

A Novel Method for Multiple Criteria Analysis: Grey Additive Ratio Assessment (ARAS-G) Method

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Abstract. The paper presents the process of the selection of a potential supplier, which have to be the most appropriate to stakeholders. The selection is based on a set of criteria: Delivery Price, Financial Position, Production Specifications, Standards and Relevant Certificates, Commercial Strength, and the Performance of supplier, etc. The criteria for evaluation and their importance are selected by taking into consideration the interests and goals of the stakeholders. The solution of problem was made by applying a new *Additive Ratio Assessment* (ARAS) method with the grey criteria scores – ARAS-G method. The proposed technique could be applied to substantiate the selection of effective alternative of sustainable development, impact on environment, structures, technologies, investments, etc.

Keywords: operation research, decision making, grey relations, multiple criteria, MCDM, alternative, supplier, selection, evaluation, weights, Additive Ratio Assessment, ARAS, ARAS-G.

1. Introduction

The Roman philosopher Seneca said “Nothing is certain except the past.” Future is uncertain. All new ideas and possible variants of decisions in real world must be compared according to a set of multiple conflicting criteria (Turskis *et al.*, 2009). The problem of decision-maker consists of evaluating a finite set of alternatives in order to find the best one, to rank them from the best to the worst, to group them into predefined homogeneous classes, or to describe how well each alternative meets all the criteria simultaneously.

The multiple criteria decision making is a model that allows the analysis of several preference criteria simultaneously. Techniques and planning methods and decision making methods develop dynamically (Kapliński, 2008; Ginevicius *et al.*, 2008; Kalibatas and Turskis, 2008; Sivilevičius *et al.*, 2008; Turskis, 2008; Zavadskas *et al.*, 2008a, 2009a). MCDM is one of the most widely used decision methodologies in the sciences, business, and government worlds (Zavadskas *et al.*, 2008b).

In an MCDM approach, first it is necessary to define the problem clearly, and then identify realistic alternatives. It is important to define the actors involved in the decision

making, select the evaluation criteria, and evaluate each alternative according to the set of criteria. Next, an MCDM method is selected to aggregate the performance of each alternative.

Different types of MCDM methods can be used for complex problem solution:

- Methods based on quantitative measurements. The methods based on multiple criteria utility theory may be referred to this group (TOPSIS – *Technique for Order Preference by Similarity to Ideal Solution* (Hwang and Yoon, 1981; Zavadskas *et al.*, 2010b), SAW – *Simple Additive Weighting* (MacCrimon, 1968; Medineckiene *et al.*, 2010), LINMAP – *Linear Programming Techniques for Multidimensional Analysis of Preference* (Srinivasan and Shocker, 1973), COPRAS – *Complex Proportional Assessment* (Zavadskas and Kaklauskas, 1996), its modification COPRAS-G (*Complex Proportional Assessment method with Grey interval numbers*; Zavadskas *et al.*, 2008c, 2008d, 2009b), and ARAS (*Additive Ratio Assessment*) method (Zavadskas and Turskis, 2010).
- Methods based on qualitative initial measurements. These include two widely known groups of methods, i.e., *Analytic Hierarchy Methods* (AHP; Saaty, 1977) and fuzzy set theory methods (Zimmerman, 2000).
- Comparative preference methods based on pair-wise comparison of alternatives. This group comprises the modifications of the ELECTRE (Roy 1990), PROMETHEE (Brans *et al.*, 1984), UTA (Jacquet-Lagrange and Siskos, 1982), MUSA (Grigoroudis and Siskos, 2002), AKUTA (Bous *et al.*, 2010), TACTIC (Vansnick, 1986), ORESTE (Roubens, 1982) and other methods.
- Methods based on qualitative measurements not converted to quantitative variables. This group includes methods of verbal decision-making analysis (Berkeley *et al.*, 1991) and uses qualitative data for decision environments involving high levels of uncertainty.

An alternative in multiple criteria evaluation is usually described by quantitative and qualitative criteria. The criteria have different units of measurement (Kersuliene *et al.*, 2010). Normalization aims at obtaining comparable scales of the criteria values. Different techniques of criteria value normalization are used. The impact of the decision-matrix normalization methods on the decision results has been investigated by many authors (Ginevičius, 2008; Noarul Haq and Kannan, 2007; Zavadskas and Turskis, 2008).

