APPLICATION OF A GENERAL MODEL OF MULTIMODAL TRANSPORTATION IN LOGISTICS

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The requirements for transport services are exogenous and can be generated by the input-output model if it is available or determined from other sources observed needs or corrected previously observed needs. The choice of a particular transport facility or a group of facilities is exogenous and multimodal transhipments are admissible.
A suggested mathematical model is aimed at optimising multimodal transportation allowing us choose the most suitable transport facilities in each particular case.

Keywords: *logistic, model, multimodal transportation, optimisation*

1. INTRODUCTION

Economic development of any country is hardly possible without an effective transport system. The manufactured goods should be transported from an enterprise to the user at minimal time and expenses. Transport makes an essential part of any logistic system, which is closely linked, with other elements of logistics. It should be effectively used to allow achieving good results to the company in business.

Most firms use two or more kinds of transport facilities to ensure safe and timely delivery of goods. Large enterprises in Western Europe have special departments for coordinating cargo transportation by various means of transport. The latter are needed to maintain close links between manufactures and users. The need for coordination arose because of different capabilities and tariffs of various transport facilities. For instance, since air transport is not easily available, the coordination of its operation with motor transport is needed while loading and distributing the delivered goods. Similar examples relating to rail, land and pipeline transport may also be provided. This does not, however, apply to motor transport because it is much more advantageous in this respect. Having various transport facilities at his disposal, a manager should try to take the main advantages, for example, low cost of transportation of a particular means of transport in order to compensate low quality of services provided by some other transport facility. This is the main reason for transporting coal or grain by rail or water transport. Rail transport can make water transport more easily available, while using water transport for transporting goods over long distances allows us to save much money due to low tariffs.

The main goal of coordinating bodies is to gain an advantage from a particular transport facility while reducing its disadvantages. They should also consider all positive and negative aspects of various means of transport. For example, coordinating the operation of rail and water transport on a particular route will reduce the cost of transportation compared to using water transport alone. On the other hand, the delivery time will be shorter in the case of using a combined system than by transporting the cargo only by water transport and longer than carrying it by rail transport. A decision to use a combined system should be made taking into account the total cost of delivery.

A transport company is usually eager to cooperate with other firms if it cannot deliver the cargo to the place of destination alone. However, if it can do it itself, the firm does not want to collaborate with others.

Another problem arises when it is necessary to transfer cargo from one transport facility to another in multimodal transportation. This increases delivery time and expenses. The considered problem can be solved by the companies, which can transfer a particular transport facility with a cargo to another transport facility.

The need for a wider use of mathematical methods and models in transport arose because more goods had to be transported and rapid scientific and technological development brought about new possibilities in this area. This, in turn, gave a strong impetus to solve the problems of planning and management in transport by new more advanced methods. Mathematical models in transport are aimed at evaluating its performance. Mathematical methods are used in the following main areas of transport now:

1. Drawing up optimal plans and controlling their fulfilment.
2. Solving the problems of mathematical programming in transportation including the following issues:
   - drawing up an optimal plan of shipments;
   - minimizing the expenses in transportation;
   - designing new traffic lines, etc.
3. Solving various technical and economic problems:
   - developing an optimal scheme using various technological systems;
   - optimal lay-out of installations;
   - choosing a rational technology;
   - planning and distributing optimal amounts of spare parts in the stockyards, etc.

2. A MODEL FOR MULTIMODAL TRANSPORTATION OF VARIOUS KINDS OF GOODS

Transport infrastructure of the state or a region is used most effectively in the context of strategic planning of traffic flows when overall expenses are the lowest. Even if an assumption that many factors do not allow goods to be transported at the lowest expenses is true, this is also valid for a case when the government controls shipments and the developed model is based on the reduced expenses. The expenses make a basis of the model and may have variable components such as costs, delays, power consumption, etc.

Let a network have a set of nodes *N*, a set of branches *A*, $A \subset N \times N \times M$ a range of transport facilities *M* and a set of cargo transfers *T*, $T \subset A \times A$. Let us denote the numerical value of the sets by n_N, n_A, n_M and n_T. Every branch *a*, $a \in A$ is associated with the cost function $s_a(.)$ which depends on the size of the cargo on the branch or, probably, on the cargo size of other network branches. In a similar way, cost function $s_t(.)$ is associated with any cargo transfer *t*, $t \in T$.

