BAYESIAN APPROACH TO FORECASTING DAMAGE TO BUILDINGS FROM ACCIDENTAL EXPLOSIONS ON RAILWAY

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A procedure for estimating potential damage to buildings induced by accidental explosions on railway is developed. By the damage are meant failures of nearby structures due to actions generated by the accidental explosions. This damage is measured in terms of probabilities of potential failures caused by the explosions. The estimation of the damage probabilities is based on a stochastic simulation of railway accident involving an explosion. The proposed simulation-based procedure quantifies epistemic (state-of-knowledge) uncertainties in the damage probabilities. These uncertainties are expressed in terms of Bayesian prior and posterior distributions. A foundation of the procedure is a computer intensive method known as a Bayesian bootstrap. It is used for approximating the posterior distributions of damage probabilities. An application of the Bayesian bootstrap makes the proposed procedure highly automatic and convenient for assessing structures subjected to the hazard of the accidental actions. In addition, it can be used for specifying safe distances between the railway and nearby buildings. Structures of these buildings can be designed for tolerable probabilities of failures induced by the accidental explosions.

Keywords: *railway, accident, damage, explosion, Monte Carlo simulation, Bayesian bootstrap*

1. INTRODUCTION

Accidental explosions (AEs) on railway are dangerous, generally large-scale phenomena. Accidents involving such phenomena include severe damage to buildings and non-structural property. A selection of such accidents on railway is described in the books [1, 2]. Industrial activities require a rail transportation of explosives and such combustible materials as liquefied gases. They constitute an inevitable potentiality of AEs. Typical AEs are bursting of explosives and physical phenomena called the unconfined vapour cloud explosions (UVCEs) and boiling liquid expanding vapour explosions (BLEVEs). On the other hand, AEs on railway is generally rare, unexpected, and difficult-to-predict phenomena. Actions induced by AEs are uncertain and so is the potential mechanical damage form AEs. Predicting this damage requires a proper dealing with uncertainties related to both blast loading from AEs and response of structures to the AEs. In addition, general knowledge and statistical data on AEs are usually limited. The presence of considerable uncertainties generates a need to apply probabilistic methods to assessing the damage from AEs.

This paper develops a procedure for estimating potential damage to buildings induced by accidental explosions on railway. By the damage are meant failures of nearby structures caused by actions generated by the AEs. This damage is measured in terms of probabilities of potential failures caused by AEs. The estimation of the damage probabilities is based on a stochastic simulation of railway accident involving an explosion. The proposed simulation-based procedure quantifies epistemic (state-of-knowledge) uncertainties in the damage probabilities. These uncertainties are expressed in terms of Bayesian prior and posterior distributions. A foundation of the procedure is a computer intensive method known as a Bayesian bootstrap. It is used for approximating the posterior distributions of damage probabilities. An application of the Bayesian bootstrap makes the proposed procedure highly automatic and convenient for assessing structures subjected to the hazard of the accidental actions. In addition, it can be used for specifying safe distances between the railway and nearby buildings. Structures of these buildings can be designed for tolerable probabilities of failures induced by AEs.

2. METHODOLOGICAL BACKGROUND

The prediction of potential damage to structures from rare AEs on railway can be formulated as a problem of statistical inference. As long as the damage probability serves as a damage measure, its estimation amounts to a statistical estimation of a mean value. However, this estimation is far from trivial. Limited knowledge and uncertainties related to AEs render an application of classical statistics of impossible. A natural approach is applying methods of Bayesian statistical theory. They do not break down in the situation of the limited knowledge about AEs. The damage probabilities can be estimated by combining Bayesian inference with methods of structural reliability analysis (SRA). Such a combination is well known in the field of SRA. Nevertheless, the author's experience suggests that standard techniques of Bayesian updating are not fully suited to the estimation of damage probabilities. These techniques are too general to take account of specificity of predicting AEs and damage from them.

Standard Bayesian updating can be enhanced by a computer intensive method of applied statistics known as "bootstrap" [3]. There is a Bayesian form of the bootstrap or the Bayesian bootstrap. Rubin introduced the latter term in 1981 [4], and he was the first, who found the connection of the bootstrap with Bayesian inference. Bayesian bootstrap is a specialized application of the bootstrap intended for simulating the posterior distribution of a parameter [3]. The Bayesian form of the bootstrap offers the scope for its application to a quantitative risk assessment (QRA), as methods of QRA are substantially based on the Bayesian statistical theory.

QRAs often deal with large-scale accidents that may include AEs. Mechanical actions imposed on structures by AEs are called in terms of structural engineering the explosive actions (e.g., [5]). Estimating probabilities of foreseeable damage events can assess a potential damage to structures due to AEs. Formally, these probabilities can be handled by means of the bootstrap within the pure frequenters framework [6]. However, QRA provides consistent means for dealing with considerable uncertainties related to AEs and, sometimes, response of structures to AEs. The QRA means allow quantifying epistemic uncertainties in probabilities of the events in question in the form of prior and posterior distributions (e.g., [7]). The Bayesian bootstrap is suitable to utilizing attractive features of data resampling techniques. It allows applying these techniques to effective Bayesian updating within a QRA that considers AEs and potential damage from them. In particular, the Bayesian bootstrap can be applied to approximating posterior distributions of damage probabilities.

