THE ANALYSIS OF APARTMENT HOUSE CONSTRUCTION INVESTMENTS EFFICIENCY APPLYING MATHEMATICAL MODELLING

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In the paper the application of mathematical modelling to evaluating the efficiency and risk of investment projects is discussed.

Keywords: *apartment house, construction, investment, mathematical modelling*

1. INTRODUCTION

In real estate sector, especially in construction or purchasing of commercial buildings or apartment house, adequate evaluation of market development and property management is of paramount importance. To ensure the reliability of the investment projects, which is closely associated with the appropriate research into the problems of market and property management and processing of the data obtained, it is necessary to define a set of criteria for the appropriate building selection. A comprehensive analysis of reliable market data is the most important factor for developing the construction investment projects [25, 27, 29] as well as for selecting a property management project and the appropriate processing of attributes by various mathematical models.

The method of simulation is widely used in solving the problems of strategic planning [16] and financing of large industrial companies [2, 4, 5, 6, 7, 14], as well as in organizing the work of small corporations [11, 23, 26], and developing strategies of construction management and in construction planning [17, 24]. The considered method is also applied to simulation of building processes [1, 3, 8, 15, 21, 22, 26, 30] and making financial solutions by construction firms [12, 19]. In each case it is necessary to analyse the initial data [9, 18].

The state of macroeconomics in the country also has an indirect impact on real estate market, implying that the individuals and firms engaged in real estate business experience its effects [20]. The economic situation and legislative basis of the state determine their activities in this area. The better the macroeconomic indices, the better the situation on real estate market.

Any particular case of mathematical modelling is very specific being dependent to a large extent on the particular area of application. In very few cases, application of a model in a particular scientific field or area of practical activities may be extended to other areas without making essential changes. Most of the economic models are discrete, implying that the initial data and the results obtained are discrete values. In the suggested model, the most probable variation intervals of the parameters are taken as a basis of modelling. Opportunities of such approach it has been proved in Lee et al. [13]. The modelling is aimed at determining zones relating to the quality of decisions in the area of investment.

2. A METHOD OF DETERMINING VARIOUS RISK ZONES OF INVESTMENTS

Mathematical modelling can be used for determining the efficiency of the investment projects [28]. A quantitative method offered for evaluating the efficiency of investment in the apartment house purchase, decoration and sale is given below. In developing this method much attention is paid to the accuracy of the initial data and investment risk analysis. Notations: k – grade (category) of a building; $1 \le k \le q$ ($k \in N$), q – the total number of a premises of various grades (categories); P (k) – building selling price; $P_{\min}(k)$, $P_{\max}(k)$ – minimum and maximum price; $A_{\min}(k)$, $A_{\max}(k)$ – minimum and maximum decoration cost (the initial purchase cost included); $\overline{P}(k)$, $\overline{A}(k)$ – average prices, respectively.

Based on the statistical data $P_{min}(k)$, $P_{max}(k)$, $A_{min}(k)$, $A_{max}(k)$, where $k=k_{min}$, $k_{min}+1$, …, k_{max}, interpolation polynomials of $n = k_{max}–k_{min}$ degree, are obtained, which are written as $a_n k^n + a_{n-1}k^{n-1} + ... + a_1 k + a_0$ for all dependent $P_{min}(k)$, $P_{max}(k)$, $\overline{P}(k)$, $A(k)$, $A_{min}(k)$, $A_{max}(k)$, where variable k is changing continuously, rather than discretely: $k_{min} \le k \le k_{max}$. This set is denoted by K. All functions given above are usually the increasing functions of the argument k.

Determining zones of various investment risk exposure. Risk zones may be defined as risk – free, standard risk, high-risk zones and a zone of losses. Their detailed analysis is given below.

Risk – free zone of investment. This is an area where the minimum selling price of premises $P_{min}(k)$ is higher than the maximum price of finished building $A_{max}(k)$ (initial purchase price included), i.e. a set K_+ of values k satisfying this inequality, $P_{min}(k) - A_{max}(k) > 0$. This set may be either empty ∅ or representing a union of separate intervals of $k \in [k_{min}, k_{max}]$.

