MODELLING FREIGHT FLOWS AT TRANSPORT TERMINAL AND VEHICLE FLEET OF OPTIMAL CARRYING CAPACITY

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Transportation of goods from the terminal to customers should be organized in such a way as to satisfy the demand of customers and to use vehicles efficiently. Freight flows distribution assignment model developed in the present investigation allows us to determine the capacity of a vehicle (i.e. its overall dimensions, carrying capacity, etc.), delivery time limits, time resources and overall costs. The methods of determining optimal lots of the transported goods as well as choosing the way and means of transportation are offered.

Keywords: *freight terminal, vehicle, lot of goods*

1. INTRODUCTION

The assignment is to take out goods from the transport terminal and to deliver them to customers in small lots, which cannot be made any smaller but can be transported together with other lots. Goods should be delivered in such a way as to satisfy the requirements of the customers and to ensure the efficient use of transport facilities. The problem of achieving more efficient freight transportation over long distances and carriage of goods about 20–40 tons by road transport have been analysed in various papers [1–4]. However, the data on the analysis of cases when the cargo of about 50–1500 kg is transported on the road network of a small area (i.e. a town), with bottlenecks on the roads and ecological limitations imposed, are scarce. Therefore, an attempt was made to investigate the outlined problem and to suggest some solutions to it.

2. MODEL OF FREIGHT FLOWS FORMATION

Let M be a fixed number of vehicles used, N – fixed number of freight delivery points; I_M – $\{1, 2, ..., M\}$ set of indices of all transport facilities, with the particular vehicles further indexed by I ; I_N – $\{1, 2, ..., N\}$ set of indices of freight delivery points, with the particular points further indexed by j . Any transport facility i $(i \in I_M)$ is given: $G_{(i)}$ – cargo-carrying capacity; $T_{(i)}$ – time resource (planned period of transportation); $D_{(i,k)}$ – size k of the cargo section of vehicle $(k \in K_L = \{1, ..., L\}$; here, L – number of sections of various cargo-carrying capacities.

It should also be noted that a lot of goods to be delivered to point j is characterized by the following parameters: $(j \in I_N)$; $g(j)$ – cargo weight; $d(j,k)$ – cargo size k; $(k \in K_L)$; $T_{(j)}$ – specified delivery time. It is assumed that each lot of goods should be delivered to the particular point, while several lots to be delivered to the same point are combined together, therefore, each point j may be associated with lot of goods j .

It may be stated that the average speed of a vehicle carrying goods between the points j_1 and j_2 depends on the profile of the road as well as on vehicle index i and weight of cargo G . Then, the average speed of transportation will depend on parameters j_1, j_2, i and G and will be denoted further by $\bar{u}$ (j_1, j_2, i, G).

A matrix of distances A between the points of each pair (j_1, j_2) is known. Relying on the distance $a(j_1, j_2)$ between the points of the pair (j_1, j_2) and the average speed of vehicle between the above points $\overline{U}(j_1, j_2, i, G)$, it is possible to determine time of freight carriage from point j_1 to point j_2 for any vehicle i:

$$t(j_1, j_2, i, G) = a(j_1, j_2)/\overline{U}(j_1, j_2, i, G). \tag{1}$$

Let us formalize a series of transportation routes aimed to embrace all points or a set of cargoes to be delivered. Let us also assume that transport facilities are at the terminal that will be assigned an index $(N+1)$. Then, let us denote by $a(j, N+1)$ and $a(j, N+1, j)$ the distances from point j to point $(N+1)$ and from $(N+1)$ to j, respectively.

The vectors x, y of integer numbers and the rearrangement $\pi = (\pi_1, \pi_2, ..., \pi_N)$ of the elements of the set I_N (i.e. the rearrangement of indices of the delivery points) characterize a series of routes. Let us assume that the vectors x belong to the sets $X(m)$, here, $m - M$ is a size vector, with the components m_i and

$$X(m) = \{x \in E^M | 1 \le x_i \le m_i,\ i \in I_m\}. \tag{2}$$

here and further, E^q denotes a set of size vectors of all numbers q.