The present paper considers a practical application of the Bayesian bootstrap to a QRA focused on assessing damage to structures. It is used for Bayesian inference based on two sources of information:

- prior knowledge existing mainly in the form of mathematical models and historical data suitable to an approximate prediction of AE characteristics and
- new information consisting of a small-size sample of measurements of AE characteristics which are highly relevant to an exposure situation (a situation in vicinity of railway where the structure under investigation stands and a potential AE can occur).

The application of the Bayesian approach falls within the general approach to QRA known as classical Bayesian approach [7]. It is shown how to utilize the prior knowledge to specifying a prior distribution of a damage probability. Then it is demonstrated how to approximate its posterior distribution by means of the Bayesian bootstrap when the new sample of AE characteristics becomes available.

3. PROBABILITY OF EXPLOSIVE DAMAGE

Damage to a structure from an AE can be represented by a finite set of n_d random events D_i (i = 1, 2, … , n_d), each standing for a foreseeable and specific mechanical damage phenomenon. If the damage is assessed in the context of QRA, probabilities of D_is can be grouped together to establish a risk profile related to a particular AE and a specific exposure situation.

The probability of the damage D_i due to an AE can be expressed in the form

$$P(D_i \mid AE) = \int_{\text{all } y} P(D_i \mid \boldsymbol{y})\, \mathrm{d}F_{\boldsymbol{Y}}(\boldsymbol{y}) = E(P(D_i \mid \boldsymbol{Y}))\,, \tag{1}$$

where D_i is the random event of damage; *EA* is the random event of imposition of AE with any characteristics; $\boldsymbol{Y}$ is the random vector of AE characteristics; $\boldsymbol{y}$ and $F_{\boldsymbol{Y}}(\boldsymbol{y})$ are the value of $\boldsymbol{Y}$ and its distribution function (d.f.), respectively.

The definition (1) is based of the fragility function $P(D_i \mid \boldsymbol{y})$ relating particular value of AE characteristics, $\boldsymbol{y}$, to probability of the damage event D_i. Fragility function is an often-used tool for describing response of structures to extreme actions (e.g., [8]).

The fragility function can be expressed as some function $p_i(\cdot)$ which relates probability of D_i to $\boldsymbol{y}$ and thus takes on probability values, namely, $p_i(\boldsymbol{y}) = P(D_i \mid \boldsymbol{y})$. This function allows introducing a random variable $\widetilde{P}$ defined as a function of the random vector $\boldsymbol{Y}$, namely, $\widetilde{P} = p_i(\boldsymbol{Y}) = P(D_i \mid \boldsymbol{Y})$. A mean value of $\widetilde{P}$ can be denoted by μ and expressed as $\mu = E(\widetilde{P}) = E(P(D_i \mid \boldsymbol{Y}))$

The problem is that the d.f. $F_{\boldsymbol{Y}}(\boldsymbol{y})$ is not known due to scarcity or irrelevance of information on characteristics of many types of AEs. However, distribution of values of $\boldsymbol{Y}$ can be approximately predicted by existing models and, in addition, data related to $\boldsymbol{Y}$ can be collected by carrying out experiments which are highly relevant to the exposure situation under investigation.

4. KNOWLEDGE AVAILABLE FOR ESTIMATING DAMAGE PROBABILITY

The expression $\mu = E(\widetilde{P})$ implies that the damage probability can be expressed as uncertain distribution parameter μ amenable to Bayesian inference. The learning process involved in Bayesian inference is one of modifying the analyst's initial probability statements about distribution parameters prior to observing data to posterior knowledge incorporating both prior knowledge and the data at hand. In a pure Bayesian analysis, the prior distribution of μ should be specified subjectively. However, the purely subjective specification does not utilize prior knowledge about many types of AEs. Such knowledge, more or less relevant to the exposure situation under investigation, is usually available for the analyst. Therefore one can make a compromise between the frequenters and Bayesian statistical analysis and specify priors for damage probabilities from the prior knowledge.

Formal means for specifying priors based on data are provided by empirical Bayes methods [9]. This paper proposes a simple heuristic approach to specifying priors from existing knowledge. The approach is based on knowledge which is highly specific to a particular AE and is expressed in the form of a mathematical model $\varphi(\cdot)$ relating characteristics of exposure situation to characteristics of AE, namely, $\boldsymbol{y} = \varphi(\boldsymbol{x} \mid \psi)$ where $\boldsymbol{x}$ is the vector describing characteristics of exposure situation in which an AE can occur; ψ is the vector used to express epistemic uncertainties in those parameters of $\varphi(\cdot)$ which are uncertain in epistemic sense. The exposure situation represented by $\boldsymbol{x}$ may be uncertain in the stochastic sense and a random vector X with an aleatory d.f can model this. $F_{\boldsymbol{X}}(\boldsymbol{x})$. A part of prior knowledge should be represented by the fragility function $p_i(\boldsymbol{y})$ which can be established for an exposed structure by methods of the structural reliability analysis (e.g., [10]). Thus the fragility function $p_i(\cdot)$ together with the model $\varphi(\cdot)$ form the main part of the prior knowledge.