2. Grey Number

Li *et al.* (2007) argued that since the decision makers such as preferences on alternatives or on the criteria of suppliers are often uncertain, supplier selection becomes more difficult. Grey theory is one of the methods used to study uncertainty, being superior in the mathematical analysis of systems with uncertain information. The advantage of grey theory over fuzzy sets theory is that grey theory can deal flexibly with the fuzziness situation. Alternative's selection can be viewed as a grey system process. We may use grey theory to resolve it. The ratings of criteria are described by linguistic variables that can be expressed in grey numbers.

Grey theory, proposed and originally developed by Deng (1982), is an effective mathematical means to:

- deal with problems described by incomplete information;
- to avoid the inherent defects of conventional, statistical methods;
- and advantage is to use a limited amount of data to estimate the behaviour of an uncertain system when the data are discrete and the information is incomplete (Wu, 2006.)

Due to presence of incomplete information and uncertain relations it is very difficult to use ordinary methods.

White number, grey number and black number are three classifications to distinguish the uncertainty level of information (Chen and Tzeng, 2004). Let

$$\otimes x = [\alpha, \gamma] = \{x | \alpha \leq x \leq \gamma, \alpha \text{ and } x \in R\}. \tag{1}$$

Then, $\otimes x$ which has two real numbers α (the lower limit of $\otimes x$) and γ (the upper limit of $\otimes x$) is defined as follows:

- if $\alpha \rightarrow -\infty$ and $\gamma \rightarrow \infty$, then $\otimes x$ is called the black number which means without any meaningful information;
- else if $\alpha = \gamma$, then $\otimes x$ is called the white number which means with complete information;
- otherwise, $\otimes x = [\alpha, \gamma]$ is called the grey number which means insufficient and uncertain information.

Nevertheless, the obtained information from real world is always uncertain or incomplete. Hence, extending the applications from white numbers (crisp values) to grey numbers is necessary for real-world applications. The basic definitions and operations of grey number are described as follows.

Let a grey number is defined to be grey number defined by two parameters (α, γ) . Let $+$, $-$, \times and \div denote the operations of addition, subtraction, multiplication and division respectively. The basic operations of grey numbers $\otimes n_1$ and $\otimes n_2$ are defined as follows:

$$\otimes n_1 + \otimes n_2 = (n_{1\alpha} + n_{2\alpha}, n_{1\gamma} + n_{2\gamma}) \quad \text{addition,} \tag{2}$$

$$\otimes n - \otimes n = (n_{1\alpha} - n_{2\gamma}, n_{1\gamma} - n_{2\alpha}) \quad \text{subtraction,} \tag{3}$$

$$\otimes n_1 \times \otimes n_2 = (n_{1\alpha} \times n_{2\alpha}, n_{1\gamma} \times n_{2\gamma}) \quad \text{multiplication,} \tag{4}$$

$$\otimes n_1 \div \otimes n_2 = \left(\frac{n_{1\alpha}}{n_{2\gamma}}, \frac{n_{1\gamma}}{n_{2\alpha}} \right) \quad \text{division,} \tag{5}$$

$$k \times (\otimes n_1) = (kn_{1\alpha}, kn_{1\gamma}) \quad \text{number product of grey numbers} \\ \text{if } k \text{ is positive real number,} \tag{6}$$

$$(\otimes n_1)^{-1} = \left(\frac{1}{n_{1\gamma}}, \frac{1}{n_{1\alpha}} \right). \tag{7}$$

3. The Proposed Grey Multiple Criteria Decision Making Model: An Additive Ratio Assessment Method with Grey Values (ARAS-G)

ARAS method (Zavadskas and Turskis, 2010, Zavadskas *et al.*, 2010a; Tupenaite *et al.*, 2010) is based on the argument that phenomena of complicated world could to be understood by using simple relative comparisons. It is argued that the ratio of the sum of normalized and weighted values of criteria, which describe alternative under consideration, to the sum of the values of normalized and weighted criteria, which describes the optimal alternative, is degree of optimality, which is reached by the alternative under comparison.