Algorithm for determining risk – free zone K_+. The algorithm considered is represented as a series of steps:

1. The real roots (if any) $k_1 < k_2 < ... < k_l$ $(l \le n)$, belonging to the interval $[k_{min}, k_{max}]$: $k_i \in [k_{min}, k_{max}]$ (i=1, …, l) are found for polynomial $P_{min}(k) - A_{max}(k) = 0$.

2. At all the intervals (k_{min}, k_1), (k_1, k_2), …, (k_l, k_{max}) signs of the difference $P_{min}(k) - A_{max}(k)$ are determined. To assess the continuity of the function $P_{min}(k) - A_{max}(k)$, it is sufficient to find the sign, i.e. to determine the expression value at the points of each interval.

3. The intervals with a positive difference $P_{min}(k) - A_{max}(k)$ (if any) are selected from the intervals obtained. It is a set K_+.

If a set K_+ coincides with the interval [kmin, kmax], then the whole interval $k_{min} \le k \le k_{max}$ is a risk – free investment zone. In this case, no other zones are sought for.

In Fig. 1 various ways of determining the particular risk-free zones are graphically illustrated.

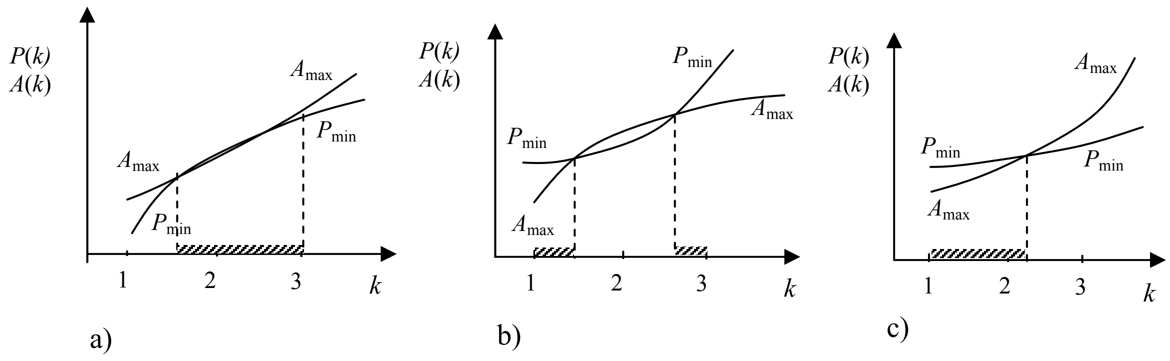


Figure 1. *The determination of risk-free zones*

Standard risk zone of investment $K_=$. This is zone $\overline{K}$, where an average selling price $\overline{P}(k)$ is higher than an average decoration price $\overline{A}(k)$ (initial purchase cost included), except for risk – free investment zone K_+: $K_= = \overline{K} \setminus K_+$.

The determining of a standard risk zone is shown in Fig. 2.

Algorithm for determining zone $K_=$. It consists of the following steps:

1. Real roots (if any) $\overline{k}_1 < \overline{k}_2 < ... < \overline{k}_m$ $(m \le n)$ of the polynomial $\overline{P}(k) - \overline{A}(k) = 0$ of the interval $[k_{min}, k_{max}]$: $\overline{k}_j \in [k_{min}, k_{max}]$ $(j = 1,...,m)$ are determined.

2. In all the intervals $(k_{min}, \overline{k}_1), (\overline{k}_1, \overline{k}_2), ..., (\overline{k_m}, k_{max})$ the differential signs of $\overline{P}(k) - \overline{A}(k)$ are determined, i.e. the differential signs at least at one point of every interval are found.

3. The intervals with a positive $\overline{P}(k)-\overline{A}(k)$ difference are chosen from the intervals obtained. This is zone $\overline{K}$.

4. The intersections of the intervals in zone K_+ are singled out from zone $\overline{K}$ intervals, i.e. $K_= = \overline{K} \setminus K_+$ is found (Fig. 2).

If zone $\overline{K}$ matches all the intervals $k_{\min} \le k \le k_{\max}$, then the search for investment zones is over.