The need to apply Bayesian inference to estimating the damage probability $P(D_i \mid AE)$ may stem mainly from a partial irrelevance of the prior knowledge to a particular exposure situation. The configuration of a structure exposed to an AE as well as the accident capable of inducing the AE may be unique by a large margin and so may not fit in the prior knowledge. The source of the partial irrelevance may lie in (i) the structure of the model $\varphi(\cdot)$ and/or (ii) data used to fit the d.f. $F_{\boldsymbol{X}}(\boldsymbol{x})$ and estimate parameters of $\varphi(\cdot)$, that is, components of ψ.

The partial irrelevance may require a correction of AE prediction by experimental data that can be considered highly relevant to an exposure situation under investigation. Clearly, these experiments can be used for improving the model $\varphi(\cdot)$ by, say, increasing its relevance to the exposure situation. However, the highly relevant (case-specific) data on AE characteristics $\boldsymbol{y}$ and, possibly, interaction of AE with the exposed structure can be used directly to estimating $P(D_i \mid AE)$.

In theory, an amount of the case-specific data may be such that the model $\varphi(\cdot)$ will no longer be needed. In practice, however, the amount of the data may be limited because experiments on AEs, especially full-scale ones, are often expensive. This may require combining the new, case-specific data with the prior knowledge behind $\varphi(\cdot)$.

The case-specific data necessary for estimating $P(D_i \mid AE)$ should be gathered and represented in the form of a sample $\mathbf{y}' = (\boldsymbol{y}_1', \boldsymbol{y}_2', \dots, \boldsymbol{y}_n')^T$ containing experimental observations of the AE characteristics. Clearly, each experiment in a series yielding the sample $\mathbf{y}'$ should imitate a potential accident and the sample $\mathbf{y}'$ itself should posses the property called by statisticians the "representativeness" (e.g., [11]). Although this property of $\mathbf{y}'$ is very important, a detailed discussion about how to ensure the representativeness is out of the scope of this paper. In subsequent discussion, it is assumed that the sample $\mathbf{y}'$ possesses this property.

Given the sample $\mathbf{y}'$ and a fragility function of interest, $p_i(\boldsymbol{y})$, one can simplify estimating $P(D_i \mid AE)$ by introducing a fictitious sample $\boldsymbol{p} = (p_1, p_2, \dots, p_n)^T$, each component of which is calculated by $p_j = p_i(\boldsymbol{y}_j') = P(D_i \mid \boldsymbol{y}_j')$ $(j = 1, 2, \dots, n)$. In this way the problem of estimating $P(D_i \mid AE)$ is made less complicated by switching from a multi-dimensional analysis to a one-dimensional case. Then the components p_j of $\boldsymbol{p}$ can be treated as realizations of the random variable $\widetilde{P}$.

Expensiveness of experiments on AEs may cause that the size n of the sample $\mathbf{y}'$ will be too small to apply methods of classical statistics for estimating $P(D_i \mid AE)$. In addition, experiments on an AE may be unique and a series of them resulting in $\mathbf{y}'$ may be carried only once. This implies that the procedure of Bayesian updating using $\mathbf{y}'$ will be a single act, rather than a more or less constant process.

5. USE OF BAYESIAN BOOTSTRAP

Epistemic uncertainties related to parameters ψ of the model $\varphi(\boldsymbol{x} \mid \psi)$ can be expressed by introducing a random vector Ψ with a d.f. $F_\Psi(\psi)$. Then $\varphi(\boldsymbol{x} \mid \psi)$ by a random function $\varphi(\boldsymbol{X} \mid \Psi)$ will yield another random variable

$$\widetilde{M} = m(\Psi) = E_{\boldsymbol{X}}\big(p_i(\varphi(\boldsymbol{X} \mid \Psi))\big) = \int_{\text{all } \boldsymbol{x}} P(D_i \mid \varphi(\boldsymbol{x} \mid \Psi))\,\mathrm{d}F_{\boldsymbol{X}}(\boldsymbol{x}), \tag{2}$$

where $m(\cdot)$ denotes some function relating $\widetilde{M}$ to Ψ. A value μ of $\widetilde{M}$ is the damage probability at given ψ, namely, $E_{\boldsymbol{X}}\big(P(D_i \mid \varphi(\boldsymbol{X} \mid \psi))\big)$. A density of $\widetilde{M}$ denoted, say, by $\pi(\mu)$ can be used as prior quantifying epistemic uncertainty in damage probability $P(D_i \mid AE)$ (Fig. 1).

