibre from the zero strain surface.

From equilibrium

$$F_{cc} + F_{sc} + F_{ct} + F_{st} = 0, \tag{9}$$

$$M_{cc} + M_{sc} + M_{ct} + M_{st} - M_0 = 0, \qquad (10)$$

where F are internal forces, M are internal moments with respect to the neutral axis, and M_0 is the external bending moment. The first subscript corresponds to either c for concrete or s for steel and the second subscript refers to compression (c) and tension (t).

Consider a case when $M - k$ relationships are available. Then, the position of the neutral axis and average strains at any fibre for all loading stages can be determined from Eqs (5)–(7). Two equilibrium equations (8) and (9) can be solved for any loading stage yielding a solution for two unknowns.

Layered model (Kaklauskas 2004) was employed for computation of internal forces in the cross-section. Computation is performed for incrementally increasing load. During the first load stage, tensile and compressive concrete stresses corresponding to the strains in the extreme fibres are computed. These stresses are then used in the equilibrium equations for the second load stage, when new stresses corresponding to larger extreme fibre strains are determined. In this way, stress-strain curves for the tensile and compressive concrete are progressively obtained from all previous stages and used in the next stage.

In this research, the above technique was applied for deriving $\sigma_t - \varepsilon_t$ relationships from $M - k$ diagrams calculated by EC2. For that purpose, a numerical experiment has been carried out. The moment-curvature diagrams were generated for a number of reinforced concrete beams according to EC2 technique. These moment-curvature diagrams were used for determining stress-strain relationships of cracked tensile concrete. The latter were applied to a non-linear calculation using finite element software ATENA and *layered model*. Similar analysis for tension reinforced concrete members was performed by Girdžius *et al.* (2007).

4. Parametric analysis

Parametric analysis has been carried out for a bending reinforced concrete section shown in Fig. 2. Variation in reinforcement ratio ($p = A_s/bd = 0{,}2; \ldots 2{,}0$ %), concrete grade (C20/25, C30/37 and C35/45) and effective depth ($d = 0{,}3; 0{,}325; 0{,}35; 0{,}37; 0{,}39$ m) has been assumed. The ratio of the area of compressive and tensile reinforcement, A_{s_2} / A_{s_1}, was taken to be 0,25.

In present investigation, influence of variation of the above parameters on curvatures of RC beams has been analysed. Moment-curvature diagrams for varying values of reinforcement ratio, compressive strength and effective depth are presented in Figs 3–5. It can be concluded that short-term curvatures are mostly influenced by the reinforcement ratio and effective depth.

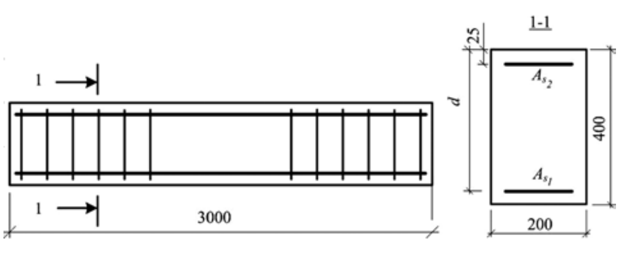

Fig. 2. Details of the beams

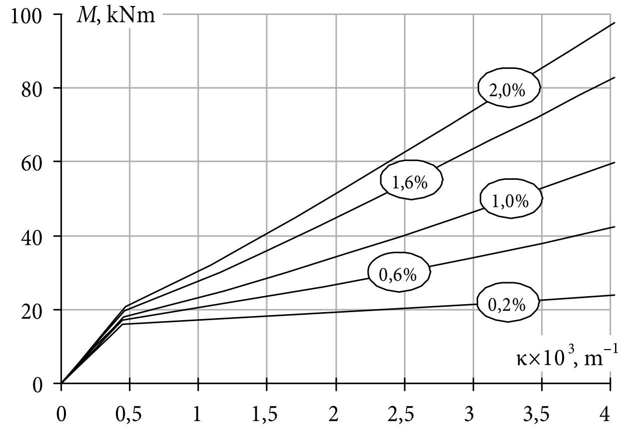

Fig. 3. Moment-curvature diagrams for ranging reinforcement ratio. Grade of concrete C30/37

