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Multi-Attribute Decision-Making Model by Applying Grey Numbers

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Abstract. Multi-attribute analysis is a useful tool in many economical, managerial, constructional, etc. problems. The accuracy of performance measures in COPRAS (The multi-attribute COmplex PRoportional ASsessment of alternatives) method is usually assumed to be accurate. This method assumes direct and proportional dependence of the weight and utility degree of investigated versions on a system of attributes adequately describing the alternatives and on values and weights of the attributes. However, there is usually some uncertainty involved in all multi-attribute model inputs. The objective of this research is to demonstrate how simulation can be used to reflect fuzzy inputs, which allows more complete interpretation of model results. A case study is used to demonstrate the concept of general contractor choice of on the basis of multiple attributes of efficiency with fuzzy inputs applying COPRAS-G method. The research has concluded that the COPRAS-G method is appropriate to use.

Keywords: multi-attribute, decision-making, model, discrete, COPRAS, COPRAS-G, grey number, contractor assessment.

1. Introduction

Multiple attribute decision-making problems are encountered under various situations where a number of alternatives and actions or candidates need to be chosen based on a set of attributes. When we consider a discrete set of alternatives described by some attributes, there are three different types of analyses that can be performed in order to provide a significant support to decision-makers:

- Ensure that the decision-maker follows a "rational" behaviour (normative option) – value functions, utility theory, distance to the ideal.
- Give some advice based on reasonable (but not indisputable) rules – The French School.
- Find the preferred solution from the partial decision hypothesis – interactive methods.

Decision-making analysis is an emerging discipline, having existed by name only since Howard (1966). It is interesting to reflect on the views of the three founders of decision-making analysis, as each (Howard, 1966; Keeney and Raiffa, 1976).

The main steps of multiple attributes decision-making are as follows:

- establishing system evaluation attributes that relate system capabilities to goals;
- developing alternative systems for attaining the goals (generating alternatives);
- evaluating alternatives in terms of attributes (the values of the attribute functions);
- applying a normative multiple attributes analysis method;
- accepting one alternative as "optimal" (preferred);
- if the final solution is not accepted, gather new information and go into the next iteration of multiple attributes optimization.

The analysis of the purpose is to be achieved by using attributes of effectiveness, which have different dimensions, different weight (Zavadskas and Vilutiene, 2006; Dzemyda *et al.*, 2007) as well as different directions of optimization (Kendall, 1970; Kaklauskas *et al.*, 2006; Viteikiene and Zavadskas, 2007). The discrete attribute values can be normalized by applying different normalization methods (Zavadskas *et al.*, 2003, 2007a; Zavadskas and Turskis, 2008; Ginevicius, 2008). The purpose of analysis also can be different (Dzemyda and Petkus, 2001; Kaklauskas *et al.*, 2007; Ginevicius *et al.*, 2007, 2008; Bregar *et al.*, 2008; Petkus and Filatovas, 2008).

The solving of each multi-attribute problem begins with constructing of decision-making matrix X. In this matrix values of the attributes x_{ji} may be: real numbers, intervals, probability distributions, possibility distributions, qualitative labels.

The problem may be (Roy, 1996):

- choice – select the most appropriate (best) alternative;
- ranking – draw a complete order of the alternatives from the best to the worst ones;
- sorting – select the best k alternatives from the list.

Multiple attributes decision aid (Hwang and Yoon, 1981) provides several powerful and effective tools (Zavadskas, 1987; Figueira *et al.*, 2005; Zavadskas and Kaklauskas, 2007; Ustinovichius *et al.*, 2007; Banaitienė *et al.*, 2008) for confronting sorting problems. There can be used very simplified techniques for the evaluation of a decision-making support methods base including methods such as the Simple Additive Weighting – SAW (Churchman and Ackoff, 1954; Tvaronavičienė *et al.*, 2008), TOPSIS – Technique for Order Preference by Similarity to Ideal Solution (Hwang and Yoon, 1981; Zavadskas *et al.*, 2006), COPRAS – COmplex PRoportional ASsessment (Zavadskas and Kaklauskas, 1996; Kaklauskas *et al.*, 2006; Zavadskas *et al.*, 2007b), fuzzy COPRAS (Zavadskas and Antuchevičienė, 2007), MOORA – Multi-Objective Optimization on basis of Ratio Analysis (Brauers and Zavadskas, 2006; Brauers *et al.*, 2008; Kalibatas and Turskis, 2008), ELECTRE – Elimination and Choice Translating Reality (Roy, 1991), Game theory methods (Peldschus and Zavadskas, 2005) and etc.

