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A Generalised Model of Ground Lease Pricing

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Abstract: In this paper, we present a generalization of Mandell's model for the estimation of ground lease pricing. We adjust the model so that it fits, in particular, the Polish legal regulations and situation of the Polish real estate market. The model involves two aspects. The first is the perpetual usufruct, a form of owning the ground similar to a long-term lease, but having some specific features. The second is allowing lease rent adjustments after some fixed period, meaning we consider the situation where the payments are fixed during certain periods as defined in the contract. The proposed model determines the minimum lease amount for the owner, which is the rate at which it is beneficial to lease the property, and the maximum for the lessee, which is the amount above which the lease is unprofitable for the leaseholder or perpetual usufruct.

Keywords: ground lease; ground rent; perpetual usufruct

1. Introduction

Public real estate management continues to be a relevant issue in the context of sustainability, partly because of the role of real estate resources in different management systems [1–3]. In many countries, municipal governments often play a key role in the development of urban areas [4,5]. It is clear that government intervention is crucial in the modern urban economy [6,7]. This means that they may create both possibilities and limitations in the real estate market mainly through local law. They create urban plans, prepare real estate for sale, or they lease properties on the open market. The subject of the lease may be real estate, machinery, equipment, means of transport, and other property. The most often leased property is a public or private real estate. In the case of public properties, given the existence of the perpetual usufruct, long-term leases of up to 30 years are occasionally used.

Ground leases are a policy instrument leading to a time-limited division of the bundle of rights between a public landowner and the ground leaseholder, who becomes the owner of the facilities on the land for specified time [8,9]. Ground leases are often used for retail, office, and industrial developments in the U.S. and many other countries. Moreover, the construction of privately owned housing on leased public land as a means of implementing land use policy is common in several countries and certain cities, including Amsterdam, Hong Kong, Stockholm, Israel, Japan, and Finland [10].

Communes usually lease land for a fixed term (up to three years) with the option of renewing the lease. Leases are usually applied to the land whose future is different than the present use—they are reserves of land for future public purposes—or not specified (no local land use plans). When

land is leased, the landlord does not bear the cost of maintaining the property or providing security, and receives rent payments for the duration of the lease.

The use of leases or detailed decisions vary from country to country and change over time. However, the most common issues that are resolved when entering into a lease agreement include the following [11]: property of buildings or objects made by the lessee, duration of the contract and the possibility of its renewal, settlements for expenditures incurred on the property, level and updates of lease rental rates, the basis for determining the rent of the lease, the possibility to buy land by the lessee, the possibility of subletting, land management responsibilites, and other arrangements such as the proper maintenance of buildings located on land and real estate insurance.

The source of potential problems associated with operating the lease may include [12] a lack of adequate change in lease rates in relation to changes in land value; resistance and opposition of tenants caused by changes in lease rates; difficulties and costs involved in valuing and securing the interest of each of the parties to the lease; problems with how to set lease rates (most often are estimates rather than market rates); the widespread presence of individual negotiations instead of market or auction procedures; costs associated with the lease being sometimes higher than the cost of selling the land; and the lack of benefit for the owner in the case of non-economic use of land by the lessee.

In the remainder of this paper, considering the legal regulations for the lease and perpetual usufruct of public land in Poland, the circumstances determining the choice of a particular use of the land taking into account the interests of each contracting party, are identified. The proposed model indicates when (at which rate of lease) it is better to sell or lease perpetual usufruct or rent or buy from the landowner's point of view, or take into perpetual usufruct or lease, from the holder of perpetual usufruct or tenant point of view. The proposed model can be used as a tool for optimizing decision-making in land management, and a fixed stake may be a benchmark when negotiating rates with a potential lessee. This problem is significant given that real estate generates significant profits for the owner of the land. On the other hand, the derived lease rates from the tenant's interests indicate the choice of a specific form of land usage: ownership, lease, or perpetual usufruct. The perpetual usufructor or lessee must include the cost of acquiring that right (first and annual fees or rent) when determining the profitability of the business. The negotiated lease rate is part of the cost of doing business on the leased property. Hence, the negotiated rate of rent should secure the interests of each party, including the owner and tenant (owner and perpetual usufructor).

2. Perpetual Usufruct—The Case of Poland

Public real estate management in Poland is governed by the Real Estate Management Act [13] and private land by the Civil Code [14]. Legal regulations provide that real estate may be the subject of sale, lease, and perpetual usufruct; this right applies only to properties owned by the public, i.e., treasury and local government units. The right of perpetual usufruct is similar to the perpetual lease rights of other countries, although these rights vary. In Poland, there is also the right of long-term lease, which is rarely used. Choice of the form of real estate management (sale, perpetual usufruct, or lease) should be made considering the legal provisions, the objectives and criteria of the landowners, and the expectations and benefits of potential buyers, perpetual usufructors, and tenants.

This study attempted to identify the parameters used for determining the choice of management form (sales, perpetual usufruct, or lease) of land in the context of sustainable management, considering the interests of each party to the transaction (owner and tenant or perpetual usufructor). The presented model enables the determination of the limit of rent amount, which is equated to updated revenues from perpetual usufruct or sale of real estate.

Perpetual usufruct is one of the forms of management of public real estate. In the Polish legal system, perpetual usufruct was legally regulated in 1961 when state ownership was the dominant form of ownership. The legislature intended this right to regulate the ownership relations between the state and the entities of the private sector. The current regulations of perpetual usufruct significantly differ from those introduced in 1961 with the Law on the Economy of Land in Cities and Settlements [15–17].

In addition, despite the decreasing presence of the political reasons for the introduction of this right, the perpetual usufruct continues to play an essential role in the system of rights in real estate management and affects various aspects of social and economic life. Perpetual usufruct is an instrument permitting long-term (40–99 years) usage of land without the need to involve capital in the amount equivalent to the acquisition of the ownership.

For land property placed into perpetual usufruct, an initial fee and annual fees are collected [13]. These fees are determined as a percentage of the land price determined by the provisions of Art. 67 of the cited Act, i.e., price determined by value. If the land property is given for perpetual usufruct by auction, the basis of the calculation of such fees is the price obtained during the auction. On the other hand, if the right of perpetual usufruct is established by open market rules, the price that is the basis for the calculation of the initial and the annual fee is the price of the property determined at a level not lower than its market value. The amount of the initial and annual fees varies and depends on the specific purpose of the contract. The initial fee amounts to 15–25% of the land price.

The percentage of annual fees may include (Art. 72, Item 3): for land properties committed to the defence and security of the state including fire protection, 0.3% of the price; for land properties committed for charity purposes and non-profit caring, cultural, curative, educational, scientific, or research and development activities, 0.3% of the price; for land properties for agricultural purposes, 1% of the price; for land properties completed for residential purposes, for the implementation of technical infrastructure, and other public purposes and sports activities, 1% of the price; for land properties for tourism, 2% of the price; and for other land properties, 2% of the price.

