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The application of FRP reinforcement in concrete structures is quite popular nowadays. In this article the cracking moment of flexural concrete structures reinforced by steel and FRP reinforcement bars is investigated. There is estimated that the behaviour of tensile concrete is nonlinear when crack opens. The nonlinearity was estimated by two different cases. In the first one the coefficient of elastic and plastic strain of tensile concrete were applied and in the second the stress-strain relationship were expressed by polynomial function. That function prepared according to compressed concrete stress-strain diagram described in design standard EN 1992-1-1. Those two different stress-strain relationships were used for cracking moment calculation. Also the cracking moment calculations were performed using mentioned standard EN and Technical building code STR 2.05.05:2005. In all cases the calculations were performed in assumption that the behaviour of compressed concrete is elastic when crack opens. The obtained results were analysed and presented base conclusions.

KEYWORDS: FRP reinforcement, cracking moment, stress-strain relationship, nonlinear behaviour, tensile concrete.

Introduction

The steel as base reinforcement in reinforced concrete structures is used many years, however it properties satisfy not all requirements to reinforcement, especially the requirements of resistance to corrosion. For this reason the durability of concrete reinforced by steel reinforcement is not so high in aggressive condition to corrosion (Abdala 2002, Al-Sunna, 2012, Toutanji 2003).

Nowadays the low durability problem could be solving applying the new type of reinforcement in concrete structures i.e. the FRP (fiber reinforced polymer) reinforcement which has higher tensile strength and higher resistance to corrosion (Barrs el.al. 2012, Barris et.al 2009, Ashour 2006).

Such type of reinforcement could be used in concrete structures as reinforcement bars, meshes or reinforcing cages. The application of FRP reinforcement in concrete structures of buildings, engineering works or underground works increase the durability of them and resistance to various damages.

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However such type of reinforcement has lower elasticity and worse anchorage with concrete. The modulus elasticity of FRP reinforcement is several or more times less than modulus of steel. For this reason the stiffness and cracking problem arises to the flexural structures because into the cracked section the tensile forces are permitted only by tensile reinforcement. In this case into the concrete structures reinforced by FRP reinforcement the cracking moment will be reached earlier and the deflection will be higher in comparing with simply reinforced structures. So, it could be concluded that high tensile strength of FRP reinforcement in flexural structures could not be exhaust because of too high deformability and application so not effective economically and structurally.

In that case FRP reinforcement could be used with steel reinforcement together in order to increase the durability of structures. Because of different elasticity the stress in steel reinforcement will be higher than in FRP and the strength of structures significantly depend on amount and strength of steel reinforcement.

But amount of steel reinforcement has no such significantly effect on cracking moment because it more depends on the concrete tensile strength and dimensions of cross section. For example according to principles of design standard EN 1992-1-1:2004 the amount of reinforcement has no effect on cracking moment (Mosley 2007)

Also the FRP reinforcement is more brittle in compare with steel. So, the application of such reinforcement into the bridges structures could be limited.

Review of main properties of FRP reinforcement

The FRP reinforcement bar consists from the material (glass or basalt) strands of fibers connected by polymers (http://www.schoeck-combar.com, http://galen.su). Such structure of fibers materials describes the high strength that could be several or more times higher than strength of solid cross section materials. The FRP reinforcement has high strength in longitudinal direction. Because of high resistance to alkalinity medium in concrete the FRP reinforcement durability is about 100 year and more. It should be mentioned that the bond between concrete and steel reinforcement is better than FRP reinforcement but in this article it is not investigated.

The reinforcement could be in different types. The most popular is GFRP (Glass fiber reinforced polymer) and Basalt fiber reinforcement, but GFRP reinforcement more use in concrete structures because of less price.

The main properties of GFRP and Basalt fiber reinforcement are presented in table 1.

Table 1
The main propeties of steel and FRP reinforcement (http://www.schoeck-combar.com, http://galen.su)

Characteristics of materials	Steel reinforcement S400 class	GFRP reinforcement „Schőck ComBAR“	GFRP reinforcement „ROCKBAR“	Bazalt fiber reinforcement „ROCKBAR“
Tensile strength MPa	360	1200	1000	1200
Density, t/m^3	7	2,2	2,0	2,0
Corrosion	Corrosion materals	No corrosion material satisfy to the first group of chemical resistance		
Length of reinforcement product, m	Bars of 6-12m length	Bars of 10-14m length	Bars until 12m length	
Elastic Modulus, N/mm^2	200000	60000	45000	50000-55000
Cover, mm	According to codes, STR	Ø+10mm, For precast reinforced concrete Ø+5mm		

The tensile concrete stress-strain relationship

In this investigation the concrete stress-strain relationship were expressed in two cases.