The vectors y belong to the sets $Y(|x|)$, here,

$$|x| = \sum_{i \in I_M} x_i \quad \text{and} \quad Y(|x|) = \{y \in E^{|x|} = y_1 < y_2 < ... < y_{|x|} < N\}. \tag{3}$$

Physical meaning of the vectors introduced is as follows: the component x_i of the vector $x(i = 1, 2, ..., M)$ denotes a number of hauls for the vehicle i (while m_i is a prescribed estimate of the above number); the component y_l of $y(l = 1, 2, ..., |x|x|)$ denotes the first position of the route l. More exactly, the total of vectors $x \in X(m)$, $y \in Y(|x|)$ and rearrangements π determine the routes $|x|x|$, while each l from π_l is expressed in the following way:

$$\Pi_l = (N+1, \pi_{y_l}, \pi_{y_{l+1}}, ..., \pi_{y_{l+1}-1}, N+1). \tag{4}$$

All the routes (u) are allotted to M groups by the components of the vector x.

Each group of routes i refers to vehicle i, while the numbers of routes of this group belong to the interval $(\bar{x}_i, \bar{x}_{i+1})$; here, the values $\bar{x}$ may be obtained via the components of the vectors x in the following way:

$$\bar{x}_1 = 1,\ \bar{x}_{i+1} = \bar{x}_i + x,\ i = 1, 2, ..., M. \tag{5}$$

Let us denote each route l by $(l = 1, 2, ..., |x|)$:

$$G(\Pi_l) = \sum_{r=y_l}^{y_{l+1}-1} g(\pi_r),\ G(\Pi_l, q) = G(\Pi_l) - \sum_{r=y_l}^{y_{l+q}} g(\pi_r),\ q = 1, 2, ..., (y_{l+1} - y_l) - 1. \tag{6}$$

Let us set the constraints to a system of routes and their allotment to the particular transport facilities. The limitations are also distributed among the groups similarly to route distribution among the vehicles. Let the route l belong to group i, i.e. $l \in [\bar{x}_i, \bar{x}_{i+1})$, here $i \in I_M$.

Firstly, the limitation on the total cargo weight is imposed on this route, implying that the above value cannot exceed carrying capacity of vehicle i:

$$G(\Pi_l) \leq G(i). \tag{7}$$

Secondly, to the route l a restriction is applied for overall freight dimensions, which cannot exceed the dimensions of freight section of the vehicle i:

$$D_k(\Pi_l) \leq D(i, k),\ k \in K_L. \tag{8}$$

The notation $D_k(\Pi_l)$ is similar to that introduced to denote the first relationship in the expression (6).

Thirdly, time limits are imposed on the route l:

$$\begin{aligned} & t\left[N+1, \pi_{r_l}, i, G(\Pi_l)\right] \leq T\left(\pi_{r_l}\right), \\ & t\left[N+1, \pi_{r_l}, i, G(\pi_l)\right] + \sum_{q=r_l}^{\bar{q}} t\left[\pi_q, \pi_{q+1}, i, G_q(\Pi_l)\right] \leq T\left(\pi_{\bar{q}}\right), \\ & \bar{q} = r_l + 1, r_l + 2, \ldots, r_{l+1} - 1. \end{aligned} \tag{9}$$

Time limits are imposed on the total of the routes:

$$\sum_{l=\bar{x}_i}^{x_{i+1}-1} \left\{ t\left[N+1, \pi_{r_l}, i, G(\Pi_l)\right] + t\left[\pi_{r_{l+1}-1}, N+1, i, 0\right] + \sum_{l=r_l}^{\bar{x}_{i+1}-1} t\left[\pi_q, \pi_{q+1}, i, G_q(\Pi_l)\right] \right\} \leq T(i); \tag{10}$$

in expressions (9) and (10) the notation found in (1) and (6) is used.