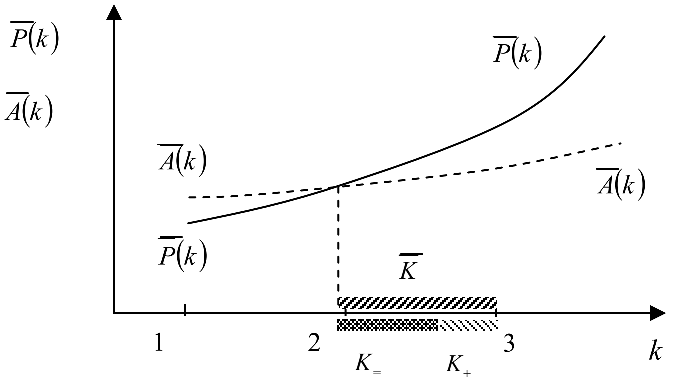


Figure 2. *Minimum risk investment zone*

<u>High-risk investment zone K_0 and zone of losses K_-</u>. These two zones may be defined simultaneously, or K_0 may be determined after K_-.

Zone of investment losses K_- (if any) is a zone where the maximum selling price of premises $P_{\max}(k)$ is lower than the minimum decoration cost (primary purchase price included) $A_{\min}$ (Fig. 3).

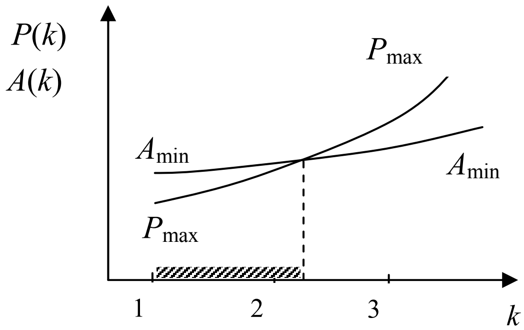


Figure 3. *Determination of zone of losses*

An algorithmn for determining zone K_- is similar to that for defining K_+: it is based on the intervals where the difference $A_{\min}(k)-P_{\max}(k)$ is positive.

A zone of high risk K_0 is such an area, where the mean selling price of the spaces $\overline{P}(k)$ is lower than the average price of the decoraited spaces $\overline{A}(k)$ with respect to the initial price (except for zone K_-). K_0 may be found as the difference between the interval $K = \{k_{\min} \le k \le k_{\max}\}$ and the earlier obtained sets $K_0 = K \setminus (K_+ \cup K_= \cup K_-)$. It is assumed that these are no other investment zones, i.e. $K = K_+ \cup K_= \cup K_- \cup K_0$. A set K_0, similar to $K_=$, may be an empty set Ø.

The above zone K_0 may be defined, and the intervals may be determined in a similar way as for zone $K_=$, where $\overline{P}(k)-\overline{A}(k)$ difference is negative. A set K_- (or, perhaps, and earlier defined sets K_+ and $K_=$) may be singled out from the obtained intervals.

3. THE ANALYSIS OF INVESTMENT IN LITHUANIA

Recently, the real estate market in Lithuania has become more brisk. The need for modern newly – built apartments is great. However, the quality of the premises (spaces) does not always meet the requirements. The rent and market price of the premises largely depend on the quality of the particular building. Buildings are divided into three groups (class) according to construction quality: A, B and C [10].

The requirements for apartment house of the class A include air conditioners, modern lifts, a large parking area, good communication lines, 24 hour guard, accessibility for transport. B class apartment house are of sufficiently high quality, however, they may lack lifts, air conditioners or sufficient parking space. C class apartment houses are of satisfactory quality, being located in old buildings. They often do not meet any of the requirements for A class buildings.

In fact, the selling price of real estate on the market had been rising for the whole period of its existence (from 1990 to 1999). It can be accounted for high inflation rate, interest rate higher than that in the bank, profitability of investments, real estate shortage, etc. Since the middle of 1999 the prices have sharply fallen. From the first quarter of 2000 they are remaining stable up to the present time.

Analysis of apartment house decoration, initial purchase and selling prices in the city of Vilnius is given. On the basis of the calculations made, the table and cost graphs were drawn showing the dependence of the estimated cost of 1 m^2 of the total apartment space on the grade of a apartment house (Fig. 4). The graph is rather subjective, because cost estimation of building finishing is a complicated problem, associated with individual, often unique features of the particular buildings. Therefore, the costs are estimated in terms of the intervals separating the maximum and minimum costs. Further, cost level in graphs is represented as bars (not lines).