1 case. Analysing the elastic plastic behaviour of tensile concrete should be note that $\sigma_c - \varepsilon_c$ relationship could have quite significant influence to the crack moment M_{crc}. The EC2 standard presents $\sigma_c - \varepsilon_c$ relation for compressed concrete but not describe the tensile concrete. If to assume that ultimate strain of tensile concrete correspond the ultimate tensile stress (f_{ctm}), and the form of tensile concrete $\sigma_c - \varepsilon_c$ curve correspond to the form of compressed concrete until the peak of stress, it is possible to apply the EC2 standard expression of stress-strain relationship.

In all figures the font of text are changed in "times new roman" and in **fig. 1** and **fig. 2** are presented units of tensile stress

So, if to calculate the tensile stress σ_{ct} directly according to the EC2 standard expression rewriting the f_{ctm} instead of f_{cm} only, we will obtain:

$$\sigma_{ct} = f_{ctm}\left(\frac{k\eta - \eta^2}{1+(k-2)\eta}\right), \quad (1)$$

where:

$k = 1{,}05\, E_{cm}\, \varepsilon_{ct,1} / f_{ctm}$;

$\varepsilon_{ct,1} = 0{,}0007\, f_{ctm}^{0.31}$.

According such way drawing up the $\sigma_c - \varepsilon_c$ relationship of tensile concrete we will obtain the curve that form at all will not be similar to the form of curve of compressed concrete **(fig. 2)**. In this case the curves forms not correspond each other because of different values of the ratios $f_{cm}/\varepsilon_{c,1}$ and $f_{ctm}/\varepsilon_{ct,1}$. For example to the concrete class $C\,20/25$ the numerical values is: $f_{cm} = 28MPa$; $\varepsilon_{c,1} = 0{,}001967$, $f_{ctm} = 2{,}21MPa$, $\varepsilon_{ct,1} = 8{,}95 \cdot 10^{-4}$. Then we obtain that the ratio of $f_{cm}/\varepsilon_{c,1} = 14234{,}875$ and $f_{ctm}/\varepsilon_{ct,1} = 2469{,}273$ differ more than six times. Also, according such case the secant modulus of compressed concrete at the peak stress $(\sigma_{ct} = f_{ctm})$ was 5,765 times bigger than to the tensile concrete. If to express the ultimate elastic plastic coefficient at the stress peak it would obtained the $\lambda = 0{,}475$ to the compressed concrete and only the $\lambda_{ct} = 0{,}0842$ to the tensile concrete. If to assume the one and the same elastic plastic coefficient to the compressed and tensile concrete, the strain at the peak stress $\varepsilon_{ct,1}$ should be expressed with different factor as $\varepsilon_{ct,1} = 0{,}000121^{0,31}_{f_{ctm}}$. This factor, of course, depend on the f_{ctm}, so, to the concrete class from $C\,8/10$ to $C\,50/60$ the strain could be expressed with average factor:

$$\varepsilon_{ct,1} = 0.00156 f_{ctm}^{1.31} / f_{cm}. \quad (2)$$

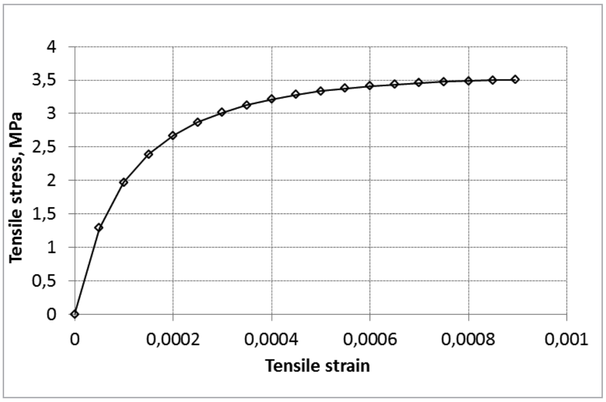

Fig. 1

The tensile concrete stress-strain relationship when the EC2 standard expression is used. Concrete class C40/50