The annual fee may be increased (Art. 76, Item 1) by the governor (in relation to property owned by the State Treasury) or by a resolution of the appropriate council or regional government. An increase in the rate may occur before the property is handed over for perpetual usufruct and is for real estate for which the annual fee, due to the purpose of the property, is 3%. In some cases, the competent authority may grant a discount (Art. 73, Paragraph 2a, Item 3).

The first payment is made in the year in which the perpetual usufruct is established, and the annual fees are paid during the perpetual usufruct, until March 31 of each year, in advance for the given year. Thus, the amount of proceeds from these charges is dependent on regulations that the municipality does not influence (interest rates), the prices obtained in auctions or the value of real estate delivered in perpetual usufruct in the open market, and the adopted resolutions on granting discounts on established fees.

Annual fees for perpetual usufruct vary over time, taking into account changes in the value of land properties. According to Art. 77, Item 1, the annual fee for a perpetual usufruct is subject to updating no more than once every three years, provided the value of the property changes. The updated annual fee is determined using the fixed percentage of the current level from the market value as of the update date.

A detailed regulation was introduced to address two cases. If the value of the property as of the date of updating the annual fee is lower than the price of that real estate determined on the date of its submission for perpetual usufruct, then the fee will not be updated. This provision shall apply to immovable property transferred for perpetual usufruct of property for five years from the date of conclusion of the contract for the transfer of land title to perpetual usufruct (Article 77, Item 2). If the updated annual fee is at least twice the current annual fee, the perpetual user pays an annual fee equal to twice the current annual fee. However, the remaining amount, more than double the current fee, is divided into two equal parts, which increase the annual fee in subsequent years. The annual fee is updated ex officio or on the perpetual usufructor application, the reason for which is usually a reduction in the value of the property. To reassume the above, the updating of the annual fees for perpetual usufruct is possible no more than once every three years, and provided that there has been a change in the market value of the land property.

Unlike perpetual usufruct, a lease institution is a form of ownership known not only in Poland but also in other European countries as well as in other continents. In Poland, legal regulations for

lease agreements, as civil law institutions, are contained in the provisions of the Civil Code Act of April 23, 1964 (Articles 659–709). As stated in Article 693 of the Civil Code for lease agreements, the landlord undertakes to give the tenant the property to use and retrieve the property for a specified or unspecified period, and the tenant agrees to pay the rentee the agreed rent. The lessee has the right to use the property, i.e., the income it earns, but is obliged to take care of the proper condition of the leased property (e.g., carry out repairs at their own expense) and return the object at the end of the lease term in a specified contract.

As mentioned earlier, the institution of a long-term lease is a form of ownership that is common not only in Europe, but also in the economies of other continents (Asia, North America, and Australia), although it is applied to a different extent and detailed regulations vary across countries and in time. The most important issue concerning long-term lease are as follows [11,18,19]:

(1) The ownership of land, buildings, and facilities. The owner of buildings or facilities is the lessee of the land under a long-term agreement and holds it as its owner without having to buy. In case the lease agreement expires or is canceled between the termination date, the lessee receives compensation equal to the value of their investment in the land. This solution is used in most of the analyzed countries (except for England Air Force Base, Louisiana, U.S., and Finland, but it refers only to industrial land).

(2) The duration and the possibility of extending the lease contract. Agreements usually cover the period of between 50 and 99 years, e.g., Canberra in Australia, or an indefinite period, as seen in the Netherlands. Contracts that concern residential properties are generally longer than those for land used for commercial or industrial purposes. For example, in Sweden, residential lease agreements are signed for a period of at least 60 years, whereas those for other purposes cover a period of a minimum 20 years. In Finland, residential lease contracts are signed for a period of 50–60 years, whereas industrial lease contracts are 20–30 years. Sometimes, land lease agreements are created for shorter periods (20–30 years). This refers to the lease of land earmarked for some commercial uses (e.g., restaurants, smaller shopping and service centers, or car showrooms). Lease contracts can cover a much longer period, i.e., 200 or even 999 years. They are created with an extension option if, for example, the owner needs land for public purposes (e.g., Finland). They can also be extended automatically (the Netherlands, Israel, Austria, or Sweden) by the same number of years, half of the years of the original contract, or by another period.

(3) Settlements for the investments in land. The landowner is usually not interested in the investment in it. They receive income in the form of a lease fee. In turn, the investment made by the lessee (user) should pay for itself before the termination of the contract. When the landowner takes over the buildings built by the lessee, the basis of settlement may be a compensation for the investment made calculated according to the market value or replacement value. Buildings may also be taken over without any compensation. For example, in Finland, in case of industrial buildings built by the land lessee. In Sweden, the lessor does not pay any compensation if a lease agreement concerns non-residential use and it contains a provision that no compensation is paid. In Israel, although buildings and other improvements cannot be the subject of separate ownership, most agreements specify that, in case a contract expires or is canceled before its termination date, the lessee has to receive compensation for the improvements. If the owner disclaims the lease before the termination date, they are usually obliged to pay compensation for taking over buildings and facilities prematurely. For example, the amount of compensation may be calculated as the difference between the market value of the developed property and the market value of the land.

(4) The level and updating of a lease fee. Regulations in this respect vary across countries. For example, in Sweden, a lease fee is paid in the form of annual land rent. The fee is adjusted every 10 or 20 years. In Finland, a lessee pays a fee of 4–5% of the estimated market value of the land. The fee is updated every 30 years according to the increase in the market value of the land, whereas, in the following years, changes in the maintenance cost index are considered. Lease

fees are usually paid on a quarterly basis. Lease contracts in which a lessee pays an initial fee are rare. In some countries (e.g., Israel, Australia, and New Zealand), there are both agreements with an initial fee and the ones without it. In Israel and the Netherlands, one may obtain a right of perpetual usufruct for the amount of the annual capitalized lease fee, but a lease fee may also be paid on a yearly basis.

Sometimes, apart from the annual rate of a lease fee (i.e., the basic fee), lessees pay additional fees calculated as the percentage of gross (2–10%) or net (10–20%) income from the lease of facilities or as a percentage of the increase in the value of a property when it is sold to another investor (e.g., Israel).

We summarized the essential features of leases and perpetual usufruct in Table 1. The analysis is based on selected countries, including Austria, Belgium, Finland, Germany, Great Britain, Israel, the Netherlands, Sweden, and Poland [11,18].

Table 1. The features of perpetual usufruct, perpetual lease (present in chosen countries), and the long-term lease of land properties.