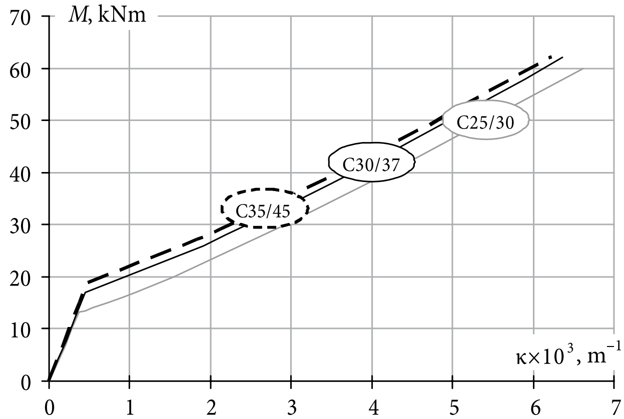

Fig. 4. Moment-curvature diagrams for ranging grade of concrete. Reinforcement ratio $p = 0{,}6$ %

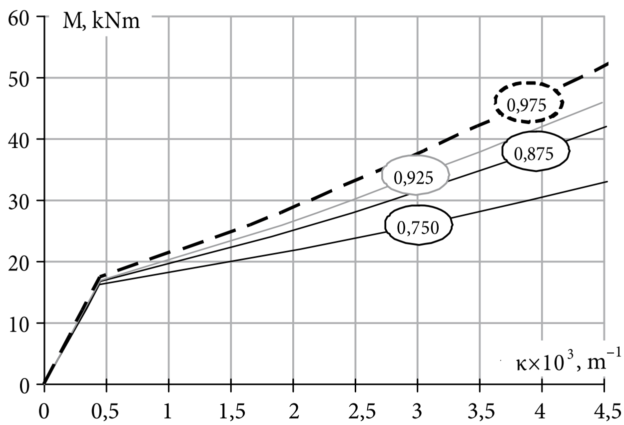

Fig. 5. Moment-curvature diagrams for ranging normalised effective depth, d/h. Reinforcement ratio $p = 0{,}6$ %, grade of concrete C30/37

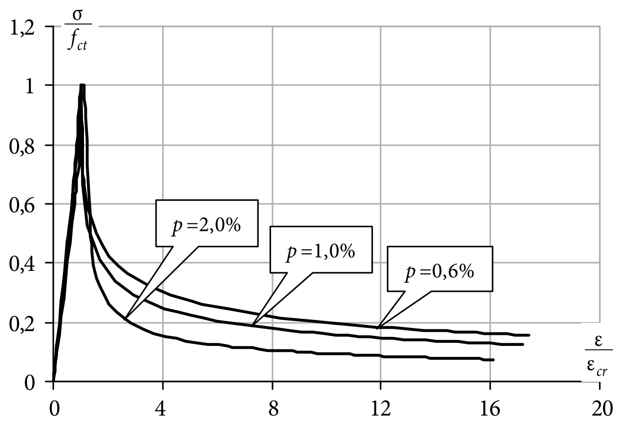

Fig. 6. Numerically derived $\sigma_t - \varepsilon_t$ relationships for varying reinforcement ratio

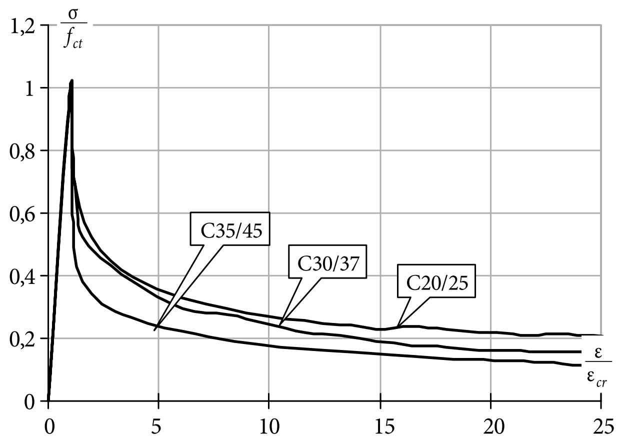

Fig. 7. Numerically derived $\sigma_t - \varepsilon_t$ relationships for varying grade of concrete