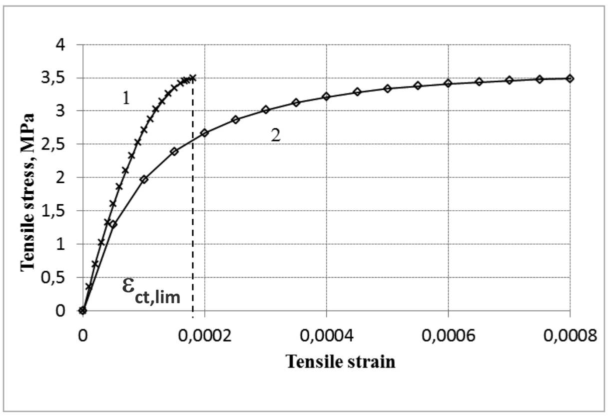

Fig. 2

The stress-strain relationship expressed according to the equation (4) (1-curve) and equation (1) (2-curve)

Then the member *k* to the tensile concrete would be obtained from the other equation

$$k = \frac{1.05E_{cm}\varepsilon_{ct,1}}{f_{ctm}} = 1.05 \cdot 22000\left(\frac{f_{cm}}{10}\right)^{0.3} \cdot 0.00156\frac{f_{ctm}^{1.31}}{f_{cm}\cdot f_{ctm}} = 18.06 f_{ctm}^{0.31} f_{cm}^{-0.7} \; . \tag{3}$$

When the ultimate concrete strain it is known the tensile stress could be expressed according to the adjusted EC2 equation:

$$\frac{\sigma_{ct}}{f_{ctm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta} = \frac{11577 f_{ctm}^{-1} f_{cm}^{0.3}\varepsilon_{ct} - 410914 f_{cm}^{2} f_{ctm}^{-2.62}\varepsilon_{ct}^{2}}{1+641\cdot\left(18.06 f_{ctm}^{0.31} f_{cm}^{-0.7} - 2\right) f_{cm} f_{ctm}^{-1.31}\varepsilon_{ct}} \; . \tag{4}$$

The tensile stress of concrete (class 40/50) obtained according to the equation (4) is presented in **fig. 2** where the peak of stress is reached at the strain equal to 0.000165.

The obtained expression (4) is not so convenient to the resultant calculation of the tensile and compressed concrete zones and to the crack moment also, because the strain member is in the denominator. Such not well on calculation of integrals. In this case more convenient the tensile stress expressed by polynomials (Židonis 2007, 2009, 2010). Then the stress of tensile concrete will be equal

$$\sigma_{ct} = a_1 \cdot \varepsilon_{ct}^2 + a_2 \cdot \varepsilon_{ct} + a_3 \, , \tag{5}$$

where:

where a_1, a_2, a_3 – free members.

The free members corresponded the stress-strain relationship of the C30/37 class concrete is ual to: $a1 = -105003697.6,\ a2 = 34987.8,\ a3 = 0.0127.$

2 case. It is also convenient to apply the elastic plastic coefficient λ to express the $\sigma - \varepsilon$ relationship. If to assume that this coefficient linear vary from the beginning of elastic plastic zone until to concrete fracture it could be expressed as (Augonis 2013)

$$\lambda = 1 - \lambda_{lim}\frac{\varepsilon_i - \varepsilon_{el}}{\varepsilon_{lim} - \varepsilon_{el}} \, , \tag{6}$$

where λ_{lim} – ultimate value of the elastic plastic coefficient at the moment of failure;

ε_i – strain;

ε_{lim} – ultimate strain (strain at the moment of failure) $\varepsilon_{lim} = f_c / (\lambda_{lim}E_c)$,

ε_{el} – ultimate elastic strain $\varepsilon_{el} \approx \frac{0,4 f_c}{E_c}$.