Feature	Perpetual Usufruct	Perpetual Lease	Long-Term Lease
Ownership of buildings	Perpetual usufructor	Lessee	Lessor (If the lessee built the building, they could claim for reimbursement)
Duration	40–99 years	Depending on the country, e.g., 20–30 years, 50–99 years, 200 or 999 years	No longer than 30 years
Possibility of extension	For the next 40–99 years	Various possibilities, depending on the contract	Various possibilities, depending on the contract. A lease concluded for a period longer than 30 years is treated as a lease concluded for an indefinite period, then termination at any moment is possible.
Method of the settlement of expenditures incurred on the ground	Remuneration defined as the market value of buildings and other equipment, and if, due to the type of real estate, market value cannot be determined, the replacement value is determined.	Compensation calculated as the difference between the market value of developed land and the value of the land without buildings	By default, without reimbursement of expenditure on the subject of the lease. Other arrangements must be included in the contract.
The rules of setting the fee/rent	First fee Annual fees as a fixed percentage of the market value of the land.	There are contracts in which the first fee is established. Most usually the rent is a constant percentage of the current market value of the land. Additional fees are often added to the basic rent.	Based on the rental rates of similar land properties in a parametric way as the percentage of market value through negotiations (arrangements) between the owner and the lessee.
The amount of the fee/rent	Differentiation according to the purpose	Differentiation according to purpose, location, form of the ownership (public or private)	Differentiation according to purpose, location, form of the ownership (public or private)
Adjusting the fee/rent	Not more often than once every three years provided that the market value of the property has increased.	Periodically update every 3 to 4 years to adjust to market rent.	As claimed in the contract, e.g., in correspondence with the CPI.
Preemption	For perpetual usufructor	To the lessee	Not present
Determining the way of using the land and the duration of development	Yes	Yes	Yes
Law established on land owned by	State Treasury or local government units	Any private or public subject	Any private or public subject

As we can see, long-term usage of land being someone else's property, without the need to purchase, as well as the ownership of buildings or facilities by a perpetual usufructor or lessee, occurs in many countries, although the details vary according to social, economic, and law determinants [17,20,21].

3. Literature Review

Various legal and financial aspects of leasing has attracted the attention of researchers in past years. Long [22] studied the relationships between leasing and the cost of capital. Lewis and Schallheim [23] studied the question of whether leases and debts should be treated as substitutes, which is a rather common assumption, or rather as complements. The results of the research demonstrated that, because leasing is a mechanism for selling excess tax deductions, it can motivate the lessee to increase the proportion of debt in its capital structure relative to an otherwise identical firm that does not use leasing. The same issue was studied by Yan [24]. However, Yan clarified that the substitutability between leases and debt arises from the fact that leases increase the marginal financing cost of debt or vice versa. The results reject the hypothesis that leases and debt are complements, but could not reject the hypothesis that they are substitutes.

Liu et al. [25] investigated the relationship between asset liquidation values and the real estate firm's financing choice. They showed that risk aversion and interest rate uncertainty could significantly bias an expectation hypothesis of lease rates, similar to that of interest rates. Clapham and Gunnelin [26] considered the term structure of lease rates, where both rent and interest rates were stochastic. In particular, they derived equilibrium relationships in a generally continuous time setting. Lally and Randal [27] investigated the influence of a ratchet clause on the rental rate for a lease of the periodically re-evaluated ground. The results of the estimations suggested that the effect of ratchet clauses upon appropriate rental rate depends primarily upon the term of the lease and the volatility in land returns. For a 20-year lease with land value reviewed every five years, the ratchet clause lowers the appropriate rental rate by around one percentage point.

Fallis [28] discussed the legal issues of leases. In particular, Fallis performed a public choice analysis of rent control. Mooradian and Yang [29] studied the influence of asymmetric information on the choice of lease type. Özdilek [30] studied property valuation with separate values for land and buildings, based on Montreal, Canada. Pope et al. [31] investigated the risk related to a leasehold property from the investor's point of view. A similar topic was investigated by Rabbani et al. [32]. Sevelka [33] studied the influence of the rent reset clause on the valuation in the ground lease problem. Yoshida et al. [34] studied the rent term premium for cancelable leases by modeling the lessor's tradeoff between the costs of leasing and the costs of cancelation.

Benjamin and Chinloy [35] developed an option-theoretic model of a retail lease. In particular, they studied economic tradeoffs between base rent, security deposit, and percentage rent provisions. Capozza and Sick [36] developed the model for valuing leased properties that involved the option to upgrade or redevelop. A similar topic was studied by Dale-Johnson [37]. Halim and Jaaman [38] developed a mathematical model for a specific type of contract, named AITAB. McCann and Ward [39] analyzed rental payments using a multi-period stock inventory model and showed that the factor that most influences the tenant's value of the land is the duration of the lease. Stanton and Wallace [40] proposed a no-arbitrage-based lease pricing model. Steele [41] considered a lease valuation model using difference equations. Wheaton [42] presented a two-period model for retail rental contracts. Country-specific models and case studies concerning chosen factors were also presented by Walters and Kent [43], Guntermann and Thomas [44], Tian [45], Kaganova et al. [46], Gholipour and Lean [47], Guarino et al. [48], and Irumba [49].

Two papers on this subject were published by Jefferies [50,51], considering the Lessor's Return Model, which was used to analyze various aspects of the real estate market, in particular in New Zealand. The model was further developed by Mandell [12,52]. A similar approach, but with simplified formulae, was used by Trybusz [53].

4. Mandell's Model

As mentioned above, the main goal of our paper was to present a model for ground lease pricing that would be applicable in countries with a legal situation similar to Poland. Our model is a

generalization of the combined model applied by Mandell [52]. In particular, it involves an additional form of ground rental, called the right of perpetual usufruct (RPU).

The model of Mandell involves two sides of the contract: lessor and lessee. Let us first recall the Lessor's Return Model (LRM) presented in this article, which was first introduced by Brown. Although not published, it was referred to by Jeffries [54], and a similar model was considered by Brown in another context [55].