According to the ARAS method a utility function value determining the complex relative efficiency of a reasonable alternative is directly proportional to the relative effect of values and weights of the main criteria considered in a project.

The first stage is grey decision-making matrix (GDMM) forming. In the GMCDM of the discrete optimization problem any problem to be solved is represented by the following DMM of preferences for m reasonable alternatives (rows) rated on n criteria (columns):

$$\tilde{X} = \begin{bmatrix} \otimes x_{01} & \cdots & \otimes x_{0j} & \cdots & \otimes x_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes x_{i1} & \cdots & \otimes x_{ij} & \cdots & \otimes x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes x_{m1} & \cdots & \otimes x_{mj} & \cdots & \otimes x_{mn} \end{bmatrix}; \quad i = \overline{0, m}; j = \overline{1, n}, \quad (8)$$

where m – number of alternatives, n – number of criteria describing each alternative, $\otimes x_{ij}$ – grey value representing the performance value of the i alternative in terms of the j criterion, $\otimes x_{0j}$ – optimal value of j criterion.

If optimal value of j criterion is unknown, then

$$\begin{aligned} \otimes x_{0j} &= \max_i \otimes x_{ij}, & \text{if } \max_i \otimes x_{ij} \text{ is preferable, and} \\ \otimes x_{0j} &= \min_i \otimes x_{ij}^*, & \text{if } \min_i \otimes x_{ij}^* \text{ is preferable.} \end{aligned} \quad (9)$$

Usually, the performance values $\otimes x_{ij}$ and the criteria weights $\otimes w_j$ are viewed as the entries of a DMM. The system of criteria as well as the values and initial weights of criteria are determined by experts. The information can be corrected by the interested parties by taking into account their goals and opportunities.

Then the determination of the priorities of alternatives is carried out in several stages.

Usually, the criteria have different dimensions. The purpose of the next stage is to receive dimensionless weighted values from the comparative criteria. In order to avoid the difficulties caused by different dimensions of the criteria, the ratio to the optimal value is used. There are various theories describing the ratio to the optimal value. However,

the values are mapped either on the interval $[0; 1]$ or the interval $[0; \infty)$ by applying the normalization of a DMM.

In the second stage the initial values of all the criteria are normalized – defining values $\otimes \bar{x}_{ij}$ of normalised decision-making matrix $\otimes \bar{X}$:

$$\otimes \bar{X} = \begin{bmatrix} \otimes \bar{x}_{01} & \cdots & \otimes \bar{x}_{0j} & \cdots & \otimes \bar{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes \bar{x}_{i1} & \cdots & \otimes \bar{x}_{ij} & \cdots & \otimes \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes \bar{x}_{m1} & \cdots & \otimes \bar{x}_{mj} & \cdots & \otimes \bar{x}_{mn} \end{bmatrix}; \quad i = \overline{0, m}; \quad j = \overline{1, n}. \quad (10)$$

The criteria, whose preferable values are maxima, are normalized as follows:

$$\otimes \bar{x}_{ij} = \frac{\oplus x_{ij}}{\sum_{i=0}^m \otimes x_{ij}}. \quad (11)$$

The criteria, whose preferable values are minima, are normalized by applying two-stage procedure:

$$\otimes x_{ij} = \frac{1}{\otimes x_{ij}^*}; \quad \otimes \bar{x}_{ij} = \frac{\otimes x_{ij}}{\sum_{i=0}^m \otimes x_{ij}}. \quad (12)$$

When the dimensionless values of the criteria are known, all the criteria, originally having different dimensions, can be compared.