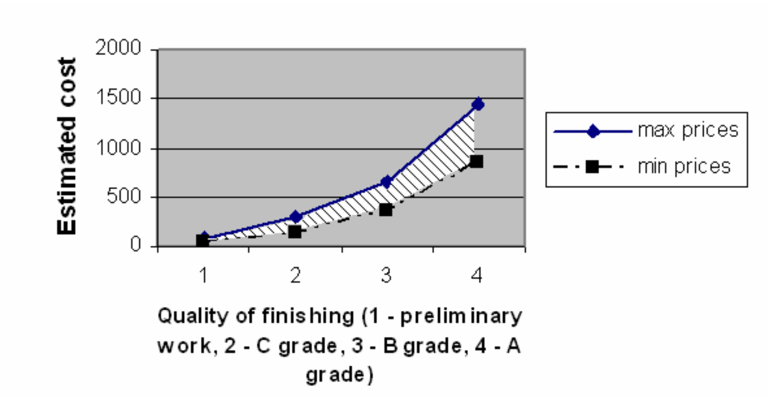


Figure 4. *Cost of apartment house decoration*

Cost analysis of apartment houses is given. The cost of 1 m^2 apartment house space in a decorated building depends on its grade (A, B, C) as well as location. The city area may be divided into zones of buildings practically of similar grades. However, there will be some differences between the buildings of the same grade, depending on the location of a particular building. This is why costs of apartment house space are represented in terms of intervals (separating the maximum and minimum prices).

Initial purchase price of non – decorat apartment house. The price of such apartments depends primarily on their location. A division of the city areas is similar to that used for selling price analysis.

4. A MATHEMATICAL MODEL FOR COST ANALYSIS OF PURCHASING, DECORATION AND SALE OF APARTMENT HOUSE SPACES IN VILNIUS

Let us demonstrate the application of the suggested method to the determining of various types of investment zones by considering the available data on various Vilnius districts. The apartment premises analysed belong to three various categories, therefore the interpolation 2-nd degree polynomials have been used. The problem with more than three categories or with other numbers may be solved in a similar way. In addition to the main variable k (grade of premises), a model may include some other variables r (e.g. a district for investment).

Example

For the values P_1, P_2, P_3 and A_1, A_2, A_3, the expression $P_{\min}(k) - A_{\max}(k)$ – was calculated and a risk-free investment zone K_+ was found.

For the values $\overline{P_1}$, $\overline{P_2}$, $\overline{P_3}$, and $\overline{A_1}$, $\overline{A_2}$, $\overline{A_3}$, the expression $\overline{P}(k) - \overline{A}(k)$ was calculated, and a zone of normal risk investment $K_=$ was found. It is evident that other investment zones do not exist.

By collecting the statistical data and grouping the districts with similar conditions of investment, it is possible to introduce a generalized district factor r into a model of identifying various investment zones, in addition to the factor K determining grade of spaces.

Based on a correlation analysis of the data, major relationships were derived, which make a model for determining the cost of decoration purchase and sale of Vilnius apartment house (Table 1).

Table 1. A set of relationship to determine the effectiveness of decoration

F-test	Total square error	Model application	Model (relationship) (In the given models k=2, 3, 4 values are used; r – district No; $1 \le r \le 14$)
12,45	92,60	Maximum decoration cost	$A_{max} = 130 + 275k$
58,,80	99,80	Minimum decoration cost	$A_{min} = 207 + 140k$
20,67	95,40	Average decoration cost	$\overline{A}$ =163+210k
53,36	94,70	Maximum selling price of premises	P_{max} = - 103 +3900r + 275k
49,85	94,30	Minimum selling price of premises	P_{min} = 1807 + 1050r + 140k
22,09	88,00	Average selling price of premises	$\overline{P}$ =1963+1400r+210k

Models for determining risk zones of investment in buildings. A set of relationships obtained can be used for developing mathematical models for determining various types of investment risk zones.