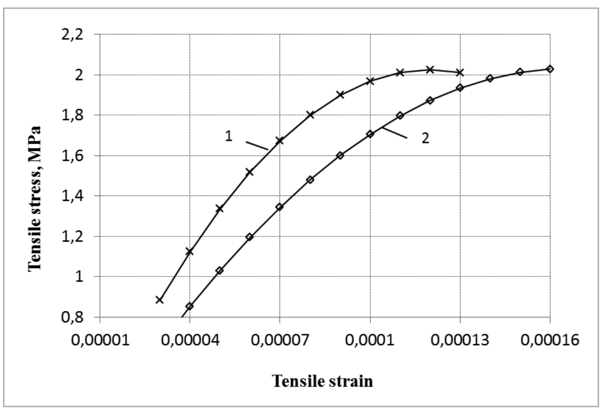

Fig. 3

The stress-strain relationship of tensile concrete ($f_{ct} = f_{ctk,005}$). 1 – when the elastic plastic coefficient $\lambda_{ct,lim} = 0,5$ is applied, 2 – when the equation (5) is applied with the factor 0,7

Then the tensile stress could be expressed as

$$\sigma_{ct} = E_c \lambda_{ct} \varepsilon_{ct} \, . \tag{7}$$

The stress-strain relationships expressed according to the equation (5) and equation (7) to the C30/37 class concrete is presented in **fig. 3** where the are used not the average values of stress (f_{ctm}) but characteristic values (fctk0,05). For this reason the factor 0,7 were applied to the values obtained by equation (5). According to that figure the curves is differ but it should be note that the ultimate strain is differ also.

The crack moment of members reinforced by steel reinforcement and FRP reinforcement

In this section the cracking moments are calculated with assumption that the behaviour of compressed concrete is elastic because not high the amount of reinforcement was predicted. For calculations the stress and strain schemes presented in **fig. 4** were used.

Calculating the cracking moment of elements reinforced by FRP reinforcement, were assumed that the bond between reinforcement and concrete is quite enough.

When the cracking moment is calculating according to the elastic plastic coefficient the ultimate tensile strain is equal to $\varepsilon_{ct,lim} = f_{ct} / \lambda E_c$ (Baikov 1991). In this case it is important right to evaluate the member λ_{lim} when crack opens.

For tensile concrete at the short term failure case this coefficient approximately equal to ~0.5, i.e., $\lambda_{lim} = 0.5$ (Baikov 1991). Of course it is average value that could vary. But if to admit that value, the ultimate strain will be equal to $\varepsilon_{ct,lim} = 2f_{ct} / E_c$. Then the resultant of tensile concrete zone would be equal:

$$F_{ct} = \frac{1}{2}\varepsilon_{el} E_c \cdot b \cdot x_{el} + \int_{x_{el}}^{h-x} \varepsilon_{ct}\lambda_{ct} E_c b \,. \tag{8}$$

So, the cracking moment will be reached then the strain of the tensile concrete will reach the ultimate strain value $\varepsilon_{ct} = \varepsilon_{ct,lim}$. According this the cracking moment could be expressed by the next equation (Augonis, 2013)

$$M_{crc} = bE_c(h-kx)\left[\frac{3\varepsilon_{ct,el}x_{el}}{4} + \frac{(\varepsilon_{ct,\lim}-\varepsilon_{ct,el})x_{el}}{3} + \frac{\varepsilon_{ct,el}(h-kx)}{3} + \frac{5(\varepsilon_{ct,\lim}-\varepsilon_{ct,el})(h-kx)}{24}\right] + \frac{1}{3}bE_c\varepsilon_c x^2 + \frac{1}{3}bE_c\frac{\varepsilon_{ct,el}^3}{\varepsilon_c^2}x^2 + \frac{\varepsilon_c(d-x)^2}{x}E_s A_s\,; \tag{9}$$

where:

$$k = 1 + \frac{\varepsilon_{ct,el}}{\varepsilon_c}\,; \quad x_{el} = \frac{\varepsilon_{ct,el}x}{\varepsilon_c}\,; \quad \varepsilon_c = \frac{\varepsilon_{ct,lim}x}{h-x}\,.$$

The height of the compressed zone is calculated according to the force equilibrium equation.