The third stage is defining normalized-weighted matrix – $\otimes \hat{X}$. It is possible to evaluate the criteria with weights $0 < \otimes w_j < 1$. Only well-founded weights should be used because weights are always subjective and influence the solution. The values of weight $\otimes w_j$ are usually determined by the expert evaluation method. The sum of weights w_j would be limited as follows:

$$\sum_{j=1}^n w_j = 1, \quad (13)$$

$$\otimes \hat{X} = \begin{bmatrix} \otimes \hat{x}_{01} & \cdots & \otimes \hat{x}_{0j} & \cdots & \otimes \hat{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes \hat{x}_{i1} & \cdots & \otimes \hat{x}_{ij} & \cdots & \otimes \hat{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \otimes \hat{x}_{m1} & \cdots & \otimes \hat{x}_{mj} & \cdots & \otimes \hat{x}_{mn} \end{bmatrix}; \quad i = \overline{0, m}; \quad j = \overline{1, n}. \quad (14)$$

Normalized-weighted values of all the criteria are calculated as follows:

$$\otimes \hat{x}_{ij} = \otimes \bar{x}_{ij} \times \otimes w_j; \quad i = \overline{0, m}, \quad (15)$$

where w_j is the weight (importance) of the j criterion and \bar{x}_{ij} is the normalized rating of the j criterion.

The following task is determining values of optimality function:

$$\otimes S_i = \sum_{j=1}^n \otimes \hat{x}_{ij}; \quad i = \overline{0, m}, \quad (16)$$

where $\otimes S_i$ is the value of optimality function of i alternative.

The biggest value is the best, and the least one is the worst. Taking into account the calculation process, the optimality function $\otimes S_i$ has a direct and proportional relationship with the values $\otimes x_{ij}$ and weights $\otimes w_j$ of the investigated criteria and their relative influence on the final result. Therefore, the greater the value of the optimality function $\otimes S_i$, the more effective the alternative. The priorities of alternatives can be determined according to the value $\otimes S_i$. Consequently, it is convenient to evaluate and rank decision alternatives when this method is used.

The result of grey decision making for each alternative is grey number $\otimes S_i$. There are several methods for transforming grey values to crisp values. The centre-of-area is the most practically and simple to apply:

$$S_i = \frac{1}{2}(S_{i\alpha} + S_{i\gamma}). \quad (17)$$

The degree of the alternative utility is determined by a comparison of the variant, which is analysed, with the ideally best one S_0 . The equation used for the calculation of the utility degree K_i of an alternative A_i is given below:

$$K_i = \frac{S_i}{S_0}; \quad i = \overline{0, m}, \quad (18)$$

where S_i and S_0 are the optimality criterion values, obtained from (16).

It is clear, that the calculated values K_i are in the interval $[0; 1]$ and can be ordered in an increasing sequence, which is the wanted order of precedence. The complex relative efficiency of the reasonable alternative can be determined according to the utility function values.

4. Case Study

The government-linked company decided to invest a sum of money for a new municipal solid waste disposal system. The percentage of sales revenues spent on purchased materials typically ranges between 50–90%. One of the most crucial is the selection of the right

supplier. The right supplier provides the right quality of materials, on time, at the right price, and the right level of service.

Selecting and evaluating the right suppliers is imperative for an organization's global marketplace competitiveness. The selection of an optimal material for an engineering design from among two or more alternative materials on the basis of two or more criteria is a multiple criteria decision-making problem. Supplier selection process represents a complex problem and thus a multiple criteria decision making (MCDM) problem.

When selecting materials for engineering designs, a clear understanding of the functional requirements for each individual component is required, and various important criteria or attributes need to be considered.

Researchers have focused on suppliers' selection problem since 1960's. Benton and Krajewski (1990) conclude that selection of poor vendors could lead to significant backlog and shortage in the quality of products delivered to customers (Noarul Haq and Kannan, 2006). Supplier selection decisions are very complicated because various actors that must be considered in the decision-making process. Over the years a number of quantitative approaches have been applied to supplier selection problems.

The basic issue in supplier selection survey is detecting of the selection criteria. The basic criteria typically utilized for this purpose are pricing structure, delivery (timeliness and costs), product quality, and service (personnel, facilities, research and development, capability, etc.; Soukup, 1987). The criteria typically are characterized with complexity, elusiveness, and uncertainty in nature. Ho *et al.* (2010) stated that supplier evaluation and selection problem has been studied extensively in the three last decades. Selecting of appropriate suppliers is a challenging issue because it complex, elusive, and uncertainty concept that is difficult to determine. Various decision making approaches have been proposed to tackle the problem. The performance of potential suppliers is evaluated against multiple criteria rather than considering a single factor-cost. Many researchers pointed out that the numbers and types of criteria totally depend on the corporate policy, objectives and strategy. The most popular criterion among hundreds of proposed ones is quality, followed by delivery, price/cost, manufacturing capability, service, management, technology, research and development, finance, flexibility, reputation, relationship, risk, and safety and environment (Ho *et al.*, 2010; Dickson, 1966).