Comparing the alternatives is the key of making the decision. However, in case of conflicting alternatives, a decision-maker must also consider imprecise or ambiguous data, which is norm in this type of decision-making problems.

2. Grey Systems and Grey Relational Analysis (GRA) in Decision-Making

Deng (1982) developed the Grey system theory and presented (Deng, 1989) grey decision-making systems. Many authors investigated grey system theory in decision-making. Zhang *et al.* (1994) analysed information entropy of discrete grey numbers, Liu and Lin (2006) – information content of grey numbers, Olson and Wu (2006) – multi-attribute models for grey relationships. The grey system has been applied in many fields. Satapathy *et al.* (2006) applied GRA to speculate the performance of grey friction materials in compliance with the existing set of incomplete data. Noorul Haq and Kannan (2007) developed a hybrid normalized multi-attribute decision-making model for evaluating and selecting the vendor using Analytical Hierarchy Process and Fuzzy Analytical Hierarchy Process and an integrated approach of GRA to a Supply Chain model. Lin and Lee (2007) proposed a novel forecasting model. Huang *et al.* (2008) examined the potentials of the software effort estimation model by integrating a genetic algorithm (GA) to the GRA. Experimental results showed that the proposed method presents more precise estimates over the results using the case-based reasoning, classification and regression trees, and artificial neural networks methods. Lin *et al.* (2008) presented an illustrative example of subcontractor selection by applying grey TOPSIS method.

A grey number (Lin *et al.*, 2004) is a number whose exact value is unknown, but a range within which the value lies is known. There are the several types of grey numbers.

- Grey numbers with only lower limits: $\otimes \in [\underline{x}, \infty)$ or $\otimes(\underline{x})$, where a fixed real value $\underline{x}$ represents the lower limit of the grey number $\otimes$.
- Grey numbers with only upper limits: $\otimes \in (-\infty, \overline{x}]$ or $\otimes(\overline{x})$, where $\overline{x}$ is a fixed real number or an upper limit of the grey number $\otimes$.
- Interval grey number is the number with both lower limit $\underline{x}$ and upper limit $\overline{x}$: $\otimes \in [\underline{x}, \overline{x}]$.
- Continuous grey numbers and discrete grey numbers. The grey numbers taking a finite number of values or a countable number of values in an interval are called discrete. The continuously taking values, which cover an interval, are continuous.
- Black and white numbers. When $\otimes \in (-\infty, \infty)$ or $\otimes \in (\otimes_1, \otimes_2)$, i.e., when $\otimes$ has not upper neither and lower limits, or the upper and the lower limits are all grey numbers, $\otimes$ is called a black number. When $\otimes \in [\underline{x}, \overline{x}]$ and $\underline{x} = \overline{x}$,$\otimes$ is called a white number.

The theory of grey systems consists of the following main concepts and results:

- foundation, consisting of grey numbers, grey elements and grey relations;
- grey systems analysis, including grey incidence analysis, grey statistics, grey clustering, etc.;
- grey systems modelling, through the use of generation of grey numbers or function so that hidden patterns can be found;
- grey prediction;
- grey decision-making;
- grey control.

Table 1
Meaning of information

	White	Grey	Black
Information	Known	Incomplete	Unknown
Appearance	Bright	Grey	Dark
Process	Old	Replace old with new	New
Property	Order	Complexity	Chaos
Methodology	Positive	Transition	Negative
Attitude	Seriousness	Tolerance	Indulgence
Conclusion	Unique solution	Multiple solution	No results

Lin *et al.* (2004) has specified four possibilities for occurrence of the grey information:

- the information of elements is grey;
- the structural information is grey;
- the boundary information is grey;
- the behaviour information of motion is grey.

The meaning of being "grey" can be as is shown in Table 1.

According to Deng (1988), the GRA has some advantages:

- It involves simple calculations and requires a smaller number of samples; a typical distribution of samples is not needed.
- The quantified outcomes from the grey relational grade do not result in contradictory conclusions to qualitative analysis.
- The grey relational grade model is a transfer functional model that is effective in dealing with discrete data.

3. Ranking of the Alternatives by Applying COPRAS Methods

The multi-attribute decision-making methods mostly deal with exactly determined information. One of such methods is COPRAS method.