According to the equation (9) obtained cracking moment results were compared with results according to the methods of standard EC2 and STR For calculations were accepted the rectangular section beam with parameters: $b = 0.2m$, $d_1 = 0.05m$, $E_s = 200GPa$, $E_k = 50GPa$, $E_c = 30GPa$, $A_s = A_k = 10cm^2$, C30/37. When the reinforcement area is constant and varies the height of cross section, the relationship of cracking moment and reinforcement ratio is presented in **fig. 5** where could be seen that the results obtained according to the equation (9) is between the results of EC2 and STR methods. In this figure the marking:

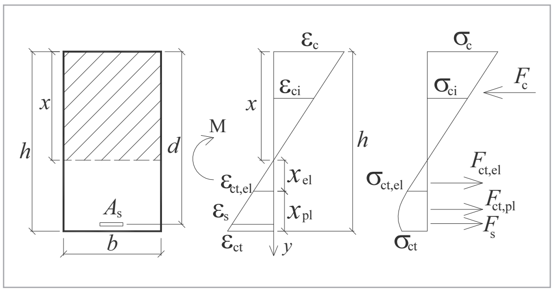

Fig. 4

Strain and stress of element when compressive concrete behaviour is elastic

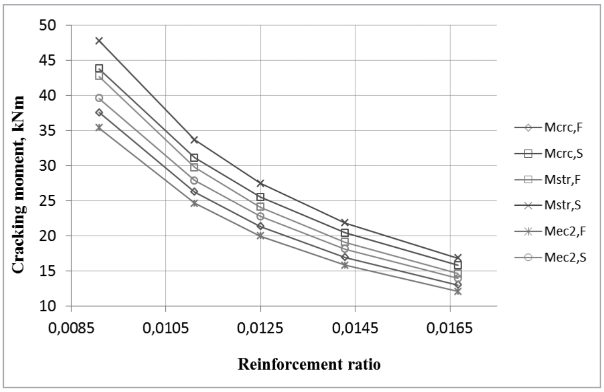

Fig. 5

The relationship of the cracking moment and the reinforcement coefficient when the area of the reinforcement is constant and varies the height of cross section

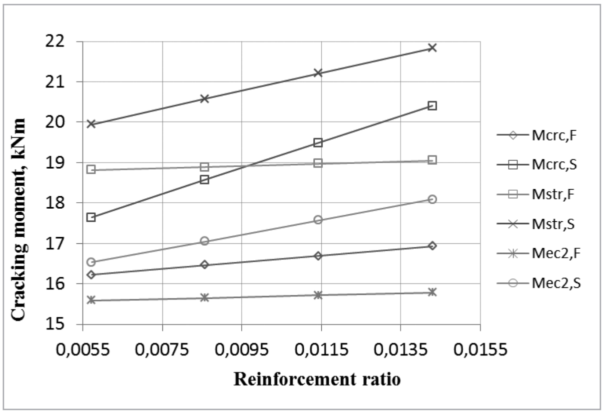

Fig. 6

The relationship of the cracking moment and the reinforcement coefficient when the area of cross section is constant and varies the area of reinforcement

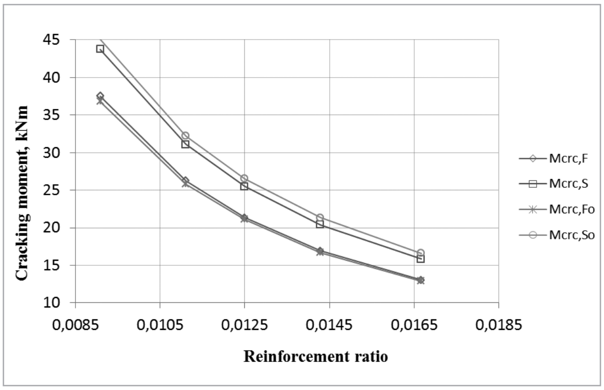

Fig. 7

The relationship of the cracking moment and the reinforcement coefficient when the area of the reinforcement is constant and varies the height of cross section. (Mcrc,Fo and Mcrc,So curves when the equation (5) is applied and Mcrc,F and Mcrc,S curves when the equation (7) is applied)