Thus, overall costs $Z(x, y, \pi)$ depending on the system of routes and their distribution among the particular facilities defined by the vectors x, y and rearrangement π may be obtained as follows:

$$\begin{aligned} Z(x, y, \pi) = \sum_{i=1}^{M} \sum_{l=\bar{x}_i}^{\bar{x}_{i+1}-1} \Big\{ & Z\left[N+1, \pi_{r_l}, i, G(\Pi_l)\right] + Z\left[\pi_{r_{l+1}-1}, N+1, i, 0\right] + \\ & + \sum_{l=r_l}^{\bar{x}_{i+1}-1} Z\left[\pi_q, \pi_{q+1}, i, G_q(\Pi_l)\right] \Big\}. \end{aligned} \tag{11}$$

In the equation (11), the costs of carrying the cargo of the weight G from the point j_1 to the point j_2 by vehicle i, are denoted by $Z(j_1, j_2, i, G)$. Generally, the costs may be expressed in terms of the distance $a(j_1, j_2)$ between the points j_1 and j_2 or by multiplying the above distance by all carried goods. In other cases, the relationship between the costs considered and the parameters j_1, j_2, i, and G may be more complicated, for example, if the costs are determined in terms of the fuel used.

In transportation, various types of costs should be taken into account. Therefore, the model considered is aimed to embrace various costs, denoting them by the index S and determining them as shown in the equation (11). Thus, let us determine the costs $(S+1)$ of the type:

$Z^{(s)}(x, y, \pi)$, here $s = 0, 1, 2, \ldots, S$

Then, let $Z^{(0)}(x, y, \pi)$ be overall costs to be minimized, while other kinds of expenses may be at the highest admissible level $Z(s)$. Then one more group of limitations referring to the overall costs will be added to the previously formulated constraints:

$$Z^{(s)}(x, y, \pi) \le Z(s),\ s = 1, 2, ..., S, \tag{12}$$

here, $Z^{(s)}(x, y, \pi)$ is found based on the values $Z^{(s)}(j_1, j_2, i, G)$ according to the formula (11), while the values $Z(j_1, j_2, i, G)$ are used to find overall costs $Z(x, y, \pi)$. Now, the problem associated with the flows of lots of goods between the terminal and customers may be formulated to find:

$$\min_{x \in X(m)} \min_{y \in Y(|x|)} \min_{\pi \in \Pi_N} \{Z^{(0)}(x, y, \pi)\} \tag{13}$$

with the limitations (7)–(10), (12).

3. DETERMINING THE STRUCTURE OF THE FLEET OF VEHICLES OF OPTIMAL CARGO-CARRYING CAPACITY

The structure of the fleet of vehicles based on their cargo-carrying capacity should meet the requirements to transporting goods in lots of various sizes.

Let cargo-carrying capacity of a vehicle be represented by a series $q_1,\ q_2, \dots,\ q_j, \dots,\ q_m$. In addition, size distribution of the lots of goods is known. The probability of a lot of goods, which would require the vehicle of $q_j\ (j = 1,\ 2,\ \dots,\ m-1)$ carrying capacity for transportation, is as follows:

$$p_j = \begin{cases} \int\limits_0^{(q_\gamma)_j} f(x)\,dx, & j = 1; \\ \int\limits_{(q_\gamma)_{j-1}}^{(q_\gamma)j} f(x)\,dx, & j > 1, \end{cases} \tag{14}$$

here, $f(x)$ – distribution density of lot sizes.