3.1. *Common COPRAS Method*

The algorithm of the COPRAS method consists of the steps as is shown in the Fig. 1.

1. Selection of the available set most important attributes, which describes alternatives.

2. Preparing of the decision-making matrix X:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix}; \quad j = \overline{1,n} \text{ and } i = \overline{1,m}; \tag{1}$$

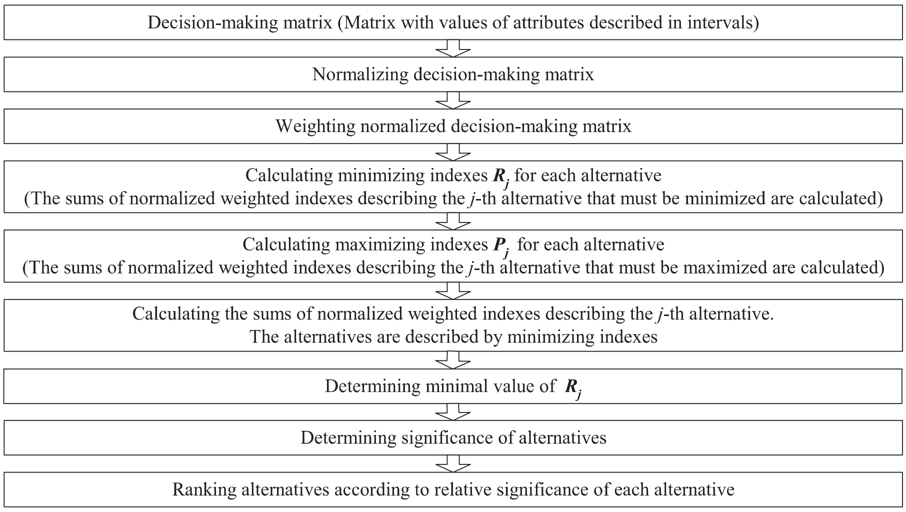

Fig. 1. Ranking of alternatives by applying COPRAS method.

where attribute i in the alternative j of a solution; m is the number of attributes; n is the number of the alternatives compared.

3. Determining weights of the attributes q_i (Kendall, 1970; Zavadskas, 1987).

4. Normalization of the decision-making matrix $\widehat{X}$. The normalized values of this matrix (Zavadskas, 1987) are calculated as

$$\overline{x_{ji}} = \frac{x_{ji}}{\sum_{j=1}^{n} x_{ji}}; \quad j = \overline{1,n} \text{ and } i = \overline{1,m}. \tag{2}$$

After this step we have normalized decision-making matrix:

$$\widehat{X} = \begin{bmatrix} \overline{x_{11}} & \overline{x_{12}} & \dots & \overline{x_{1m}} \\ \overline{x_{21}} & \overline{x_{22}} & \dots & \overline{x_{2m}} \\ \vdots & \vdots & \dots & \vdots \\ \overline{x_{n1}} & \overline{x_{n2}} & \dots & \overline{x_{nm}} \end{bmatrix}. \tag{3}$$

5. Calculation of the weighted normalized decision-making matrix $\widehat{X}$. The weighted normalized values $\hat{x}_{ji}$ are calculated as

$$\hat{x}_{ji} = \overline{x_{ji}} \cdot q_i; \quad j = \overline{1,n} \text{ and } i = \overline{1,m}. \tag{4}$$

In formula (4) q_i is weight of the ith attribute.

After this step we have weighted normalized decision-making matrix:

$$\widehat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \dots & \hat{x}_{1m} \\ \hat{x}_{21} & \hat{x}_{22} & \dots & \hat{x}_{2m} \\ \vdots & \vdots & \dots & \vdots \\ \hat{x}_{n1} & \hat{x}_{n2} & \dots & \hat{x}_{nm} \end{bmatrix}; \quad j = \overline{1,n} \text{ and } i = \overline{1,m}. \tag{5}$$

6. Sums P_j of attributes values which larger values are more preferable (optimization direction is maximization) calculation for each alternative (line of the decision-making matrix):

$$P_j = \sum_{i=1}^{k} \hat{x}_{ji}. \tag{6}$$

In formula (6) k is number of attributes which must to be maximized (it is assumed that in the decision-making matrix columns first of all are placed attributes with optimization direction maximum and ones with optimization direction minimum are placed after).