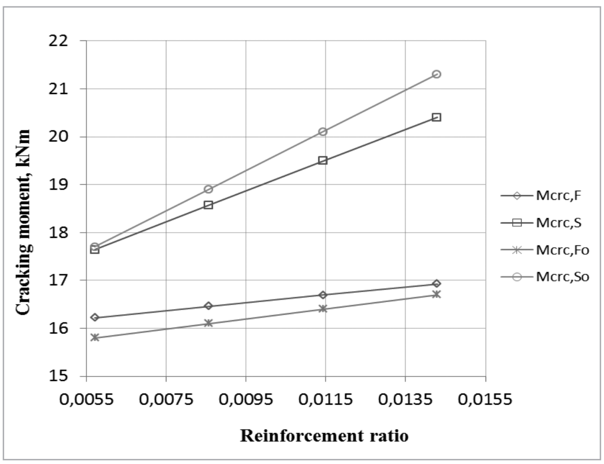

Fig. 8

The relationship of the cracking moment and the reinforcement coefficient when the area of cross section is constant and varies the area of reinforcement. (Mcrc,Fo and Mcrc,So curves when the equation (5) is applied and Mcrc,F and Mcrc,S curves when the equation (7) is applied)

"Mcrc,F" means the cracking moment of element reinforced by FRP reinforcement, "Mcrc,F" – reinforced by steel reinforcement, "Mec2" – cracking moment calculated according to EC2 method and "Mstr" – cracking moment calculated according to STR method.

When the cross section area is constant and varies the reinforcement area, the relationship of cracking moment and reinforcement ratio is presented in **fig. 6**. At this case also the results obtained according to the equation (8) is between the results of EC2 and STR methods but there is higher increase of cracking moment when the amount of reinforcement arises.

When the cracking moment is calculating according to the stress-strain expression (5) the resultant of the tensile concrete zone will be equal:

$$F_{ct} = \frac{1}{2}\varepsilon_{el} E_c \cdot b \cdot x_{el} + \int_{x_{el}}^{h-x} b\left(a_1 \cdot \varepsilon_{ct}^2 + a_2 \cdot \varepsilon_{ct} + a_3\right). \quad (10)$$

The calculation of cracking moment was performed analogically as before using the scheme presented in **figure 4**. The calculated cracking moment values according to the stress-strain expression (5) and expression (7) is presented in **figure 7** and **figure 8**.

The cracking moment obtained some higher when the polynomial stress-strain relationship were used. This increase is evaluated by higher value of the ultimate strain that is $\varepsilon_{ct,1} = 0,0001656$. In case when the expression (7) were used the ultimate strain value is $\varepsilon_{ct,lim} = 0,000133$. i.e., some lower.

Analysing the influence of stress-strain relationship to the cracking moment it is important to evaluate the elastic modulus of reinforcement.

Calculated the cracking moment values according to the standard EC2 and STR methods and equation (9) with vary elastic modulus of reinforcement is presented in **figure 9**.

In this figure could be seen that curves obtained using different expressions of stress-strain diagram of tensile is quite similar.

In these calculations were accepted such parameters: $C30/37$, $b = 0.2m$, $h = 0.4m$, $d_1 = 0.05m$, $A_s = 10cm^2$.

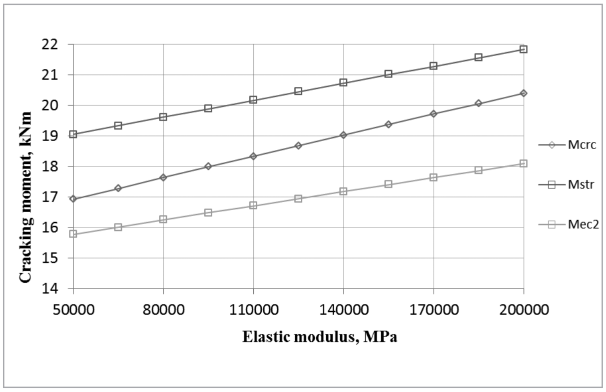

Fig. 9

The relationship of the cracking moment and the elastic modulus of reinforcement

If some part of steel reinforcement to change by FRP reinforcement the cracking moment will be obtained between values of element reinforced only by steel reinforcement and reinforced only by FRP reinforcement.

Conclusions

1 The application of FRP reinforcement has significant influence to the cracking moment of flexural elements in compare with steel reinforcement and it influence increase when the area of FRP reinforcement arises.

2 The application of different stress-strain diagrams has no significant influence to the cracking moment calculated to the flexural elements reinforced by FRP and steel reinforcement.

3 In all investigated cases the calculated cracking moment values varied between values obtained by standard EC2 and STR methods. This is because of rectangular diagram that is described in STR. Also it could be state that EC2 method describes highest reserve.

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