Let A denote the initial ground rent that is fixed for p periods and then may be changed according to a growth rate that lessors expect to be equal to g. Then, the present value of the future stream of perpetual payments is as shown in Mandell [52], Equations (1) and (2):

$$V_{LRM} = \frac{A}{r}\left(1-(1+r)^{-p}\right)\sum_{i=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{ip} = \frac{A}{r}\times\frac{(1+r)^p-1}{(1+r)^p-(1+g)^p} \tag{1}$$

where r denotes the discount rate assumed by the lessor. In the LRM, the lessor has to choose between selling and leasing the property. Let us denote the present market value of the property with X. The ground rent is usually defined as a percentage a of the market value:

$$A = aX. \tag{2}$$

The lessor prefers leasing the land to selling it if the present value of all the payments is not less than the present market price, i.e., when

$$V_{LRM} \geq X. \tag{3}$$

This means that the ground rent rate a must satisfy the condition (Mandell [52], Equation (5))

$$a \geq a_{LRM} = r\frac{(1+r)^p-(1+g)^p}{(1+r)^p-1}. \tag{4}$$

On the other side of the market, there is the lessee, who also considers two possible decisions: leasing or buying the property. In the Lessee's Affordability Model (LAM) presented by Jefferies [54], the present value of leasing the land and the present value of freehold are compared. It is assumed that the buyer needs to finance buying through loans and the interest rate (i.e., the cost of capital), which equals c. It is also assumed, however, that in the case when there is a choice between buying the property for X and leasing it for cX, the psychological aspects will favor the choice of the freehold. For that reason, Jefferies assumed that the discount rate on the leasing side of the equation should be less than c. To be more specific, it is defined as

$$d = Dc \tag{5}$$

where $D \in (0,1]$ is the reduction ratio (also called the D-factor). The present value of leasing for one period is equal to (c.f. Mandell [52], Equation (8))

$$PVL = bX\frac{1-(1+d)^{-p}}{d} \tag{6}$$

where b is the rent expressed as the percentage of the market value X. The present value of the freehold financed with a loan (for one period) is in turn expressed by (c.f. Mandell [52], Equations (9) and (10))

$$PVF = cX\frac{1-(1+c)^{-p}}{c} = X\left(1-(1+c)^{-p}\right). \tag{7}$$

The lessee will prefer the leasing variant if

$$PVL \leq PVF \tag{8}$$

i.e., when (c.f. Mandell [52], Equation (13))

$$b \leq b_0 = d\frac{1-(1+c)^{-p}}{1-(1+d)^{-p}}. \tag{9}$$

As indicated by Mandell [52], this model is not satisfactory. The main disadvantage is that it is not completely clear how to compute the D-factor. To be more specific, it is necessary to introduce LAM to derive the correct value of D, but it is necessary to know the value of D in order to introduce LAM (see Mandell [52] for details and for the discussion of other disadvantages of Jefferies' approach).

This is why another model was proposed. We fully agree with Mandell that this model much better reflects reality. Let us assume that the lessee expects that the future growth rate (in the period of length p) of the market value X of the property equals h. They also expect that the present value of the benefits of possessing (not necessarily of being the owner of) the property is equal to U. Under such assumptions, the present value of the net benefits of the freehold equals

$$PVFB = U + \left(\frac{(1+h)^p}{(1+c)^p} - 1\right)X, \tag{10}$$

whereas the present value of the leasing is (c.f. Mandell [52], Equation (14))

$$PVLB = U - bX\frac{1-(1+c)^{-p}}{c}. \tag{11}$$

Since the lessee chooses the lower value, the present value of the benefits from leasing must not be less than the present value of the benefits from the freehold, which means that the following condition must hold (c.f. Mandell [52], Equation (16)):

$$b \leq b_{LAM} = c\frac{(1+c)^p - (1+h)^p}{(1+c)^p - 1}. \tag{12}$$

The combined model assumes that the payment rate lies between a_{LRM} and b_{LAM}. Let us present its generalization, involving specific legal conditions present in Poland.

5. The Generalized Model

As mentioned above, the third variant of possessing the land occurs in Poland—the perpetual usufruct. In this case, not only the regular rent but also the initial payment $Y = kX$ has to be paid (here k is the initial payment expressed as the ratio of the property's market price X). The yearly rent is then calculated as the ratio of the market price X. The contract ends after n years and may be renewed. In the remainder of this section, we present the estimations of the present value of all the income from the perspective of the property's owner and then from the perspective of the potential user. In the last subsection, we compare the results in order to define the intervals to which the payment rates should belong. We present here only the main results in a short form, with details in Appendixes A.1 and A.2.

5.1. Owner's Perspective

The first possible decision of the owner may be to sell the property. In such cases, the present value of the income may be estimated by the market price of the property (X):

$$PV_s = X. \tag{13}$$

In the case of perpetual usufruct, the present value of the income takes the form

$$PV_u = kX + a_u f_u X + \left(\frac{1+i}{1+r}\right)^n X \tag{14}$$

where the factor f_u is defined as

$$f_u = \frac{1-(1+r)^{-t_u}}{r} \times \frac{1-\left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}}}{1-\left(\frac{1+i_u}{1+r}\right)^{t_u}} + \frac{1-(1+r)^{-n_{u1}}}{r} \times \left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}} \tag{15}$$

where k is the rate of initial payment, a_u is the perpetual usufruct annual rate, n is the duration of perpetual usufruct, i is the expected average annual increase in the property's value, r is the expected average annual discount rate, t_u is the number of years for which the payment is fixed, and i_u is the average annual rate of the payment's increase.

Moreover, the number of periods with fixed payment is derived with

$$n_{u0} = \left\lfloor \frac{n}{t_u} \right\rfloor = n \text{ div } t_u, \tag{16}$$

and the number of remaining years (i.e., the number of years in the last period not longer than t_u, where the payment remains fixed) is then equal to

$$n_{u1} = n - t_u n_{u0} = n \bmod t_u. \tag{17}$$

In the case of lease, the present value derived with

$$PV_d = a_d f_d X + \left(\frac{1+i}{1+r}\right)^n X \tag{18}$$

where the factor f_d is defined as

$$f_d = \frac{1-(1+r)^{-t_d}}{r} \times \frac{1-\left(\frac{1+i_d}{1+r}\right)^{t_d n_{d0}}}{1-\left(\frac{1+i_d}{1+r}\right)^{t_d}} + \frac{1-(1+r)^{-n_{d1}}}{r} \times \left(\frac{1+i_d}{1+r}\right)^{t_d n_{d0}} \tag{19}$$

where a_d is the lease annual rate, t_d is the number of years for which the payment is fixed, and i_d is the average annual rate of the payment's increase.