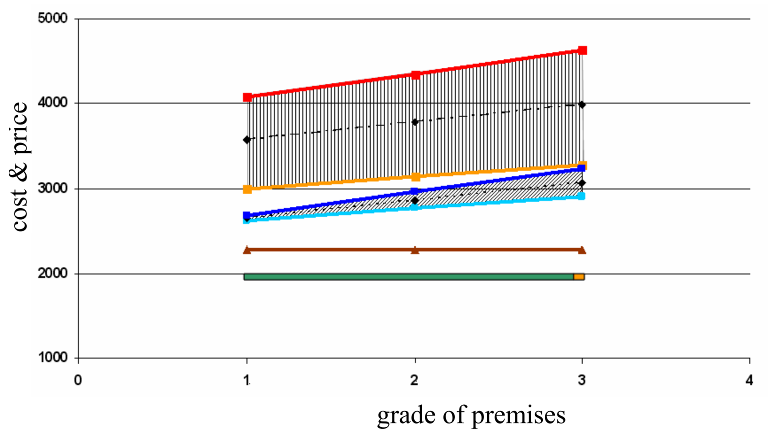
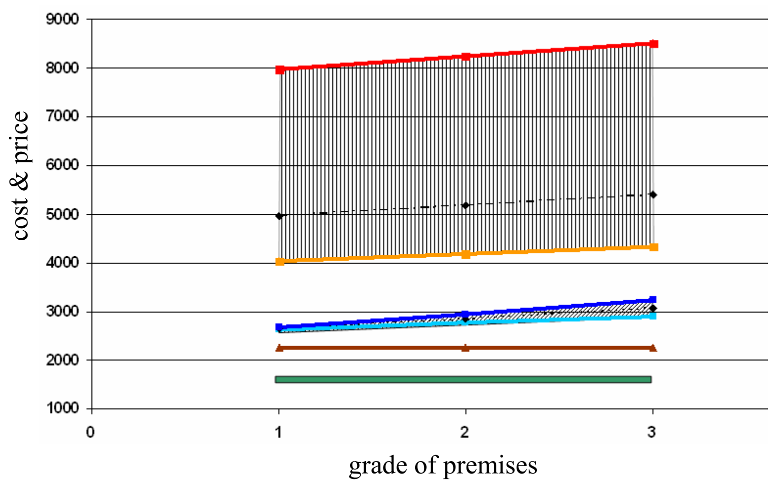
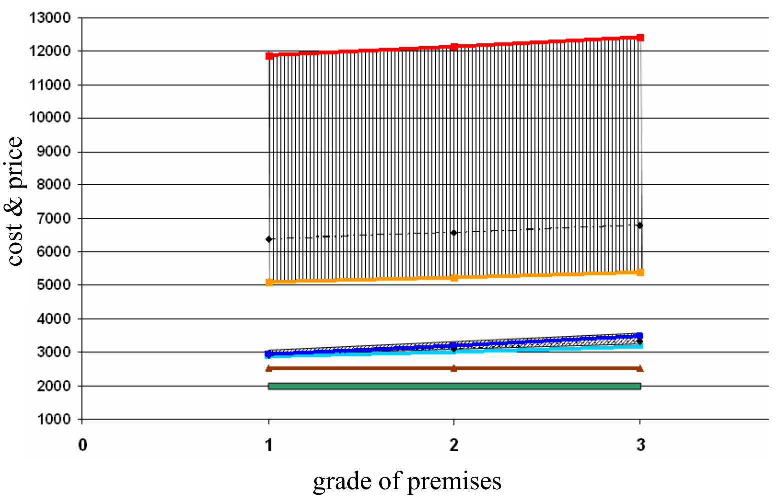
By determining a risk-free investment zone based on the main factor expressed as $P_{\min}(k) - A_{\max}(k) > 0$. In a similar way, models for determining normal risk, high risk and unprofitable (losses) zones were derived (Table 2).