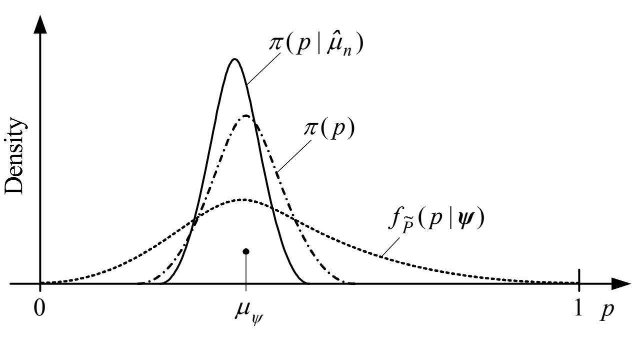

Figure 1. *Schematic representation of densities related to probabilistic damage assessment in Bayesian context: $f_{\widetilde{P}}(p \mid \psi)$ = density of the random variable $\widetilde{P} = p_i(\varphi(\boldsymbol{x} \mid \psi))$ with the mean μ_ψ; $\pi(p)$ = prior distribution of μ; $\pi(p \mid \hat{\mu}_n)$ = posterior distribution of μ*

The source of the epistemic uncertainty do not necessarily may be only on the side of the model $\varphi(\boldsymbol{x} \mid \psi)$ used to predicting AE characteristics. Epistemic uncertainty can also be related to values of the fragility function $p_i(\boldsymbol{y})$ at given $\boldsymbol{y}$. However, this part of epistemic uncertainty can be handled in the framework of (2) and is leaved out for brevity.

The usual Bayesian posterior has the form $\pi(\mu \mid \text{data}) \propto \pi(\mu)\, L(\text{data} \mid \mu)$ where "data" is represented eventually by the samples $\boldsymbol{p}$ or $\mathbf{y}'$. The main idea followed in paper is to replace the usual Bayesian posterior $\pi(\mu \mid \text{data})$ by an estimated posterior $\hat{\pi}(\mu \mid \text{data}) \propto \pi(\mu)\, \hat{L}_B(\text{data} \mid \mu)$ where $\hat{L}_B(\text{data} \mid \mu)$ is an estimate of the likelihood function based on bootstrap estimation of the density of the pivotal quantity $\hat{\mu}_n - \widetilde{M}$ with $\hat{\mu}_n = n^{-1} \sum_{j=1}^{n} p_j$. Boos and Monahan [12] suggested a possibility of such a replacement.

The first step is to estimate the distribution function of the data $\boldsymbol{p}$ using the empirical d.f. $\hat{F}_n$ of the p_j's. In the second step, a set of B random samples of size n from $\hat{F}_n$ is generated and a mean $\hat{\mu}'_{nb}$ is calculated for each sample b ($b = 1, 2, \ldots, B$). From the B simulated estimates $\hat{\mu}'_{n1}, \hat{\mu}'_{n2}, \ldots, \hat{\mu}'_{nB}$, one can compute a kernel density estimate.

$$\hat{k}_B(u) = \frac{1}{B\,w} \sum_{b=1}^{B} \kappa\left(\frac{u - (\hat{\mu}'_{nb} - \hat{\mu}_n)}{w} \right),$$

where w is a bandwidth (window width, smoothing parameter) and $\kappa(\cdot)$ is a kernel function. Since the function $\hat{k}_B(u - \mu)$ is an estimate of the sampling density of $\hat{\mu}_n$ given μ, the likelihood function of $\hat{\mu}_n$ can be estimated by

$$\hat{L}_B(\hat{\mu}_n \mid \mu) = \hat{k}_B(\hat{\mu}_n - \mu) = \frac{1}{B\,w} \sum_{b=1}^{B} \kappa\left(\frac{2\hat{\mu}_n - \mu - \hat{\mu}'_{nb}}{w} \right).$$

The resulting estimate of posterior of the damage probability is

$$\hat{\pi}(\mu \mid \hat{\mu}_n) = C(\hat{\mu}_n)\pi(\mu)\, \hat{L}_B(\hat{\mu}_n \mid \mu),$$

where the normalizing constant $C(\hat{\mu}_n)$ can be found by numerical integration.

Practical implementation of the bootstrap-based updating procedure is relatively simple, as the estimates $\hat{L}_B(\hat{\mu}_n \mid \mu)$ and $\hat{\pi}(\mu \mid \hat{\mu}_n)$ can be computed almost automatically. The estimate $\hat{L}_B(\hat{\mu}_n \mid \mu)$ is relatively insensitive to the choice of the kernel function $\kappa(\cdot)$ [3]. The only implementation problem is associated with the choice of the bandwidth w that can have considerable influence on $\hat{L}_B(\hat{\mu}_n \mid \mu)$. Shao and Tu [3] provide a review of approaches to choosing w.

6. EXAMPLE: ASSESSING EXPLOSIVE DAMAGE TO INDUSTRIAL BUILDING

6.1. Exposure Situation

Consider an industrial building to be constructed in vicinity of a 500 m section of railway (Fig. 2). This transportation facility is used to carry commercial explosives that can detonate in consequence of a railway accident. The accidental explosion will generate a shock front resulting in an impulsive loading imposed on the building. Characteristics of this loading can be described by the vector $\boldsymbol{y} = (y_1, y_2)^T$, where y_1 (MPa) and y_2 (MPa s/m^2) are the peak positive overpressure and positive impulse of the incident shock front, respectively.