Goods transported in a multimodal network are denoted by the index *p*, $p \in P$, where *P* is a set of all goods having the value of n_p. Any product is sent from the node of origin *o*, $o \in O \subseteq N$ to the destination node *d*, $d \in D \subseteq N$. The demand for any kind of goods for all pairs of (O/*D)* nodes is determined in the O/*D* matrix including a subset of various transport facilities to be used only for a considered case. It is assumed that the demand for and the choice of particular transport facilities are determined exogenously. Let $g^{m(p)}$ be a matrix of the demand for the product $p \in P$, where $m(p)$ is a subset of means of transport belonging to a set $M(p)$, which is a set of subsets of all transport facilities used to deliver product *p*.

The flow of goods *p* in a multimodal network is denoted by *vp* and is made up of the flows of the considered goods found on the network branches and cargo transfer branches.

$$v = \begin{pmatrix} (v_a^p), \ a \in A \\ (v_t^p), \ t \in T \end{pmatrix} \tag{1}$$

The total flow of goods in a multimodal network is denoted by $v = (vp)$, $p \in P$ and is $n_p(n_A + n_T)$ – dimensional vector.

Average cost functions, $s_a^p(v)$ on branches and $s_t^p(v)$ on cargo transfer branches, correspond to a given flow vector *v*. Average cost functions of the goods *p* are denoted similarly to those of the flow *vp*, *sp*, $v \in p$, where

$$s^{p} = \begin{pmatrix} \left(s_{a}^{p}\right), \ a \in A \\ \left(s_{t}^{p}\right), \ t \in T \end{pmatrix} \tag{2}$$

and $s = (sp), \ p \in P$ is $n_p \left(n_A \times n_T\right)$ – dimensional vector of average cost functions.

Overall costs of the flow of goods $p, \ p \in P$ on cargo transfer branch $t, \ t \in T$ are the product $s_a^p(v) \ v_a^p$; overall costs of the flow on cargo transfer branch $t, \ t \in T$, are $s_t^p(v) \ v_t^p$. Overall costs of all flows of goods in a multimodal network are the function F that is to be minimized:

$$F = \sum\nolimits_{p \in P} \left(\sum\nolimits_{a \in A} s_a^p(v) v_n^p + \sum\nolimits_{t \in T} s_t^p(v) v_t^p \right) \tag{3}$$

satisfying the stability and non-negative character of the flows in the set. The following notation is used to define the constraints of the above multimodal network for transporting various kinds of goods. Let $K_{od}^{m(p)}$ be a set of routes leading from the node of origin $o, \ o \in O$ to a destination node d, $d \in D$, when only transport facilities from $m(p) \in M(p), \ p \in P$ are used. Then, the equation for the flows will be as follows:

$$\begin{gathered} \sum\nolimits_{k \in K_{od}^{m(p)}} h_k = g_{od}^{m(p)}, \\ o \in O, \quad d \in D, \quad m(p) \in M(p), \quad p \in P, \end{gathered} \tag{4}$$

where h_k is the flow for the route k, non-negativity constrains:

$$h_k \geq 0, \ k \in K_{od}^{m(p)}, \ o \in O, \ d \in D, \ m(p) \in M(p), \ p \in P. \tag{5}$$

Let Ω be a set of flows v satisfying (4) and (5). Since the flow equation is defined in the space of flow routes, let us use the following relationship between the branch and route flows for the sake of convenience:

$$v_a^p = \sum\nolimits_{k \in K^p} \delta_{ak} h_k, \ a \in A, \ p \in P, \tag{6}$$

where $K^p = \bigcup_{m(p) \in M(p)} \bigcup_{o \in O} \bigcup_{d \in D} K_{od}^{m(p)}$ is a set of routes o that may be used for delivering p

$$\delta_{ak} = \begin{cases} 1, \text{ when } a \in k \\ 0 \text{ differently} \end{cases}$$

is an indicator function defining the branches of a particular route. It also applies to the branches of cargo transfer:

$$v_t^p = \sum\nolimits_{k \in K^p} \delta_{tk} h_k, \ t \in T, \ p \in P, \tag{7}$$

where

$$\delta_{ak} = \begin{cases} 1 \text{ when } t \in T \\ 0 \text{ differently.} \end{cases}$$

Cargo transfer t is on the route k, if two branches determining the transfer refer to this route. Finally, minimizing the objective function (3), and satisfying the constraints of (4) – (5) as well as the constraints according to the definition in (6) – (7) obtain an optimal multimodal distribution model of various kinds of goods.

A described model is of a general character and may be applied to define the demand in various ways. Though the model is used when the choice of transport facilities allowing referring the demand for product g^p to a particular subset of a set of transport facilities has been made 'a priori', a possibility remains to transport goods by all available means of transport, if the need arises. This means that $m(p)$ is a set of all transport facilities of a network, while $M(p)$ has only one component, which is $m(p)$ in this case. The model is flexible allowing us to describe intermodal traffic as well. The transfer of cargo from one transport facility to another may be made only at particular nodes of a network and the particular means of transport may be used. The model is also adequate when various large investment scenarios are compared.