7. Sums R_j of attributes values which smaller values are more preferable (optimization direction is minimization) calculation for each alternative (line of the decision-making matrix):

$$R_j = \sum_{i=k+1}^{m} \hat{x}_{ji}. \tag{7}$$

8. Determining the minimal value of R_j:

$$R_{\min} = \min_j R_j; \quad j = \overline{j,n}. \tag{8}$$

9. Calculation of the relative weight of each alternative Q_j:

$$Q_j = P_j + \frac{R_{\min} \sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} \frac{R_{\min}}{R_j}}. \tag{9}$$

Formula (9) can to be written as follows:

$$Q_j = P_j + \frac{\sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} \cdot \frac{1}{R_j}}. \tag{9*}$$

10. Determination of the optimality criterion K:

$$K = \max_j Q_j; \quad j = \overline{1,n}. \tag{10}$$

11. Determination of the priority of the project. The greater weight (relative weight of alternative) Q_j, the higher is the priority (rank) of the project. In the case of $Q_{\max}$, the satisfaction degree is the highest.

12. Calculation of the utility degree of each alternative:

$$N_j = \frac{Q_j}{Q_{\max}} 100\%, \tag{11}$$

where Q_j and $Q_{\max}$ are the weight of projects obtained from Eq. (9*).

3.2. *COPRAS Method with Grey Numbers*

Most of multi-attribute decision-making problems must be determined not with exact attribute values, but with fuzzy values or with values in some intervals. Zavadskas *et al.* (2008a) presented the main ideas of COPRAS-G method. The idea of COPRAS-G method with attribute values expressed in intervals is based on the real conditions of decision-making and applications of the Grey systems theory.

The procedure of using the COPRAS-G method consists in the following steps:

1. Selecting the set of the most important attributes, describing the alternatives.

2. Constructing the decision-making matrix $\otimes X$:

$$\otimes X = \begin{bmatrix} [\otimes x_{11}] & [\otimes x_{12}] & \dots & [\otimes x_{1m}] \\ [\otimes x_{21}] & [\otimes x_{22}] & \dots & [\otimes x_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [\otimes x_{n1}] & [\otimes x_{n2}] & \dots & [\otimes x_{nm}] \end{bmatrix}$$

$$= \begin{bmatrix} [w_{11}; b_{11}] & [w_{12}; b_{12}] & \dots & [w_{1m}; b_{1m}] \\ [w_{21}; b_{21}] & [w_{22}; b_{22}] & \dots & [w_{2m}; b_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [w_{n1}; b_{n1}] & [w_{n2}; b_{n2}] & \dots & [w_{nm}; b_{nm}] \end{bmatrix}; \quad j = \overline{1,n}; \quad i = \overline{1,m}, \tag{12}$$

where $\otimes x_{ji}$ is determined by w_{ji} (the smallest value, the lower limit) and b_{ji} (the biggest value, the upper limit).

3. Determining weights of the attributes q_i.

4. Normalizing the decision-making matrix $\otimes X$:

$$\begin{aligned} \overline{w_{ji}} &= \frac{w_{ji}}{\frac{1}{2}\left(\sum_{j=1}^{n} w_{ji} + \sum_{j=1}^{n} b_{ji}\right)} = \frac{2w_{ji}}{\sum_{j=1}^{n} w_{ji} + \sum_{j=1}^{n} b_{ji}}; \\ \overline{b_{ji}} &= \frac{b_{ji}}{\frac{1}{2}\left(\sum_{j=1}^{n} w_{ji} + \sum_{j=1}^{n} b_{ji}\right)} = \frac{2b_{ji}}{\sum_{j=1}^{n}(w_{ji} + b_{ji})}; \\ & i = \overline{1,n} \ \text{ and } \ j = \overline{1,m}. \end{aligned} \tag{13}$$

In formula (13) w_{ji} is the lower value of the i attribute in the alternative j of the solution; b_{ji} is the upper value of the attribute i in the alternative j of the solution; m is the number of attributes; n is the number of the alternatives compared.