Moreover, the number of periods with a fixed payment is derived with

$$n_{d0} = \left\lfloor \frac{n}{t_d} \right\rfloor = n \text{ div } t_d, \tag{20}$$

and the number of remaining years (i.e., the number of years in the last period not longer than t_d, where the payment remains fixed) is then equal to

$$n_{d1} = n - t_d n_{d0} = n \bmod t_d. \tag{21}$$

The lease is an attractive option for the owner if

$$a_d \geq \max\left\{ \frac{1-\left(\frac{1+i}{1+r}\right)^n}{f_d}, \frac{k + a_u f_u}{f_d} \right\}. \tag{22}$$

5.2. User's Perspective

The user, when choosing the option, analyses the present value of the costs. They can finance part of the cost from a loan with interest rate i_h, which must be paid back in n_h years. In the model, we assumed that the loan is paid with decreasing payments with n_h fixed principal payments and respective interest payments (other payment schedules may be easily applied as well). Part of the cost, $w_s < 1$, must be paid in cash. The total present value of the cost of buying the property is equal to

$$PV_s^\star = w_s X + (1 - w_s) \times \frac{X}{n_h} \times \frac{1 - \left(\frac{1+i_h}{1+r}\right)^{n_h}}{1 - \frac{1+i_h}{1+r}}. \tag{23}$$

We may observe that

$$PV_s^\star = PV_s(1 + q_s) \tag{24}$$

where

$$q_s = w_s + \frac{(1 - w_s)}{n_h} \times \frac{1 - \left(\frac{1+i_h}{1+r}\right)^{n_h}}{1 - \frac{1+i_h}{1+r}} - 1 > 0 \tag{25}$$

(i.e., $PV_s^\star > PV_s$).

In the case of perpetual usufruct, we assume that part of the initial payment (w_u) must be paid in cash, but the remaining part may be paid using a bank loan. The present value of the cost is then equal to

$$PV_u^\star = kPV_s(1 + q_u) + a_u f_u X \tag{26}$$

where

$$q_u = w_u + \frac{(1 - w_u)}{n_h} \times \frac{1 - \left(\frac{1+i_h}{1+r}\right)^{n_h}}{1 - \frac{1+i_h}{1+r}} - 1 > 0. \tag{27}$$

The present value of the total cost corresponding with lease equals

$$PV_d^\star = a_d f_d X = PV_d - \left(\frac{1+i}{1+r}\right)^n X. \tag{28}$$

This reasoning, similar to that in the previous section, leads to the conclusion that the user will accept the form of leasing if

$$a_d \leq \min\left\{\frac{1 + q_s}{f_d}, \frac{k(1 + q_u) + a_u f_u}{f_d}\right\}. \tag{29}$$

5.3. Range of Acceptable Leasing Rates

Taking together the inequalities deduced in the two preceding subsections, we obtain the following interval for the leasing rate:

$$a_d \in \left[\max\left\{\frac{1 - \left(\frac{1+i}{1+r}\right)^n}{f_d}, \frac{k + a_u f_u}{f_d}\right\}, \min\left\{\frac{1 + q_s}{f_d}, \frac{k(1 + q_u) + a_u f_u}{f_d}\right\}\right]. \tag{30}$$

Only if the above interval is nonempty, leasing may occur. This is not obvious. However, it is easy to see that

$$\frac{1 - \left(\frac{1+i}{1+r}\right)^n}{f_d} < \frac{1 + q_s}{f_d} \tag{31}$$

and

$$\frac{k + a_u f_u}{f_d} < \frac{k(1 + q_u) + a_u f_u}{f_d}. \tag{32}$$

The relations between all four parts of the formula may vary in the general setting. This means that, under certain conditions, leasing is not acceptable.

5.4. Perpetual Usufruct Payments

Although the yearly payments in perpetual usufruct are fixed by the law and thus not negotiable, one could wonder whether they are set at the right level. Of course, changing the law is much more difficult than signing a contract, but it is still possible. The reasoning similar to the above allows us to define the acceptable range for the yearly payment in a perpetual usufruct. If it is possible to choose between selling or buying the property, then the following must hold:

$$PV_u \geq PV_s \tag{33}$$

$$PV_u^\star \leq PV_s^\star. \tag{34}$$

The first inequality is equivalent to

$$kX + a_u f_u X + \left(\frac{1+i}{1+r}\right)^n X \geq X, \tag{35}$$

i.e.,

$$a_u \geq \frac{1 - k - \left(\frac{1+i}{1+r}\right)^n}{f_u}. \tag{36}$$

Similarly, the second equation leads us to

$$kX(1 + q_u) + a_u f_u X \leq X(1 + q_s) \tag{37}$$

$$a_u \leq \frac{1 + q_s - k(1 + q_u)}{f_u}. \tag{38}$$

It follows that the rate of yearly payment in the perpetual usufruct should belong to the following interval:

$$a_u \in \left[\frac{1 - k - \left(\frac{1+i}{1+r}\right)^n}{f_u}, \frac{1 + q_s - k(1 + q_u)}{f_u}\right]. \tag{39}$$

6. Simulations

In this section, we present the results of the simulations. First, we assumed the following about the parameters, corresponding with the current situation in Polish market:

(1) expected average increase of the property's price: 1 or 2% (i.e., $i \in \{1\%, 2\%\}$);
(2) duration of the contract: 15, 20, 25, 30, 35, 40, or 99 years (i.e., $n \in \{15, 20, 25, 30, 35, 40, 99\}$);
(3) duration of the period, where the annual payments are fixed: 5 or 10 years (i.e., $t_d = t_u \in \{5, 10\}$);
(4) expected average increase in the lease rate and perpetual usufruct annual payment rate is equal to the increase in the property's value (i.e., $i_d = i_u = i$);
(5) loan's duration: 30 years (i.e., $n_h = 30$);
(6) share of equity: from 0% (the whole financed by a loan), by 25, 50, 75, to 100% (no loan), i.e., $w_s = w_u \in \{0\%, 25\%, 50\%, 75\%, 100\%\}$;
(7) expected discount rate: 3.5 or 5% (i.e., $r \in \{3.5\%, 5\%\}$);

(8) annual effective interest rate for the loan: discount rate plus 0.5 pp. (i.e., $i_h \in \{4\%, 5.5\%\}$, depending on r);
(9) initial payment rate for perpetual usufruct: 25% (i.e., $k = 25\%$); and
(10) annual payment rate for perpetual usufruct: 1% (i.e., $a_u = 1\%$).

The results of the simulations are presented in Appendix A.3. The main conclusion from the simulation is that the existence of perpetual usufruct influences the possible lease rate more than the possible sale or purchase. In every case, the interval is much smaller in this case. Moreover, in the case of long-term leases, the existence of the perpetual usufruct makes the lease impossible. For example, from Table A8, for 20 years, we have a minimum possible lease rate deduced from the sale or purchase equal to 3.26%, whereas the maximum possible rate allowed by the perpetual usufruct is 2.99%. This shows that the perpetual usufruct is the main reason why long-term leases are not popular in Poland.

7. Conclusions

The generalization of Mandell's model presented in the article enables combining various additional elements affecting the Polish real estate market. In particular, we considered the third possible form of possessing property, which is the perpetual usufruct. We also analyzed the more general example, where the yearly payments are not adjusted every year but may be fixed for some period, defined in the contract, which also occurs in the Polish market.