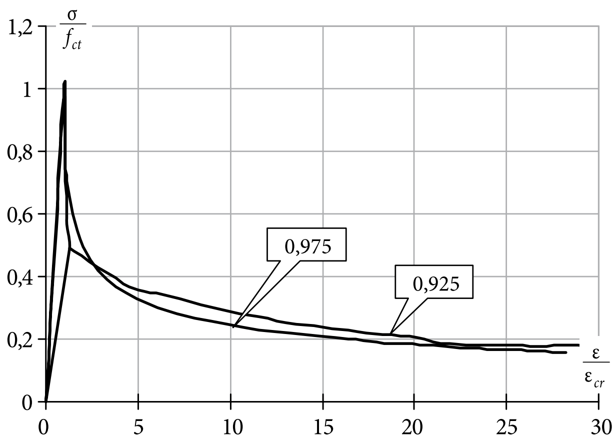

Fig. 8. Numerically derived $\sigma_t - \varepsilon_t$ relationships for varying normalised effective depth, *d*/*h*

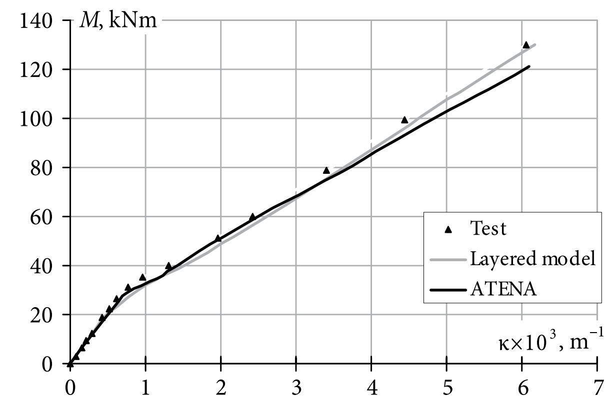

Fig. 9. Experimental and calculated moment-curvature diagrams for the beam 1R

The derived stress-strain relationships for tensile concrete obtained from the numerical tests are presented in Figs 6–8.

5. Application of the derived σ_t – ε_t relationships for the analysis of experimental RC members

This section discusses application of the derived stress-strain relationships in modelling curvature analysis of experimental RC beams tested by Clark and Speirs (1978). For that purpose, the σ_t – ε_t diagrams were incorporated into non-linear finite element software ATENA and *layered* model (Kaklauskas 2004). The latter is a simplified numerical method based on classical expressions of mechanics of materials and use of material diagrams.

Present investigation employs test data of two beams, namely 1R and 2 (Clark, Speirs 1978). The beams were nominally 3,5 m long, 400 mm high and 200 mm wide. They were tested under a four-point loading system which gave a constant moment zone of 1,2 m and two shear spans of 1,0 m. Each beam in tension zone was reinforced with three bars. However, the specimens had different reinforcement diameter and, therefore, different ratio. Main parameters of the beams are given in Table 1.

Table 1. Main characteristics of beams tested by Clark and Speirs (1978)

Beam	Depth, mm	Width, mm	Tensile reinforcement		Concrete cube strength, MPa
			Diameter, mm	Area, mm^2	
1R	412	202	25	1472	34,7
2	408	203	20	943	33,3

Moment-curvature diagrams calculated by *the layered* model and ATENA software are shown in Figs 9 and 10 along with test data. It can be concluded that good agreement has been achieved between the predictions by ATENA and *the layered* model. It can be also noted that the theoretical results fitted well the test data.

6. Moment-curvature analysis of bridge concrete girder

The derived stress-strain relationships for tensile concrete were applied for deformation analysis of bridge concrete girder. A girder shown in Fig. 11 is widely used in Lithuania and other post Soviet countries. One recent application of such girder in bridge engineering is a continuous reinforced concrete overpass in Vilnius. Grade of concrete was C30/37 and reinforcement yield strength, f_y, and modulus of elasticity, E_s, were 500 MPa and 200 GPa, respectively. Tensile and compressive reinforcement of the girder consisted of 10∅32 (A_{s1} = 80,42 cm^2) and of 2∅16 (A_{s2} = 4,02 cm^2) bars, respectively. All needed for the analysis geometrical and material characteristics are presented in Fig. 11.