Then, the decision-making matrix is normalized:

$$\otimes\widehat{X} = \begin{bmatrix} [\otimes\overline{x_{11}}] & [\otimes\overline{x_{12}}] & \dots & [\otimes\overline{x_{1m}}] \\ [\otimes\overline{x_{21}}] & [\otimes\overline{x_{22}}] & \dots & [\otimes\overline{x_{2m}}] \\ \vdots & \vdots & \dots & \vdots \\ [\otimes\overline{x_{n1}}] & [\otimes\overline{x_{n2}}] & \dots & [\otimes\overline{x_{nm}}] \end{bmatrix}$$
$$= \begin{bmatrix} [\overline{w_{11}};\overline{b_{11}}] & [\overline{w_{12}};\overline{b_{12}}] & \dots & [\overline{w_{1m}};\overline{b_{1m}}] \\ [\overline{w_{21}};\overline{b_{21}}] & [\overline{w_{22}};\overline{b_{22}}] & \dots & [\overline{w_{2m}};\overline{b_{2m}}] \\ \vdots & \vdots & \dots & \vdots \\ [\overline{w_{n1}};\overline{b_{n1}}] & [\overline{w_{n2}};\overline{b_{n2}}] & \dots & [\overline{w_{nm}};\overline{b_{nm}}] \end{bmatrix}. \tag{14}$$

5. Calculating the weighted normalized decision-making matrix $\otimes\widehat{X}$. The weighted normalized values $\otimes\hat{x}_{ji}$ are calculated as follows:

$$\otimes\hat{x}_{ji} = \otimes\overline{x}_{ji} \cdot q_i; \quad \hat{w}_{ji} = \overline{w}_{ji} \cdot q_i; \quad \hat{b}_{ji} = \overline{b_{ji}} \cdot q_i. \tag{15}$$

In formula (15), q_i is the weight of the ith attribute.

Then, the decision-making matrix is normalized:

$$\otimes\widehat{X} = \begin{bmatrix} [\otimes\hat{x}_{11}] & [\otimes\hat{x}_{12}] & \dots & [\otimes\hat{x}_{1m}] \\ [\otimes\hat{x}_{21}] & [\otimes\hat{x}_{22}] & \dots & [\otimes\hat{x}_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [\otimes\hat{x}_{n1}] & [\otimes\hat{x}_{n2}] & \dots & [\otimes\hat{x}_{nm}] \end{bmatrix}$$
$$= \begin{bmatrix} [\hat{w}_{11};\hat{b}_{11}] & [\hat{w}_{12};\hat{b}_{12}] & \dots & [\hat{w}_{1m};\hat{b}_{1m}] \\ [\hat{w}_{21};\hat{b}_{21}] & [\hat{w}_{22};\hat{b}_{22}] & \dots & [\hat{w}_{2m};\hat{b}_{2m}] \\ \vdots & \vdots & \dots & \vdots \\ [\hat{w}_{n1};\hat{b}_{n1}] & [\hat{w}_{n2};\hat{b}_{n2}] & \dots & [\hat{w}_{nm};\hat{b}_{nm}] \end{bmatrix}. \tag{16}$$

6. Calculating the sums P_j of the attribute values, whose larger values are more preferable:

$$P_j = \frac{1}{2}\sum_{i=1}^{k}(\hat{w}_{ji} + \hat{b}_{ji}). \tag{17}$$

7. Calculating the sums R_j of attribute values, whose smaller values are more preferable:

$$R_j = \frac{1}{2} \sum_{i=k+1}^{m} \left(\hat{w}_{ji} + \hat{b}_{ji}\right); \quad i = \overline{k, m}. \tag{18}$$

8. Determining the minimal value of R_j:

$$R_{\min} = \min_j R_j; \quad j = \overline{j, n}. \tag{19}$$

9. Calculating the relative weight of each alternative Q_j:

$$Q_j = P_j + \frac{\sum_{j=1}^{n} R_j}{R_j \sum_{j=1}^{n} \cdot \frac{1}{R_j}}. \tag{20}$$

10. Determining the optimality criterion K:

$$K = \max_j Q_j; \quad j = \overline{1, n}. \tag{21}$$

11. Determining the priority of the project.
12. Calculating the utility degree of each alternative:

$$N_j = \frac{Q_j}{Q_{\max}} 100\%, \tag{22}$$

where Q_j and $Q_{\max}$ are the weight of projects obtained from Eq. (20).