The first conclusions from the experiments prove our suppositions that the leasing rates used in Poland in the leasing contracts between local communities and individuals have been improperly defined. Another conclusion is that the yearly payment rates in perpetual usufructs are often incorrect. We are aware of the fact that any changes in the law are difficult and time-consuming in estimation, but our conclusions could be a starting point for further considerations in this area.

There are some limitations of the presented model. For instance, we assumed that the durations of leasing and perpetual usufruct are equal. This could be overcome by introducing two possibly distinct durations. Another limitation consists in the assumptions about the constant characteristics of the parameters. Of course, parameters such as interest rates change over time, but it is almost impossible to predict their future values, especially in emerging markets like Poland. For that reason, some simplifications were necessary. However, we discussed the present situation and decisions of the owner and possible user, so analyzing the possible scenarios based on the current market situation seems to be reasonable, if we assume symmetry of information.

As mentioned at the beginning of this section, we performed numerical experiments. We think that performing simulations for other countries that have a similar legal situation would be interesting. We also think that the model itself could be further developed by introducing additional parameters, such as expected income of the user resulting from the possession of the property compared with other possible forms of earning. Although the conducted simulations refer to Polish conditions, the model presented can be used in any other economy, demonstrating the value of our study. These models consider many economic parameters and quantities that affect the decisions made in sustainable real estate management.

Author Contributions: M.A. and M.T. together designed research and wrote the paper. In particular, M.T. presented the concept of perpetual usufruct in Section 2, whereas M.A. developed the mathematical model in Section 5 and the appendix. A.B. and R.T. provided extensive advice throughout the study regarding the abstract, introduction, literature review, research methodology and data, and the results of the manuscript. The discussion was a team task. All authors have read and approved the final manuscript.

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Appendix A

In this appendix, we present the extended version of the reasoning that allows for one to derive the equations presented in Sections 5.1 and 5.2.

Appendix A.1. Owner's Perspective

The first possible decision of the owner may be to sell the property. In such cases, the present value of the income may be estimated by the market price of the property:

$$PV_s = X. \tag{A1}$$

The next possible choice is to lease the property. This may take two forms: the perpetual usufruct or the regular lease. Let us start with analyzing leasing. In the beginning, the initial payment is made. As mentioned before, it is equal to $Y = kX$, i.e.,

$$Y = kPV_s. \tag{A2}$$

Then, each year, the lessee needs to pay the yearly rent $A = a_u X$, where a_u denotes the rate and X the market price. The payment is fixed during some period expressed in a number of years (denoted by t_u), which is then adjusted according to the yearly increase in rent i_u, often deduced from the increase in the property's market price. This means that the yearly payment (in constant prices) increases after each t_u years by the growth factor:

$$g = \left(\frac{1+i_u}{1+r}\right)^{t_u} \tag{A3}$$

where r is the expected discount rate. Since the total number of years n of the perpetual usufruct does not need to be divisible by the number of years where the rent is fixed, we need to distinguish the number of full periods and the remainder. The number of full periods during which the payment is fixed is denoted by n_{u0} and may be computed from the formula:

$$n_{u0} = \left\lfloor \frac{n}{t_u} \right\rfloor = n \text{ div } t_u \tag{A4}$$

where $\lfloor \cdot \rfloor$ denotes the *floor* function, i.e., the biggest integer not greater than the argument (e.g., $\lfloor 1.2 \rfloor = 1$, $\lfloor 13.77 \rfloor = 13$), whereas div is the integer division function. The number of remaining years (i.e., the number of years in the last period, not longer than t_u, where the payment remains fixed) is then equal to

$$n_{u1} = n - t_u n_{u0} = n \bmod t_u \tag{A5}$$

where mod denotes the remainder of the integer division. This means that the value of the yearly payments during t_u years after t changes of the payment rate ($t = 0, 1, \ldots, n_{u0} - 1$), at tt_u years from the beginning (i.e., after t fixed payment periods) equals

$$\sum_{j=1}^{t_u} \frac{a_u X}{(1+r)^j} = a_u X \times \frac{1-(1+r)^{-t_u}}{r}. \tag{A6}$$

This means that the present value of the stream of all the payments made during all the complete fixed payment periods is

$$\begin{aligned} a_u X \times \tfrac{1-(1+r)^{-t_u}}{r} \times \sum_{j=0}^{n_{u0}} g^j &= a_u X \times \tfrac{1-(1+r)^{-t_u}}{r} \times \sum_{j=0}^{n_{u0}} \left(\tfrac{1+i_u}{1+r}\right)^{t_u j} \\ &= a_u X \times \tfrac{1-(1+r)^{-t_u}}{r} \times \tfrac{1-\left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}}}{1-\left(\frac{1+i_u}{1+r}\right)^{t_u}}. \end{aligned} \tag{A7}$$

The present value of the stream of the payments made during the remaining n_{u1} years equals

$$\sum_{j=1}^{n_{u1}} \frac{a_u X(1+i_u)^{t_u n_{u0}}}{(1+r)^{t_u n_{u0}+j}} = a_u X \times \frac{1-(1+r)^{-n_{u1}}}{r} \times \left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}}. \tag{A8}$$

For simplicity of the further notation, let us define the factor f_u as

$$f_u = \frac{1-(1+r)^{-t_u}}{r} \times \frac{1-\left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}}}{1-\left(\frac{1+i_u}{1+r}\right)^{t_u}} + \frac{1-(1+r)^{-n_{u1}}}{r} \times \left(\frac{1+i_u}{1+r}\right)^{t_u n_{u0}}. \tag{A9}$$

The total income of the property's owner is equal to the sum of the present values of the initial payment, yearly payments, and the discounted future value of the property, including the expected average increase of its value i. Considering the above, the present value of the income after choosing the variant of the perpetual usufruct is

$$PV_u = kX + a_u f_u X + \left(\frac{1+i}{1+r}\right)^n X. \tag{A10}$$

Note that, if the payments can be adjusted each year, i.e., when $t_u = 1$, the formula for f_u simplifies to

$$f_u = \frac{1-\left(\frac{1+i_u}{1+r}\right)^n}{r-i_u}. \tag{A11}$$

Similar considerations allow us to analyze the case of the regular lease contract. This time, the number of years in a fixed payment period is denoted by t_d. The number of such complete periods equals

$$n_{d0} = \left\lfloor \frac{n}{t_d} \right\rfloor = n \text{ div } t_d, \tag{A12}$$

whereas the number of years in the remainder is

$$n_{d1} = n - t_d n_{d0} = n \bmod t_d. \tag{A13}$$

As before, if we define the factor

$$f_d = \frac{1-(1+r)^{-t_d}}{r} \times \frac{1-\left(\frac{1+i_d}{1+r}\right)^{t_d n_{d0}}}{1-\left(\frac{1+i_d}{1+r}\right)^{t_d}} + \frac{1-(1+r)^{-n_{d1}}}{r} \times \left(\frac{1+i_d}{1+r}\right)^{t_d n_{d0}}, \tag{A14}$$