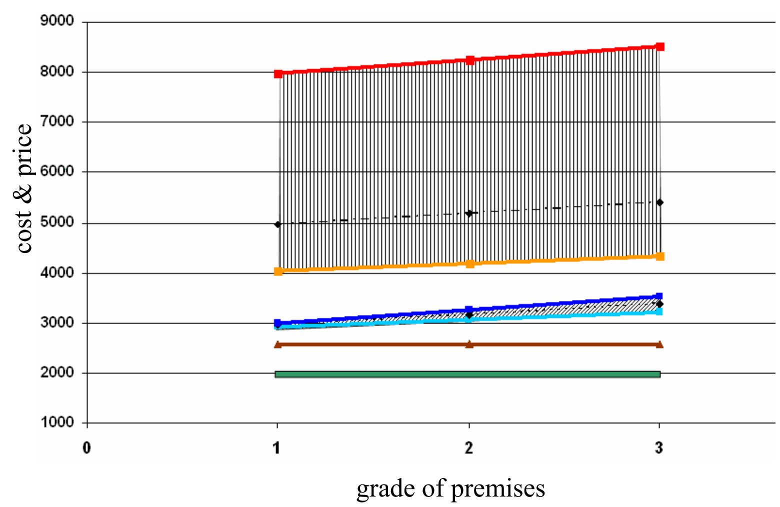
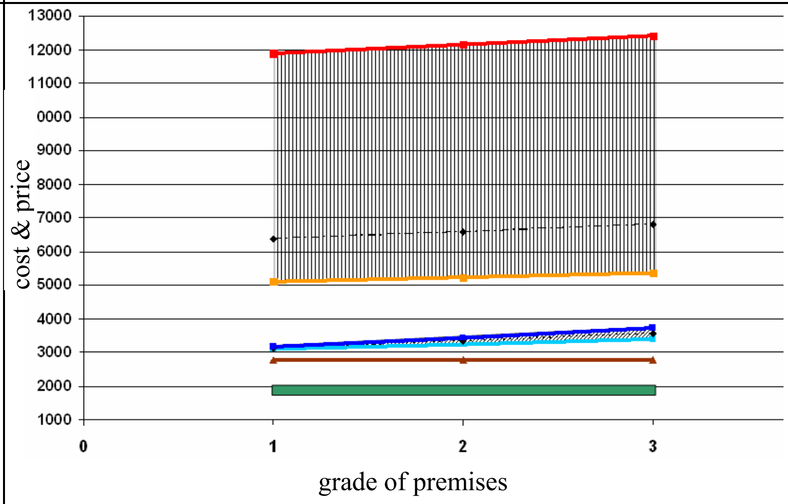
Table 2. Models for determining risk zones

Nr.	Model application	Model expression
1.	Risk – free investment zones	P_{min} - A_{max} = 1807 + 1050r + 140k - 130 + 275k;
2.	Normal risk zones	$\overline{P}(k) - \overline{A}(k) = 1963 + 1400r - 163 + 210k$
3.	High risk and losses zones	A_{min} - P_{max} =207 + 140k –(- 103 +3900r + 275k)

By applying the defined mathematical models, the graphs of distribution of various investment risk zones in Vilnius districts were plotted (graphs for individual districts are shown in Table 3). Optimisation of the received mathematical models is possible by applying multiattribute decision-making approach [31]

Table 3. Graphs of distribution of investment risks in different districts

R	District	The graphs
1	Modern residential districts (Seskine, Baltupiai, Lazdynai, Karoliniskes)	
2	Antakalnis	
3	Other short streets in Senamiestis district	

4	Zverynas	
5	Gediminas Av., Didzioji, Pilies, Vokieciu str.	
	selling price of space	risk-free investments
	cost of decoration	normal risk investments
	average cost of decoration	high risk investments
	average cost of sold premises	unprofitable investments (losses)

The analysis of the graphs obtained allows us to conclude that, taking into account the investment risks, the effect of investments may be determined. Several groups of districts may be identified with respect to the type of investment:

- for districts r = 8;10;13;14 (which are the most fashionable and prestigious districts of Vilnius) practically any kind of decoration of the purchased spaces of any rate is effective, because all investments are in a risk-free zone. The first-rate decoration work is preferable, however, medium-rate work will also pay;
- for districts r = 2;3;4;5;9, a risk-free zone embraces spaces of grade A. However, the investment into a medium-rate decoration will also pay off;
- in districts r = 7;11;12 risk-free and profitable investments are associated with average grade spaces. The investments into higher grade spaces are also effective, however, in this case, the initial investments are considerably larger;
- for the investments into buildings in districts r = 1,6 to be effective, a more detailed analysis of the particular cases is needed. Zones of normal risk are characteristic of these districts.

5. CONCLUSION

The suggested technique allows for computer-aided determination of risk zones in making investments in profitable real estate. The available database of various types of premises and typical districts of the city provides a possibility of determining the most probable risk zones for investment in particular projects quickly and reliably.

The model used in the present work allows for including other criteria determining the investment risk. The calculations made for various districts of Vilnius proved the effectiveness of the described method.

Solving actual problems of selecting the best variants of construction and reconstruction investment projects validated the developed methods. The investment projects selected have been implemented.

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