# 485. Bessel functions, time averaged fringes and dynamical systems

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**Abstract.** Bessel functions play a primary role in experimental mechanics whenever the mathematical description of time-averaged optical processes is considered. Nevertheless, numerical modelling of time-averaged fringes is a complex task usually involving the Finite Element Method, the physical model describing interference effects, and a graphical model comprising all interacting objects. We propose a simplified approach for visualization of time-averaged geometric moiré fringes which can have a definite educational value and can complement courses comprising Bessel functions and exemplify the role of these functions in experimental mechanics.
**Key words:** Bessel function, Finite Element Method, moiré fringes, time-averaged fringes

## Introduction.

In mathematics, Bessel functions, first defined by the mathematician Daniel Bernoulli and generalized by Friedrich Bessel, are canonical solutions of Bessel's differential equation [1]:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - \alpha^2\right) y = 0 \qquad (1)$$

for an arbitrary real or complex number $\alpha$. The most common special case is where $\alpha$ is an integer; then $\alpha$ is referred to as the order of the Bessel function.

Bessel's equation arises when finding separable solutions to Laplace's equation and the Helmholtz equation in cylindrical or spherical coordinates. Bessel functions are therefore especially important for many problems of wave propagation, static potentials, electromagnetic waves in a cylinder waveguide, diffusion problems on a lattice, digital signal processing, etc [2, 3, 4].

Since Bessel's differential equation is a second-order differential equation, there must be two linearly independent solutions. Depending upon circumstances, however, various formulations of these solutions are convenient. Different variations comprise Bessel function of the first kind, Bessel functions of the second kind, Modified Bessel functions, Hankel functions, Spherical Bessel functions, Riccati-Bessel functions [1].

Students with deeper mathematical interests usually go through the formulations, definitions and properties of Bessel functions in their basic curriculum, and analyze integral, asymptotic and series expansion forms of the Bessel functions. On the other hand, students with