then the present value of the total income of the owner is

$$PV_d = a_d f_d X + \left(\frac{1+i}{1+r}\right)^n X. \tag{A15}$$

In particular, if $t_d = 1$ (i.e., if the yearly payments are adjusted each year), then

$$f_d = \frac{1-\left(\frac{1+i_d}{1+r}\right)^n}{r-i_d}. \tag{A16}$$

Given the three variants, the owner chooses the lease if $PV_d \geq \max\{PV_s, PV_u\}$. This allows us to derive the minimum payment rate that is acceptable for the owner. First, let us compare lease with selling the property:

$$PV_d \geq PV_s \tag{A17}$$

$$a_d f_d X + \left(\frac{1+i}{1+r}\right)^n X \geq X \tag{A18}$$

$$a_d \geq \frac{1-\left(\frac{1+i}{1+r}\right)^n}{f_d}. \tag{A19}$$

Similarly, the comparison with the perpetual usufruct allows us to find the following lower bound:

$$PV_d \geq PV_u \tag{A20}$$

$$a_d f_d X + \left(\frac{1+i}{1+r}\right)^n X \geq kX + a_u f_u X + \left(\frac{1+i}{1+r}\right)^n X \tag{A21}$$

$$a_d \geq \frac{k + a_u f_u}{f_d}. \tag{A22}$$

Finally, the yearly payment rate must satisfy the condition

$$a_d \geq \max\left\{\frac{1-\left(\frac{1+i}{1+r}\right)^n}{f_d}, \frac{k + a_u f_u}{f_d}\right\} \tag{A23}$$

if the owner is supposed to choose the leasing.

Appendix A.2. User's Perspective

The calculations similar to those performed in the previous subsection also allow us to estimate the costs of the potential user (buyer or lessee). We assume that the user may cover part of the cost using the loan with interest rate i_h that must be paid back in n_h years. We assume that the loan is paid with decreasing instalments n_h fixed principal payments and respective interest payments. In case of a different payment schedule, the formulae given below may be easily transformed using the elementary rules of financial mathematics. We assume that part of the cost, $w_s < 1$, must be paid in cash. Then, the total present value of the cost of buying the property is

$$PV_s^\star = w_s X + (1-w_s) \times \frac{X}{n_h} \times \frac{1-\left(\frac{1+i_h}{1+r}\right)^{n_h}}{1-\frac{1+i_h}{1+r}}. \tag{A24}$$

We may observe that

$$PV_s^\star = PV_s(1+q_s) \tag{A25}$$

where

$$q_s = w_s + \frac{(1-w_s)}{n_h} \times \frac{1-\left(\frac{1+i_h}{1+r}\right)^{n_h}}{1-\frac{1+i_h}{1+r}} - 1 > 0; \tag{A26}$$

(i.e., $PV_s^\star > PV_s$).

Similarly, we can estimate the present value of the cost of perpetual usufruct and lease. In both cases, we consider the fact that, when the contract ends, the property goes back to the owner. Let us consider the perpetual usufruct first. We assume that the potential user may cover part of the initial payment (w_u) using the bank loan. This means that the present value of the cost of the perpetual usufruct is

$$PV_u^\star = kPV_s(1+q_u) + a_u f_u X \tag{A27}$$

where

$$q_u = w_u + \frac{(1-w_u)}{n_h} \times \frac{1-\left(\frac{1+i_h}{1+r}\right)^{n_h}}{1-\frac{1+i_h}{1+r}} - 1 > 0. \tag{A28}$$

Finally, the present value of the total cost corresponding with lease equals

$$PV_d^\star = a_d f_d X = PV_d - \left(\frac{1+i}{1+r}\right)^n X. \tag{A29}$$

The reasoning is similar to that of the previous section, leading us to the conclusion that the user will accept the form of leasing if $PV_d^\star \leq \min\{PV_s^\star, PV_u^\star\}$. This allows us to estimate the highest payment rate acceptable by the user. We have

$$PV_d^\star \leq PV_s^\star \tag{A30}$$

$$a_d f_d X \leq X(1+q_s) \tag{A31}$$

$$a_d \leq \frac{1+q_s}{f_d}, \tag{A32}$$

and

$$PV_d^\star \leq PV_u^\star \tag{A33}$$

$$a_d f_d X \leq kX(1+q_u) + a_u f_u X \tag{A34}$$

$$a_d \leq \frac{k(1+q_u) + a_u f_u}{f_d}. \tag{A35}$$

Thus, if the user is supposed to choose the form of leasing, the following condition must be satisfied:

$$a_d \leq \min\left\{\frac{1+q_s}{f_d}, \frac{k(1+q_u) + a_u f_u}{f_d}\right\}. \tag{A36}$$

Appendix A.3. Simulations Results

In Tables A1 and A2, we present the results of the simulations for $r = 3.5\%$, $i = 1\%$.

Table A1. Results of the simulations for $r = 3.5\%$, $i = 1\%$, $t_d = t_u = 5$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	2.55%	2.55%	2.55%	2.55%	2.55%	2.55%	2.55%
Max. (purchase)	0%	8.91%	7.07%	5.98%	5.26%	4.76%	4.38%	3.00%
	25%	8.76%	6.95%	5.88%	5.17%	4.68%	4.31%	2.95%
	50%	8.61%	6.83%	5.78%	5.08%	4.59%	4.23%	2.90%
	75%	8.45%	6.71%	5.68%	4.99%	4.51%	4.16%	2.85%
	100%	8.30%	6.59%	5.57%	4.90%	4.43%	4.08%	2.80%
Min. (p.u.) [1]	arbitrary	3.08%	2.65%	2.39%	2.23%	2.11%	2.02%	1.70%
Max. (p.u.)	arbitrary	3.23%	2.77%	2.50%	2.32%	2.19%	2.10%	1.75%

Source: own calculations. [1] p.u.—perpetual usufruct.

Table A2. Results of the simulations for $r = 3.5\%$, $i = 1\%$, $t_d = t_u = 10$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	2.59%	2.61%	2.60%	2.61%	2.60%	2.61%	2.61%
Max. (purchase)	0%	9.06%	7.24%	6.10%	5.39%	4.86%	4.49%	3.07%
	25%	8.90%	7.11%	5.99%	5.29%	4.77%	4.41%	3.02%
	50%	8.75%	6.99%	5.89%	5.20%	4.69%	4.33%	2.97%
	75%	8.59%	6.87%	5.79%	5.11%	4.61%	4.26%	2.91%
	100%	8.44%	6.74%	5.68%	5.02%	4.53%	4.18%	2.86%
Min. (p.u.)	arbitrary	3.11%	2.69%	2.42%	2.25%	2.13%	2.05%	1.72%
Max. (p.u.)	arbitrary	3.26%	2.81%	2.52%	2.35%	2.21%	2.12%	1.77%

Source: own calculations.