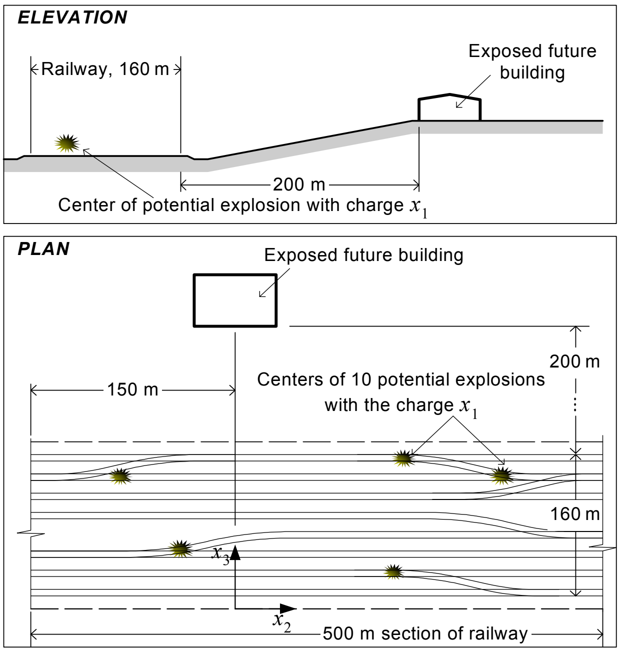

Figure 2. *Exposure situation involving accident with accidental explosion on railway*

The damage to building is represented by the random event D_i consisting in a flexural failure or a shear failure of reinforced concrete panels to be used for external envelope of the building. These panels should resist not only wind pressure but also the pressure arising from a reflection of the shock front by the façade of the future building.

The exposure situation is represented by the vector $\boldsymbol{x} = (x_1, x_2, x_3)^T$, where is x_1 (kg) is the weight of transported explosives (charge weight); x_2 (m) and x_3 (m) are the coordinates of the centre of accidental explosion in the coordinate system shown in Fig. 2. Thus the expression $r = (x_2^2 + (360 - x_3)^2)^{1/2}$ is the standoff of the explosion to the façade and the expression $s = x_1^{1/3} r^{-1}$ is the so-called inverse scaled distance (kg$^{1/3}$/m) (inverse normalized standoff, e.g. [1]).

6.2. Prior knowledge

The model $\boldsymbol{y} = \varphi(\boldsymbol{x} \mid \psi) = (\varphi_1(\boldsymbol{x} \mid \psi), \varphi_2(\boldsymbol{x} \mid \psi))$ represents the prior knowledge with the model components

$$y_1 = \varphi_1(\boldsymbol{x} \mid \boldsymbol{\psi}) = \psi_1 (\psi'_1 s + +\psi'_2 s^2 + \psi'_3 s^3), \tag{3}$$

$$y_2 = \varphi_2(\boldsymbol{x} \mid \boldsymbol{\psi}) = \psi_2 \psi'_4 x_1^{2/3} r^{-1} \tag{4}$$

and the vector of regression parameters

$$(\psi'_1, \psi'_2, \psi'_3, \psi'_4)^T = (0{,}1\,\text{MPa} \times \text{m/kg}^{1/3}; 0{,}43\,\text{MPa} \times \text{m}^2/\text{kg}^{2/3};\ 1{,}4\,\text{MPa} \times \text{m}^3/\text{kg};\ 6{,}3\,\text{MPa} \times \text{s/(m kg}^{2/3}))^T$$

Components of the vector $\boldsymbol{\psi} = (\psi_1, \psi_2)^T$ are dimensionless adjustment factors (relative overpressure and relative impulse of the commercial explosive, which can detonate in consequence of the railway accident, compared to an equivalent weight of TNT explosive).

Stochastic uncertainty related to the exposure situation and so arguments of the model $\varphi(\boldsymbol{x} \mid \psi)$ are expressed by a random vector $\boldsymbol{X} = (X_1, X_2, X_3)^T$ with components distributed as follows $X_1 \sim$ N(500 kg, 30 kg), $X_2 \sim$ U(0 m, 160 m), $X_3 \sim$ U(0 m, 500 m), where L and U denote the normal distribution and the uniform distribution, respectively. The uniform distribution of X_2 and X_3 implies that the accidental explosion can occur with the same probability within the 500 m ×160 m area of the railway. The uniform distribution of the explosion point coordinates X_2 and X_3 is considered only the first approximation to forecasting the position of explosion centre.