4. Case Study of Contractors' Selection

4.1. *Contractors' for Construction Selection Problem*

Contractors' selection in construction problem was selected to illustrate newly proposed method. Construction projects are complicated, unique and built only once. The designing and construction process always deals with uncertainty, risk (Zou *et al.*, 2007) and risk management (Schieg, 2007; Savčuk, 2007; Shevchenko *et al.*, 2008). The right selection of a qualified contractor depends on many attributes and gives confidence to the stakeholder that the selected contractor can achieve the project goals (Turskis, 2008). The efficiency of a construction process is often associated with the successful choice of a contractor. The construction industry contractors should employ solid and reliable strategies to establish their profit and risk margins for their offers. The importance of non-price attributes is well recognised in the literature. In order to select the most appropriate contractor for the project and prepare the most realistic and accurate bid proposal, general contractors have to know all financial, technical and general information about these contractors (Yang *et al*., 2007; Arslan *et al*., 2008). Various procedures for a contractor's selection are analysed and applied in practice (Dikmen *et al.*, 2007; Egemen and Mohamed, 2008; Mitkus and Trinkūnienė, 2008). Dikmen *et al*. (2008) and Zavadskas *et al*. (2008b) described the models of the contractor selection by discrete values of attributes.

4.2. *Problem Solution*

The set of attributes and initial values of attributes are determined on the basis of expert, normative and calculation methods. The selected attributes for contractor assessment are as follows: x_1 – standard of quality ([score] includes adherence to specifications, adherence to contractual obligations, standard of workmanship, any remedial work, attendance to remedial work; quality of material); x_2 – financial ([score] includes financial soundness, financial control, difficulties of obtaining bond and price of contract); x_3 – ([score] includes completion of contract on time; causes of delay; adherence to programme; attitude in progressing contract; achievement of key dates; continuity of work; amount of liquidated damages); x_4 – complaints in communications with partners ([score] includes relationship with client's representative; attitude and care for client or tenant; readiness to advise build ability; enthusiasm; difficulties in settling claims; amount of claims, amount of unjustified claims; regularity of interim valuation; attitude to claims and disputes; relationship with design team; cooperation with designers and other contractors; relationship with subcontractors; reputation). In order to establish the attribute weights a survey has been carried out and 25 experts have been questioned. The weights of attributes were established according to the rating methods (Zavadskas, 1987). They are presented in Table 2.

Table 2

Initial decision-making matrix with values of the attributes describing the compared alternatives in intervals

	Attributes							
Contractor No.	Standard of quality (score)		Financial (score)		Progress of work (score)		Complaints in communications with partners (scores)	
Optimization direction	*max*		*max*		*max*		*min*	
Attribute weight $-q_i$	0.15		0.40		0.20		0.25	
	$\otimes x_1$		$\otimes x_2$		$\otimes x_3$		$\otimes x_4$	
	w_1	b_1	w_2	b_2	w_3	b_3	w_4	b_4
1	64	85	50	55	60	80	80	75
2	57	81	52	56	62	76	75	70
3	61	78	55	58	53	61	80	70
4	59	93	54	62	55	72	90	80
5	63	89	61	68	54	63	75	65

If we look to the initial decision-making matrix values we can to say according to the Standard of quality optimistic values the best alternative is 4th alternative and the worst is third alternative. According to the pessimistic values the best alternative is the first alternative and the worst alternative is 4th alternative. If we want to select the best alternative according to financial assessment, then we according to the optimistic values must to select first alternative and the worst alternative will be fifth alternative. But if selection will be according to the pessimistic values, then we first of all will select also firs alternative and the worst one will be also 5th alternative.

In case selection according to the Complaints in communications with partners, the best optimistic is second (5th alternative), while worst is 4th one. According to the pessimistic values the best is second or third alternative, while the worst is 4th alternative. We can't to decide what alternative is the best and which of values – pessimistic or optimistic must to be taken in to account. For solution of this problem are applied COPRAS (with pessimistic values and another case with optimistic values) and COPRAS-G methods. After 5th step of solution we have weighted normalized matrix as is shown in Table 3.

After 12-th solution step we have final solution results as are shown in Table 4 and Fig. 2.

According to solution results we can to state, if we will perform calculations with optimistic, pessimistic and grey values the results will be different (Table 4 graphical chart). According to the optimistic values alternatives ranks as follows: $A_5 \succ A_4 \succ A_2 \succ A_1 \succ A_3$. According to the pessimistic values alternatives ranks as follows: $A_5 \succ A_2 \succ A_3 \succ A_1 \succ A_4$. We can to certain that the best alternative is the same. If will be selected 3rd alternative, then according to the optimistic strategy it will be the worst and according