In Tables A3 and A4, we present the results of the simulations for $r = 3.5\%$, $i = 2\%$.

Table A3. Results of the simulations for $r = 3.5\%$, $i = 2\%$, $t_d = t_u = 5$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	1.56%	1.56%	1.56%	1.56%	1.56%	1.56%	1.56%
Max. (purchase)	0%	8.51%	6.61%	5.47%	4.72%	4.18%	3.78%	2.19%
	25%	8.36%	6.50%	5.38%	4.64%	4.11%	3.72%	2.15%
	50%	8.22%	6.38%	5.29%	4.56%	4.04%	3.65%	2.11%
	75%	8.07%	6.27%	5.19%	4.48%	3.97%	3.59%	2.08%
	100%	7.93%	6.16%	5.10%	4.40%	3.90%	3.52%	2.04%
Min. (p.u.)	arbitrary	2.98%	2.54%	2.27%	2.10%	1.97%	1.88%	1.51%
Max. (p.u.)	arbitrary	3.13%	2.65%	2.37%	2.18%	2.05%	1.95%	1.55%

Source: own calculations.

Table A4. Results of the simulations for $r = 3.5\%$, $i = 2\%$, $t_d = t_u = 10$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	1.61%	1.63%	1.62%	1.63%	1.62%	1.63%	1.63%
Max. (purchase)	0%	8.78%	6.92%	5.69%	4.94%	4.36%	3.96%	2.29%
	25%	8.63%	6.80%	5.59%	4.86%	4.28%	3.90%	2.25%
	50%	8.48%	6.69%	5.49%	4.77%	4.21%	3.83%	2.21%
	75%	8.33%	6.57%	5.40%	4.69%	4.13%	3.76%	2.18%
	100%	8.18%	6.45%	5.30%	4.61%	4.06%	3.69%	2.14%
Min. (p.u.)	arbitrary	3.05%	2.61%	2.32%	2.15%	2.02%	1.92%	1.53%
Max. (p.u.)	arbitrary	3.20%	2.73%	2.42%	2.24%	2.09%	1.99%	1.57%

Source: own calculations.

Finally, in Tables A5–A8, we assume that the discount rate is higher (r = 5%; a higher cost of the loan was assumed), and the remaining assumptions are the same as in the previous cases.

Table A5. Results of the simulations for $r = 5\%$, $i = 1\%$, $t_d = t_u = 5$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	4.08%	4.08%	4.08%	4.08%	4.08%	4.08%	4.08%
Max. (purchase)	0%	9.90%	8.09%	7.04%	6.35%	5.88%	5.54%	4.47%
	25%	9.73%	7.96%	6.92%	6.25%	5.78%	5.45%	4.39%
	50%	9.57%	7.82%	6.80%	6.14%	5.68%	5.36%	4.32%
	75%	9.40%	7.68%	6.68%	6.03%	5.58%	5.26%	4.24%
	100%	9.23%	7.55%	6.56%	5.92%	5.49%	5.17%	4.17%
Min. (p.u.)	arbitrary	3.31%	2.89%	2.64%	2.48%	2.37%	2.29%	2.04%
Max. (p.u.)	arbitrary	3.47%	3.02%	2.76%	2.59%	2.47%	2.39%	2.12%

Source: own calculations.

Table A6. Results of the simulations for $r = 5\%$, $i = 1\%$, $t_d = t_u = 10$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	4.14%	4.17%	4.16%	4.17%	4.16%	4.17%	4.17%
Max. (purchase)	0%	10.06%	8.27%	7.17%	6.49%	6.00%	5.67%	4.57%
	25%	9.89%	8.14%	7.05%	6.39%	5.90%	5.57%	4.49%
	50%	9.72%	8.00%	6.93%	6.28%	5.80%	5.48%	4.41%
	75%	9.55%	7.86%	6.81%	6.17%	5.70%	5.38%	4.34%
	100%	9.38%	7.72%	6.69%	6.06%	5.60%	5.29%	4.26%
Min. (p.u.)	arbitrary	3.35%	2.93%	2.67%	2.51%	2.40%	2.32%	2.06%
Max. (p.u.)	arbitrary	3.52%	3.07%	2.79%	2.62%	2.50%	2.42%	2.14%

Source: own calculations.

Table A7. Results of the simulations for $r = 5\%$, $i = 2\%$, $t_d = t_u = 5$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%	3.12%
Max. (purchase)	0%	9.48%	7.59%	6.48%	5.75%	5.24%	4.87%	3.54%
	25%	9.32%	7.47%	6.37%	5.66%	5.15%	4.79%	3.48%
	50%	9.16%	7.34%	6.26%	5.56%	5.07%	4.70%	3.42%
	75%	9.00%	7.21%	6.15%	5.46%	4.98%	4.62%	3.36%
	100%	8.84%	7.08%	6.05%	5.36%	4.89%	4.54%	3.30%
Min. (p.u.)	arbitrary	3.21%	2.77%	2.51%	2.34%	2.22%	2.14%	1.83%
Max. (p.u.)	arbitrary	3.37%	2.90%	2.62%	2.44%	2.31%	2.22%	1.89%

Source: own calculations.

Table A8. Results of the simulations for $r = 5\%$, $i = 2\%$, $t_d = t_u = 10$.

Lease Rate	Rate of Equity	Lease Duration (Years)						
		15	**20**	**25**	**30**	**35**	**40**	**99**
Min. (sale)	arbitrary	3.22%	3.26%	3.24%	3.26%	3.25%	3.26%	3.26%
Max. (purchase)	0%	9.78%	7.94%	6.73%	6.02%	5.46%	5.09%	3.70%
	25%	9.62%	7.81%	6.62%	5.91%	5.37%	5.01%	3.64%
	50%	9.45%	7.67%	6.51%	5.81%	5.28%	4.92%	3.58%
	75%	9.29%	7.54%	6.39%	5.71%	5.18%	4.83%	3.52%
	100%	9.12%	7.41%	6.28%	5.61%	5.09%	4.75%	3.45%
Min. (p.u.)	arbitrary	3.28%	2.85%	2.57%	2.40%	2.27%	2.19%	1.86%
Max. (p.u.)	arbitrary	3.45%	2.99%	2.68%	2.50%	2.36%	2.27%	1.93%

Source: own calculations.

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