Epistemic uncertainty can be introduced into the model $\varphi(\boldsymbol{x} \mid \boldsymbol{\psi})$ by assuming components ψ_1 and ψ_2 of $\boldsymbol{\psi}$ to be random variables. The adjustment factors ψ_1 and ψ_2 are conventionally expressed as fixed values or deterministic function of the so-called scaled distance [1]. However, the nature of explosive actions is highly random and adjustment factors may be expressed in the form of random variables Ψ_1 and Ψ_2 having epistemic distributions. In this example, a lognormal distribution is assumed for the random adjustment factors: Ψ_1 ~L(0,15842; 0,09975) with the mode of 1,16; Ψ_2 ~L(0,17551; 0,09975) with the mode of 1.18.

Further part of prior knowledge is represented by the fragility function $p_i(\cdot)$ which relates the peak positive overpressure y_1 and the positive impulse y_2 to failure probability $P(D_i \mid \boldsymbol{y})$. The form of $p_i(\boldsymbol{y})$ can be established using results obtained by Low & Hao [13], who investigated the reliability of RC slabs under impulsive loads. The function $p_i(\boldsymbol{y})$ is represented in the form $p_i(\boldsymbol{y}) = P(D_i \mid \varphi'(x_1, x_2, y_1), \varphi''(y_1, y_2))$ where $\varphi'(.)$ and $\varphi''(.)$ are two deterministic functions which relate the peak overpressure and impulse of reflected shock front to the respective characteristics of the incident shock front. The function $\varphi'(.)$ is called the “reflected pressure factor versus angle of incidence” and is usually represented in the graphical form [14]. The angle of incidence of the shock front can be simply determined from the explosion point coordinates x_2 and x_3. The function $\varphi''(.)$ is a simple formula allowing estimating reflected impulse from the incident impulse y_2 [1].

In this example, the fragility function $p_i(\boldsymbol{y})$ is approximated by a d.f. $F(z_1, z_2 \mid \mu_{P1}, \mu_{P2}, \sigma_{P1}, \sigma_{P2}, \rho)$ of a bivariate normal distribution, namely,

$$p_i(z_1, z_2) = F(z_1, z_2 \mid \mu_{P1}, \mu_{P2}, \sigma_{P1}, \sigma_{P2}, \rho), \qquad (5)$$

where the arguments z_1 and z_2 are defined as $z_1 = \varphi'(x_1, x_2, y_1)$ and $z_2 = \varphi''(y_1, y_2)$ and the parameters μ_{P1} =3,2×10^{-3} MPa, μ_{P2} =1,45 MPa×s/m^2, σ_{P1} =0,64×10^{-3} MPa, σ_{P2} = 0,29 MPa×s/m^2, ρ = 0,2.

Clearly, the analyst may have uncertainties in epistemic sense related to elements of the model $p_i(\boldsymbol{y})$ as well as further elements of the model $\varphi(\boldsymbol{x} \mid \boldsymbol{\psi})$, first and foremost, the vector of the regression parameters, $(\psi'_1, \psi'_2, \psi'_3, \psi'_4)^T$; however, these uncertainties are ignored in the present example for simplicity.

The formulas (3) and (4) are standard relations obtained by experiments on TNT. They are, strictly speaking, valid only for a distant explosion of a charge positioned on the ground that forms a horizontal plane. In addition, the model represented by (3) and (4) assumes that the shock front generated by a TNT explosion does not encounter any obstacles. A gentle slope of the ground between the railway and the exposed future building makes the model $\varphi(\boldsymbol{x} \mid \boldsymbol{\psi})$ only partially relevant to the exposure situation shown in Fig. 2. In addition, the assumption of the uniform distribution of the explosion point coordinates X_2 and X_3 may be considered sound; however, a detailed analysis of railway traffic by means of QRA may introduce corrections in this assumption.

Despite the irrelevance of the model represented by (3) and (4), it can be used for specifying a prior distribution of $P(D_i \mid AA)$.

6.3. Specifying Prior from Existing Knowledge

Propagating epistemic uncertainty related to the parameters ψ_1 and ψ_2 can specify the prior distribution of the damage probability $P(D_i \mid AE)$. Thus uncertainty can be transformed into uncertainty in $P(D_i \mid AE)$ by (2). The density $\pi(\mu)$ of the random variable $\widetilde{M}$ defined by (2) will serve as the prior distribution.

The density $\pi(\mu)$ can be specified by fitting it to the sample μ_k (k = 1, 2, … , n_k), the elements of which are estimates of the mean values $E_{\boldsymbol{X}}(p_i(\varphi(\boldsymbol{X} \mid \psi_k)))$ for given ψ_k. The values ψ_k is generated from the epistemic d.f. $F_\Psi(\psi)$ by means of stochastic (Monte Carlo) simulation. The mean values $E_X(\cdot)$ can also be estimated by simulation, namely, by $\mu_k = n_e^{-1}\sum_{l=1}^{n_e} p_i(\varphi(\boldsymbol{x}_l \mid \psi_k))$ where $\boldsymbol{x}_l$ is the value of $\boldsymbol{X}$ generated from the aleatory d.f. $F_X(\boldsymbol{x})$. Fig. 3 shows a density of normal distribution fitted to the generated sample μ_k (k = 1, 2, … , 1000) obtained using $n_e = 1\times10^5$. Consequently, the prior density $\pi(\mu)$ is specified as a density of a normal distribution N(0,353; 0,09196) with a 90 % confidence interval]0,202; 0,504[.