It is assumed that the demand for goods is exogenously assigned but the choice of the haulier or a particular transport has not been introduced in the expression yet. Therefore, we assume that the behaviour of the sender is reflected in the matrix of origin and destination nodes and in the details of transport facility selection. However, it should be noted that if some information about the senders and their transport facilities is available, it could be introduced into the model, with the appropriately defined subsets of admissible means of transport in the respective O/D matrix. The ultimate flows of transport facilities are known, while precise data about the hauliers is actually not available. The above model allows us to describe the infrastructure, transport means and services and prescribe the particular kinds of goods to be carried by several means of transport simultaneously. This allows us to describe a competition in terms of the goods to be carried and the available services, which is relevant for determining network extension scenarios. Though the goal is to offer a method of strategic planning available to the whole country, the model is sufficiently flexible to be used in describing the infrastructure for a single haulier. In other words, a suggested model is capable of describing a large multimodal system for transportation of various goods for strategic planning. However, it may also be used for the analysis of the cargo carried by one haulier.

3. A GENERAL MATHEMATICAL MODEL FOR MULTIMODAL TRANSPORTATION

A mathematical model is aimed at optimising multimodal transportation based on using various means of transport. The objective function of the model used to determine the minimal costs and minimal time of delivery as well as minimal pollution of the environment as a constant for particular transport facilities and minimal costs of cargo insurance will be as follows:

$$Z_{\min} = \min\left(\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} + \sum_{k=1}^{l} \overline{V}_k + \sum_{k=1}^{l} \overline{T}_k + \sum_{k=1}^{l} \overline{D}_k \right) \quad (8)$$

where C_{ij} is the cost (Lt) of delivering a container from the i-th loading place to the j-th place of destination $(i = 1 \div m, \quad j = 1 \div n)$;

x_{ij} the number of containers (units) to be carried from the i-th loading place to the j-th place of destination $(i = 1 \div m, \quad j = 1 \div n)$;

$\overline{V}_k$ is the average cost of an hour of container transportation (Lt) $(k = 1 \div l)$;

$\overline{T}_k$ is the average cost of transport facility (Lt) in terms of the environment pollution $(k = 1 \div l)$;

$\overline{D}_k$ is the average insurance cost of cargo (Lt) carried by a particular kind of transport facilities $(k = 1 \div l)$.

The limiting conditions of the mathematical model for a balanced transportation problem are expressed in the following way:

$$\begin{cases} \sum_{j=1}^{n} x_{1j} \le b_1 \\ \sum_{j=1}^{n} x_{2j} \le b_2 \\ \dots\dots\dots\dots\dots \\ \sum_{j=1}^{n} x_{mj} \le b_m \end{cases} \quad (9) \qquad \begin{cases} \sum_{i=1}^{m} x_{i1} = a_1 \\ \sum_{i=1}^{m} x_{i2} = a_2 \\ \dots\dots\dots\dots\dots \\ \sum_{i=1}^{m} x_{in} = a_n \end{cases} \quad (10)$$

$$x_{ij} \ge 0, \quad \text{when } i = 1 \div m \\ j = 1 \div n,$$

moreover,

$$\sum_{i=1}^{m} b_i = \sum_{j=1}^{n} a_j \quad (11)$$

where b_1, b_2, ... b_m are containers (units) at destination places (demand).

If the condition (11) is not satisfied, then an imaginary place of loading or destination is introduced to obtain a balanced transportation problem.

$\sum_{K=1}^{l} \overline{V}_k$ is the average cost of container delivery;

$\sum_{K=1}^{l} \overline{T}_k$ is the average cost of using a particular kind of transport facilities in terms of the environment pollution;

$\sum_{K=1}^{l} \overline{D}_k$ is the average cost of cargo insurance for a particular kind of transport facilities, i.e. all three components of an objective function depend on x_{ij} which is the number of containers (units) carried from the i-th loading place to the j-th place of destination and, thus, on the number of hauls.

For example, containers may be carried by rail, motor or sea transport, while the number of containers in a haul varies. Therefore, when calculating average expenses for different modes of transportation, the number of hauls should be taken into consideration.

4. CONCLUSION

A mathematical model aimed at optimising multimodal transportation based on the use of various kinds of transport facilities and allowing us to choose the most suitable means of transport and services in any particular case is offered in the present paper.

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