The main steps of problem solution are as follows: determine aim and scope → generate criteria set → generate set of alternatives → determine criteria weights → determine criteria scores → selection of aggregation model → evaluation, priority setting and improving decisions → implementing selection.

By interviewing and surveying purchase managers of firm, 6 criteria were identified for supplier selection (Table 1). A set of experiments were performed to develop and evaluate an empirical methodology to convert ordinal criteria rankings from several DMs into aggregate criteria weights. The experts sample was composed of 27 experts from different fields of stakeholders. The effects of the different decision makers on the aggregate weights were found to be insignificant at significance level $\alpha = 0.05$. The importance weights of criteria are presented in Table 1.

Linguistic variables for grey weighting criteria are shown in Table 2.

Table 1
The criteria for evaluation of raw materials potential suppliers' selection

	Criteria	Measure units	Opt.	Weight	
				α	γ
x_1^*	Delivery price	10^3 €/m^3	min	0.195	0.210
x_2	Financial position	Points	max	0.195	0.195
x_3	Performance	Points	max	0.054	0.132
x_4	Standards and relevant certificates	Points	max	0.132	0.195
x_5	Production specifications	Points	max	0.171	0.210
x_6	Commercial strength	Points	max	0.117	0.195

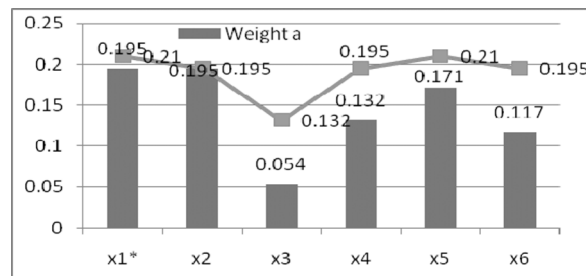


Fig. 1. Graphic representation of criteria weights in terms of α and γ scores.

Table 2
The linguistic variables or the ratings

Linguistic variables	Grey numbers	
	α	γ
Very low (VL)	0.00	0.20
Low (L)	0.10	0.30
Medium low (ML)	0.20	0.40
Medium (M)	0.35	0.65
Medium high (MH)	0.60	0.80
High (H)	0.70	0.90
Very high (VH)	0.80	1.00

The four of possible alternatives of suppliers under consideration A_i ($i = \overline{1, 5}$) comparing against 6 criteria are presented in Table 3. Solution process and ranking of alternatives is presented in Tables 4 – 6.

Table 3
Initial grey decision making matrix

Alt.	Criteria											
	$\otimes x_1^*$		$\otimes x_2$		$\otimes x_3$		$\otimes x_4$		$\otimes x_5$		$\otimes x_6$	
Opt.	min		max		max		max		max		max	
	α	γ	α	γ	α	γ	α	γ	α	γ	α	γ
w	0.195	0.210	0.195	0.195	0.054	0.132	0.132	0.195	0.171	0.210	0.117	0.195
A_0	0.541	0.541	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
A_1	0.591	0.882	0.470	0.600	0.663	0.810	0.430	0.750	0.650	0.860	0.315	0.615
A_2	0.544	0.673	0.750	0.960	0.512	0.740	0.340	0.550	0.660	0.820	0.555	0.975
A_3	0.541	0.831	0.550	0.780	0.659	0.830	0.550	0.860	0.830	0.930	0.375	0.690
A_4	0.706	1.102	0.670	0.900	0.709	0.920	0.320	0.670	0.720	0.960	0.405	0.735