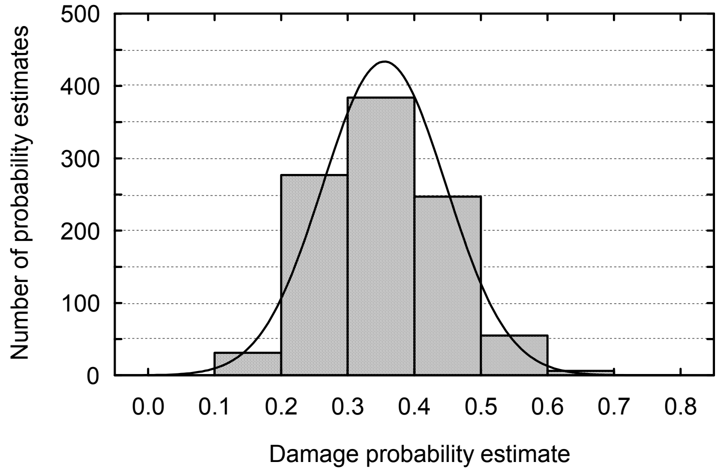

Figure 3. *Histogram of the sample μ_k (k = 1, 2, … , 1000) and a transformed density of a normal distribution N(0.353, 0.09196) fitted to the sample*

6.4. New Data on Possible Railway Accident

The new data may be obtained by an experiment that imitates potential accidents on the railway section. A detailed analysis of traffic in the railway section can yield, say, ten potential centres of accidental explosion (Fig. 2). Then a series of ten explosions can be carried out by detonating charges x_1 of the explosive under investigation in a blast measuring facility that imitates the ground surface of the exposure situation. The weight of charges, x_1, can be chosen by chance from the distribution of X_1.

The series of experiments will yield a sample $\mathbf{y}' = (\boldsymbol{y}'_1, \boldsymbol{y}'_2, \dots, \boldsymbol{y}'_{10})^T$. Let us say that ten pairs of measured overpressures and impulses given in Table 1 represent results of the experiment.

Table 1. Observed elements $\boldsymbol{y}'_i$ of the sample $\mathbf{y}'$ and computed elements p_j of the sample $\boldsymbol{p}$

j	1	2	3	4	5
$\boldsymbol{y}'_i$ (MPa, MPa×s/m^2)	(2,06; 0,960)	(2,99; 1,38)	(2,68; 1,26)	(2,73; 1,27)	(3,41; 1,59)
p_j	0,001674	0,1484	0,05482	0,06283	0,4286
j	6	7	8	9	10
$\boldsymbol{y}'_i$ (MPa, MPa×s/m^2)	(2,66; 1,23)	(2,74; 1,24)	(3,29; 1,46)	(2,92; 1,30)	(3,82; 1,74)
p_j	0,04543	0,0545	0,2861	0,09814	0,7022

A small size of the sample $\mathbf{y}'$ is likely, as the experiment imitating the accident may be expected to be expensive and time consuming. Clearly, the ten-element sample $\mathbf{y}'$ is too small to estimate the damage probability $P(D_i \mid AE)$ using $\mathbf{y}'$ alone, that is, without prior knowledge. However, this sample may be used to update the prior distribution N(0,353; 0,09196) of $P(D_i \mid AE)$ specified on the basis of prior knowledge. To do this, the fragility function (5) should be applied to transform $\mathbf{y}'$ into the fictitious sample $\boldsymbol{p}$ given in Table 1.

The mean $\hat{\mu}_{10}$ of the sample ***p*** is equal to 0,1882. The sample ***p*** can be applied to estimating the likelihood function $L(\hat{\mu}_{10} \mid \mu)$ and approximating posterior distribution of $P(D_i \mid AE)$.

6.5. Posterior Distribution as an Epistemic Estimate of Damage Probability

The estimate of the likelihood function, $\hat{L}_B(\hat{\mu}_{10} \mid \mu)$, was obtained using the Gaussian kernel function $\kappa(\cdot)$. The number of bootstrap replications, *B*, necessary to generate the sample ($\hat{\mu}'_{n1}$, $\hat{\mu}'_{n2}$, … , $\hat{\mu}'_{nB}$) was taken to be equal to 1000. The choice of *B* was based on the rules of thumb suggested in [3].

The choice of the bandwidth *w* was not investigated in detail. Three values 0,01, 0,02, and 0,03 of *w* were chosen manually to assess the influence of *w* on the posterior density $\hat{\pi}(\mu \mid \hat{\mu}_n)$. As expected, the largest value of *w* produced the smoothest estimate of the likelihood function. Fig. 4a shows a graph of $\hat{L}_B(\hat{\mu}_{10} \mid \mu)$ at $w = 0.03$.