Moment-curvature diagrams for the mid-span section were calculated using the EC2 method, ATENA and the *layered* model. Analysis results are shown in Fig. 12. It can be seen that the shape of the calculated load-deflection diagrams was well captured in the present analysis. Agreement of the calculations using different techniques were within reasonable limits, whereas the predictions by the numerical techniques almost coincided. Taking the predictions by the EC2 as a reference, it can be concluded that the curvatures calculated by ATENA and *the layered* model were slightly underestimated.

It should be noted that *M* – *k* diagrams were obtained only for the mid-span section of the girder. Analogous computations can be carried out for other sections. Such sectional analysis may be used for a general analysis of any indeterminate bridge structure (continuous beam, frame etc.), being thus capable to calculate stresses, strains and deflections at any point of bridge structure.

7. Concluding remarks

Present research aimed at deriving tension stiffening relationships based on EC2 provisions for deformation analysis of bending RC structures. For that purpose, a parametric study has been carried out. For a number of reinforced concrete sections, moment-curvature diagrams were generated according to EC2 technique. These diagrams were used for deriving stress-strain relationships of cracked tensile concrete. The obtained constitutive relationships were applied for curvature analysis of experimental RC beams reported in the literature. For that purpose, the $\sigma_t - \varepsilon_t$ diagrams were incorporated into non-linear finite element software ATENA and *layered* model. Good agreement has been achieved between the predictions by ATENA and *the layered* model. It can be also noted that the theoretical results fitted well the test data. The defined stress-strain relationships for tensile concrete were also applied for deformation analysis of bridge concrete girder. The shape of the calculated load-deflection diagrams was well captured. Agreement of the calculations using different techniques were within reasonable limits whereas the predictions by the numerical techniques almost coincided.

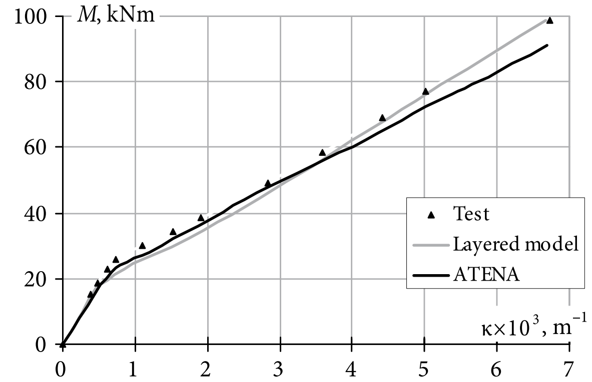

Fig. 10. Experimental and calculated moment-curvature diagrams for the beam 2

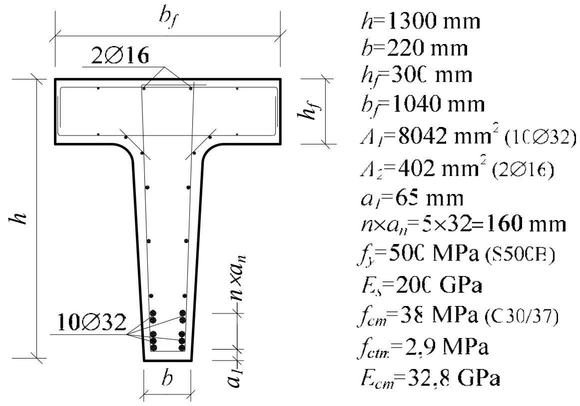

Fig. 11. Cross-section of bridge girder at the mid-span

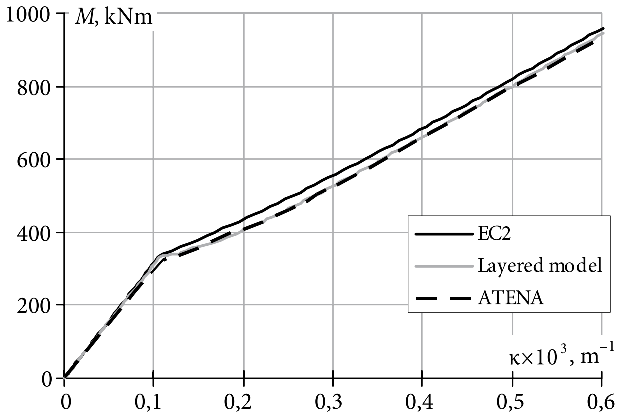

Fig. 12. Moment-curvature diagrams obtained by EC2, software ATENA (solid black line) and *the layered* model

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