Table 4
Changed-initial grey decision making matrix

Opt.	Criteria											
	$\otimes x_1$		$\otimes x_2$		$\otimes x_3$		$\otimes x_4$		$\otimes x_5$		$\otimes x_6$	
	max		max		max		max		max		max	
	α	γ	α	γ	α	γ	α	γ	α	γ	α	γ
w	0.195	0.210	0.195	0.195	0.054	0.132	0.132	0.195	0.171	0.210	0.117	0.195
A_0	1.848	1.848	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
A_1	1.134	1.692	0.470	0.600	0.663	0.810	0.430	0.750	0.650	0.960	0.315	0.615
A_2	1.486	1.838	0.750	1.260	0.512	0.540	0.340	0.550	0.660	0.960	0.555	0.975
A_3	1.203	1.848	0.550	0.780	0.659	0.930	0.550	0.860	0.830	1.320	0.375	0.690
A_4	0.907	1.416	0.670	0.900	0.709	1.020	0.320	0.670	0.720	0.960	0.405	0.735
\sum	6.579	8.644	3.440	4.540	3.543	4.300	2.640	3.830	3.860	5.200	2.650	4.015

5. The Results of Multiple Criteria Analysis

The criteria, whose preferable values are minima, are changed (formula 4, Table 4), The initial values of criteria were recalculated by applying normalization (formulae 20 and 21), That way the discrepancy between the different dimensions of the optimal values was eliminated, Normalized decision-making matrix (Table 5) was processed applying the ARAS-G method (formulae 11–18).

The solution results are presented in Table 6. The most reasonable alternative according to calculation results is second (A_3). The priority order of the investigated alternatives can be represented as (Fig. 3).

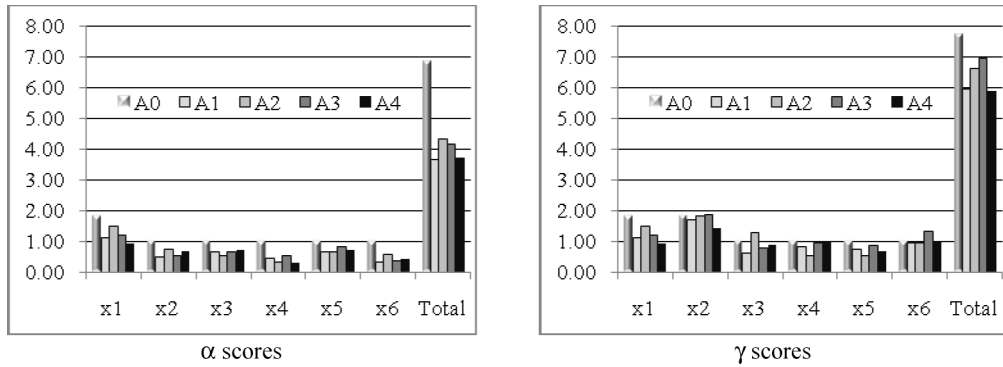


Fig. 2. Graphic representation of alternatives in terms of α and γ scores of criteria.

Table 5
Normalised grey decision making matrix

Opt.	$\otimes \bar{x}_1$		$\otimes \bar{x}_2$		$\otimes \bar{x}_3$		$\otimes \bar{x}_4$		$\otimes \bar{x}_5$		$\otimes \bar{x}_6$	
	max		max		max		max		max		max	
	α	γ	α	γ	α	γ	α	γ	α	γ	α	γ
w	0.195	0.210	0.195	0.195	0.054	0.132	0.132	0.195	0.171	0.210	0.117	0.195
A_0	0.214	0.281	0.220	0.291	0.233	0.282	0.261	0.379	0.192	0.259	0.249	0.377
A_1	0.131	0.257	0.104	0.174	0.154	0.229	0.112	0.284	0.125	0.249	0.078	0.232
A_2	0.172	0.279	0.165	0.366	0.119	0.152	0.089	0.208	0.127	0.249	0.138	0.368
A_3	0.139	0.281	0.121	0.227	0.153	0.262	0.144	0.326	0.160	0.342	0.093	0.260
A_4	0.105	0.215	0.148	0.262	0.165	0.288	0.084	0.254	0.138	0.249	0.101	0.277