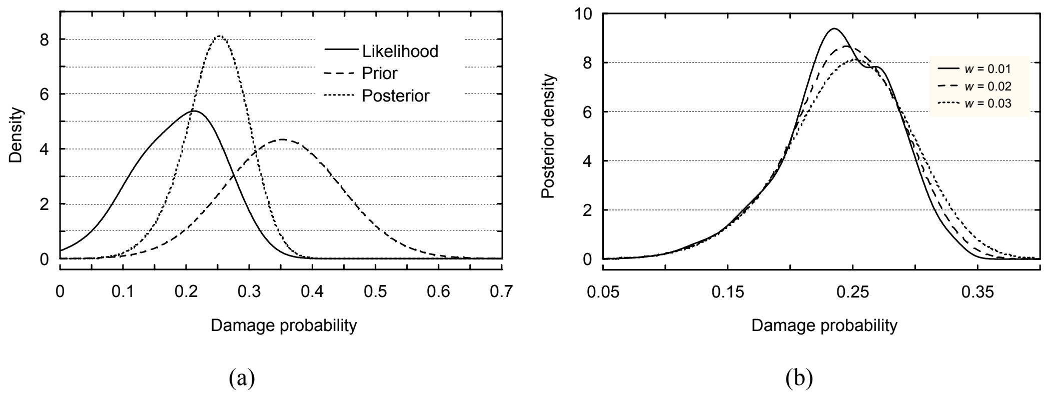

Figure 4. *Graphs of the functions related to updating via Bayesian bootstrap: (a) graph of the likelihood function estimate* $\hat{L}_B(\hat{\mu}_{10} \mid \mu)$*, the prior density* $\pi(\mu)$ *and estimate of posterior density* $\hat{\pi}(\mu \mid \hat{\mu}_{10})$ *obtained with the bandwidth w = 0,03; (b) Posterior density of the damage probability at three values of the bandwidth w*

Three approximations of the posterior density $\hat{\pi}(\mu \mid \hat{\mu}_{10})$ computed at the three values of *w* are show in Fig. 4b. These approximations were obtained by a numerical calculation. The approximations $\hat{\pi}(\mu \mid \hat{\mu}_{10})$ express the epistemic uncertainty related to $P(D_i \mid AE)$. The three approximations $\hat{\pi}(\mu \mid \hat{\mu}_{10})$ can be compared and so the influence of the bandwidth *w* on the posterior density assessed by calculating approximate confidence intervals for each of the three $\hat{\pi}(\mu \mid \hat{\mu}_{10})$. Table 2 shows three 90 % confidence intervals at the three values of *w*. These intervals can be easily computed during the to numerical evaluation of $\hat{\pi}(\mu \mid \hat{\mu}_{10})$. The confidence intervals given in Table 2 can be compared with a 90 % bootstrap confidence interval computed using the ten-element sample ***p***. The limits of the latter interval can be taken as the 5th and 95th percentiles of an ordered sample obtained from the bootstrap sample ($\hat{\mu}'_{10,1}$, $\hat{\mu}'_{10,2}$, … , $\hat{\mu}'_{10,1000}$) [6]. The bootstrap confidence interval is]0.0902, 0.304[. This interval is based on the new data only (ignores the prior knowledge). It is apparent that the width of the intervals given in Table 2 is considerably smaller than the one of the bootstrap confidence interval.

Table 2. Approximate 90 % confidence intervals calculated for the damage probability $P(D_i \mid AE)$ from posterior densities

Bandwidth *w*	Constant $C(\hat{\mu}_{10})$	Confidence interval
0,01	1,2311	] 0,158; 0,308 [
0,02	1,2470	] 0,159; 0,315 [
0,03	1,2713	] 0,160; 0,326 [

The approximate confidence intervals given in Table 2 may be used for making a decision concerning whether the resistance of the wall panels is sufficient to resist the damage D_i or, alternatively, to design the panels for a tolerable value of the damage probability $P(D_i \mid AE)$.

7. CONCLUSION

The paper presented an approach to assessing damage to buildings due to accidental explosions (AEs) on railway. The damage can be caused to nearby structures by the blast loading generated by an AE. The attention was focussed on quantifying uncertainties related to both characteristics of AEs and damage from them. The damage was understood as structural failures caused by blast loading. Probabilities of these failures (damage probabilities) were taken as damage measures. The prediction of damage was realised as estimating the damage probabilities. A procedure for estimating the damage probabilities has been proposed. The basis of the procedure is an application of a computer intensive method of applied statistics that is called the Bayesian bootstrap. This method is used for expressing estimates of the damage probabilities in terms of Bayesian posterior distributions. These distributions were treated as measures of epistemic uncertainty in the damage probabilities.

Formally, the Bayesian bootstrap was applied to Bayesian inference using prior knowledge. It consists of mathematical models and historical data suitable to an approximate prediction of loading from an AE. Another part of this knowledge is new information. It is represented by a small-size sample of highly case-specific measurements of the AE. The procedure is to a large measure automatic. It does not require any statistical derivations. Therefore, it can be applied to a practical assessment of existing structures build in vicinity of railway and exposed to the hazard of AEs. The procedure can also be applied to specifying safe distances between railway and nearby future buildings.

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