Table 3

Weighted normalized matrix according to a COPRAS and COPRAS-G method

Contractor No.	Pessimistic values				Optimistic values			
	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$
1	0.032	0.074	0.042	0.050	0.200	0.184	0.227	0.208
2	0.028	0.076	0.044	0.047	0.190	0.187	0.216	0.194
3	0.030	0.081	0.037	0.050	0.187	0.194	0.173	0.194
4	0.029	0.079	0.039	0.056	0.218	0.207	0.205	0.222
5	0.031	0.090	0.038	0.047	0.209	0.227	0.179	0.181

Table 3 (continuation)

Contractor No.	Values in intervals							
	$\otimes\hat{x}_1$		$\otimes\hat{x}_2$		$\otimes\hat{x}_3$		$\otimes\hat{x}_4$	
1	0.026	0.035	0.070	0.077	0.038	0.050	0.053	0.049
2	0.023	0.033	0.073	0.078	0.039	0.048	0.049	0.046
3	0.025	0.032	0.077	0.081	0.033	0.038	0.053	0.046
4	0.024	0.038	0.076	0.087	0.035	0.045	0.059	0.053
5	0.026	0.037	0.085	0.095	0.034	0.040	0.049	0.043

Table 4

Solution results (S_j – ascending rank of alternatives. The smallest is the best)

Contractor No.	Alternative's weight Q_j			Alternative's degree of efficiency N_j			Rank S_j		
	Pessimistic	Optimistic	Interval	Pessimistic	Optimistic	Interval	Pessimistic	Optimistic	Interval
1	0.197	0.197	0.197	93.0	92.3	92.7	4	4	$3 = 4$
2	0.201	0.198	0.200	95.0	92.8	94.0	2	3	2
3	0.198	0.191	0.194	93.5	89.5	91.3	3	5	5
4	0.192	0.201	0.197	90.4	94.4	92.7	5	2	$3 = 4$
5	0.212	0.213	0.213	100	100	100	1	1	1

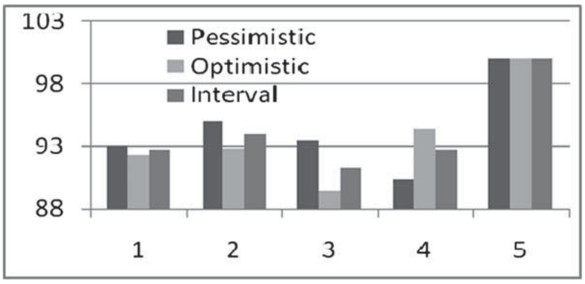

Fig. 2. Graphic representation of alternatives' ranking according to COPRAS methods.

to the pessimistic strategy it will be 3rd. The best 5th alternative is selected according to the COPRAS-G method. Alternatives ranks as follows: $A_5 \succ A_2 \succ A_1 = A_4 \succ A_3$.

5. Conclusions

In real life multi-attribute modelling of multi-alternative assessment problems attribute values, which deals with the future, can to be expressed in intervals.

COPRAS-G is newly developed method for assessment of alternatives by multiple-attribute values determined in intervals.

This approach is intended to support the decision-making process and increase the efficiency of the resolution process.

COPRAS-G method can be applied to the solution of wide range discrete multi-attribute assessment problems in construction.

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Daugiatikslis sprendimų priėmimo modelis taikant pilkuosius skaičius

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Daugiatikslę analizę tikslinga taikyti ekonomikos, valdymo, konstrukcijų ir kitokių uždavinių sprendimui. COPRAS metode alternatyvos aprašomos diskrečiomis rodiklių reikšmėmis. Tikslo funkcija yra tiesiogiai proporcingai priklausoma nuo rodiklių aprašančiųjų alternatyvas reikšmių ir tų rodiklių svorių. Visuomet egzistuoja tam tikras neapibrėžtumas sudarant tokių uždavinių modelius. Neapibrėžtumai ypatingai būdingi rodiklių reikšmėms. Aprašant galimas planuojamas sprendimų alternatyvos negali būti tiksliai nustatytos ar apskaičiuotos rodiklių reikšmės. Šio darbo tikslas – parodyti galimus modeliavimo rezultatus, kai alternatyvių sprendimų rodiklių reikšmės yra pateikiamos intervaluose, t.y. duomenys pateikiami apibrėžtuose intervaluose. Nauja koncepcija pritaikyta statybos rangovo parinkimui. Tokiu būdu parodomas naujai pasiūlyto COPRAS-G metodo tinkamumas įvairių ekonomikos ir vadybos uždavinių sprendimui.