Table 6
Normalised-weighted grey decision making matrix and solution results

	$\otimes \hat{x}_1$	$\otimes \hat{x}_2$	$\otimes \hat{x}_3$	$\otimes \hat{x}_4$	$\otimes \hat{x}_5$	$\otimes \hat{x}_6$	
A_0	0.042	0.059	0.043	0.057	0.013	0.037	
A_1	0.026	0.054	0.020	0.034	0.008	0.030	
A_2	0.034	0.059	0.032	0.071	0.006	0.020	
A_3	0.027	0.059	0.024	0.044	0.008	0.035	
A_4	0.020	0.045	0.029	0.051	0.009	0.038	
	$\otimes S$	S	K				
A_0	0.548	0.274	1				
A_1	0.371	0.185	0.676				
A_2	0.437	0.218	0.796	Alternatives ranks as follows: $A_3 \succ A_2 \succ A_4 \succ A_1$			
A_3	0.440	0.220	0.803				
A_4	0.395	0.197	0.720				

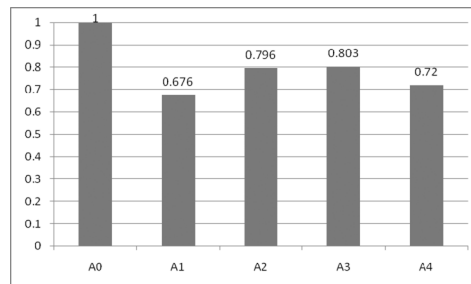


Fig. 3. Comparison of suppliers' alternatives performance level.

It means that the best alternative is third, and the worst alternative is the first. It can be stated that the alternative 3 is only 80% of optimal alternative performance level, and the performance of the worst alternative 1 is only 68%.

According to the given data on the criteria describing the alternatives, rational solutions about supplier's selection can be made. With an illustrative example: selection of a potential supplier the proposed methodology is validated, It is found that there are six main criteria which must to be considered: Delivery Price (γ weight equals to 0.210), Production Specifications (γ weight equals to 0.210), Financial Position (γ weight equals to 0.195), Standards and Relevant Certificates (γ weight equals to 0.195), Commercial Strength (γ weight equals to 0.195), and the Performance (γ weight equals to 0.132).

6. Conclusions

Traditional optimization, statistical and econometric analysis approaches used within the engineering context are often based on the assumption that the considered problem is well formulated and decision-makers usually consider the existence of a single objective, evaluation criterion or point of view that underlies the conducted analysis, In such a case the solution of engineering problems is easy to obtain.

But in reality, the modelling of engineering problems is based on a different kind of logic taking into consideration the existence of multiple criteria, the conflicting aims of decision maker, the complex, subjective and different nature of the evaluation process, and the participation of several decision makers.

In this paper it is supposed to deal with grey data to help the model to be very applicable due to lack of certainty and crisp data in real word situations especially about qualitative variables.

In this paper is developed new additive ratio analysis method with grey criteria scores – ARAS-G. Researchers and stakeholders found a way to evaluate and rank alternatives by applying grey values, and to compare scores of alternatives with the ideal possible alternative.

An example of a supplier selection problem was used to illustrate the proposed approach. The experimental result shows that the proposed approach is reliable and reasonable. Weights results show that stakeholders are more concern about the delivery price

and the production specification than the commercial strength and the performance of suppliers.

In conclusion, ARAS-G method has a promising future in the decision making field, because he offers a highly methodological basis for decision support.

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Naujas daugiakriterinės analizės metodas: pilkojo suminio santykinio vertinimo (ARAS-G) metodas

Zenonas TURSKIS, Edmundas Kazimieras ZAVADSKAS

Straipsnyje pateikiamas proceso modelis tinkamausiam tiekėjui iš potencialių tiekėjų aibės atrinkti, kuris tenkintų suinteresuotųjų grupių interesus. Pasirinkimas yra grindžiamas tokiais kriterijais: pristatymo kaina, finansinė tiekėjo padėtis, gamybos specifikacija, standartų ir reikiamų sertifikatų tenkinimas, prekybinė galia ir tiekėjo patrauklumas ir kt. Vertinimo kriterijai ir jų svarba yra pasirinkti atsižvelgiant į suinteresuotųjų šalimių interesus ir tikslus. Problemos sprendimui buvo sukurtas ir pateiktas naujas daugiakriterinės analizės metodas: Pilkojo suminio santykinio vertinimo (ARAS-G) metodas. Siūlomas metodas galėtų būti taikomas pagrįsti tvarios plėtros alternatyvų vertinimą, poveikio aplinkai atranką, sprendžiant statybos konstrukcijų, technologijos, investicijų ir t.t. parinkimo uždavinius.

Table 1
The criteria for evaluation of raw materials potential suppliers' selection

	Criteria	Measure units	Opt.	Weight	
				α	γ
x_1^*	Delivery price	10^3 €/m^3	min	0.195	0.210
x_2	Financial position	Points	max	0.195	0.195
x_3	Performance	Points	max	0.054	0.132
x_4	Standards and relevant certificates	Points	max	0.132	0.195
x_5	Production specifications	Points	max	0.171	0.210
x_6	Commercial strength	Points	max	0.117	0.195

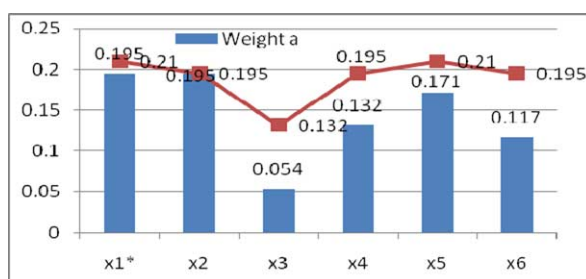


Fig. 1. Graphic representation of criteria weights in terms of α and γ scores.

Table 2
The linguistic variables or the ratings

Linguistic variables	Grey numbers	
	α	γ
Very low (VL)	0.00	0.20
Low (L)	0.10	0.30
Medium low (ML)	0.20	0.40
Medium (M)	0.35	0.65
Medium high (MH)	0.60	0.80
High (H)	0.70	0.90
Very high (VH)	0.80	1.00

The four of possible alternatives of suppliers under consideration A_i ($i = \overline{1, 5}$) comparing against 6 criteria are presented in Table 3. Solution process and ranking of alternatives is presented in Tables 4 – 6.

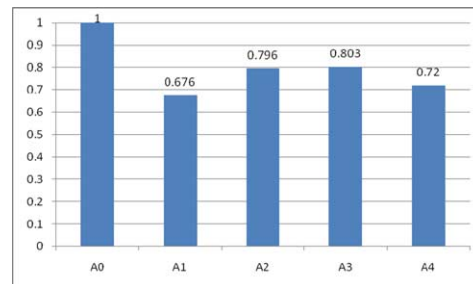


Fig. 3. Comparison of suppliers' alternatives performance level.

It means that the best alternative is third, and the worst alternative is the first. It can be stated that the alternative 3 is only 80% of optimal alternative performance level, and the performance of the worst alternative 1 is only 68%.

According to the given data on the criteria describing the alternatives, rational solutions about supplier's selection can be made. With an illustrative example: selection of a potential supplier the proposed methodology is validated, It is found that there are six main criteria which must to be considered: Delivery Price (γ weight equals to 0.210), Production Specifications (γ weight equals to 0.210), Financial Position (γ weight equals to 0.195), Standards and Relevant Certificates (γ weight equals to 0.195), Commercial Strength (γ weight equals to 0.195), and the Performance (γ weight equals to 0.132).

6. Conclusions

Traditional optimization, statistical and econometric analysis approaches used within the engineering context are often based on the assumption that the considered problem is well formulated and decision-makers usually consider the existence of a single objective, evaluation criterion or point of view that underlies the conducted analysis, In such a case the solution of engineering problems is easy to obtain.

But in reality, the modelling of engineering problems is based on a different kind of logic taking into consideration the existence of multiple criteria, the conflicting aims of decision maker, the complex, subjective and different nature of the evaluation process, and the participation of several decision makers.

In this paper it is supposed to deal with grey data to help the model to be very applicable due to lack of certainty and crisp data in real word situations especially about qualitative variables.

In this paper is developed new additive ratio analysis method with grey criteria scores – ARAS-G. Researchers and stakeholders found a way to evaluate and rank alternatives by applying grey values, and to compare scores of alternatives with the ideal possible alternative.

An example of a supplier selection problem was used to illustrate the proposed approach. The experimental result shows that the proposed approach is reliable and reasonable. Weights results show that stakeholders are more concern about the delivery price