The probability of occurrence of a lot of goods requiring q_m capacity vehicle, which would transport a lot of goods by, i hauls $(i + 1,\ 2,\ \dots)$ is:

$$p_{m,i} = \begin{cases} \int\limits_{(q_\gamma)_{m-1}}^{(q_\gamma)_m} f(x)\,dx, & j = 1; \\ \int\limits_{(i-1)q_{\gamma_m}}^{i(q_\gamma)_m} f(x)\,dx, & i > 1. \end{cases} \tag{15}$$

A number of vehicles j of the type $(j = 1, 2, \dots, m-1)$ needed is as follows:

$$A_{ej} = \frac{\overline{N}_{v.r.} p_j}{T_{nj}} \left(\frac{l_{g.ej}}{\upsilon_{tj} \beta_j} + t_{npj} \right) \tag{16}$$

here, $\overline{N}_{v.r.}$ – average number of requests for goods transportation per 24 hours.

A required number of q_m capacity vehicles:

$$A_m = \frac{\overline{N}_{v.r.} \sum_{i=1}^{\infty} ip_{m,i}}{T_{nm}} \left(\frac{l_{g.em}}{\upsilon_{tm}\beta_m} + t_{npm} \right). \tag{17}$$

Total number of vehicles:

$$A_e = \sum_{j=1}^{m} A_{ej} = \overline{N}_{v.r} \left[\sum_{j=1}^{m-1} \frac{p_j}{T_{nj}} \left(\frac{l_{g.ej}}{\upsilon_{tj}\beta_j} + t_{t_{npj}} \right) + \frac{\sum_{i=1}^{\infty} ip_{m,i}}{T_{nm}} \left(-\frac{l_{g.em}}{\upsilon_{tm}\beta_m} + t_{npm} \right) \right]. \tag{18}$$

By dividing the left and the right sides of the equations (16) and (18), we get that:

$$\frac{A_{ej}}{A_e} = \frac{p_j}{T_{nj}B} \left(\frac{l_{g.ej}}{\upsilon_{tj}\beta_j} + t_{hpj} \right), \quad j = 1, 2, \ldots, m-1. \tag{19}$$

Similarly, from the equations (17) and (18) we obtain that:

$$\frac{A_{ej}}{A_e} = \frac{\sum_{i=1}^{\infty} ip_{m,i}}{T_{nj}B} \left(\frac{l_{g.ej}}{\upsilon_{tj}\beta_j} + t_{hpj} \right). \tag{20}$$

and from (19) and (20) we get:

$$B = \frac{A_e}{\overline{N}_{v.r}} = \sum_{j=1}^{m-1} \frac{p_j}{T_{nj}} \left(\frac{l_{g.ej}}{\upsilon_{tj}\beta_j} + t_{npj} \right) + \frac{\sum_{i=1}^{\infty} ip_{m,i}}{T_{nm}} \left(\frac{l_{g.em}}{\upsilon_{tm}\beta_{mj}} + t_{npm} \right). \tag{21}$$

If $T_{nj} = T_{nm} = T_n$, then we should calculate:

$$T_n B = \sum_{j=1}^{m-1} p_j \left(\frac{l_{g.ej}}{\upsilon_{tj}\beta_j} + t_{npj} \right) + \left(\frac{l_{g.em}}{\upsilon_{tm}\beta_m} + t_{npm} \right) \sum_{i=1}^{\infty} ip_{m,i}. \tag{22}$$

Therefore, to determine the probability of requests for transporting goods by various capacity vehicles means to find the type of size distribution of lots and the average output of the above vehicles per 24 hours.

Exponential distribution of lot sizes can be expressed in the following way:

$$f(x) = \frac{1}{\overline{g}} e^{-\frac{x}{\overline{g}}},$$

here, $\overline{g}$ – average lot size of goods, t.

$$p_1 = \frac{1}{\overline{g}} \int_0^{(q_\gamma)_1} e^{-\frac{x}{\overline{g}}} dx = 1 - e^{-\frac{(q_\gamma)_1}{\overline{g}}}, \tag{23}$$

$$p_j = \frac{1}{\bar{g}} \int_{(q_\gamma)_{j-1}}^{(q_\gamma)_j} e^{-\frac{x}{\bar{g}}} dx = e^{-\frac{(q_\gamma)_{j-1}}{\bar{g}}} - e^{-\frac{(q_\gamma)_j}{\bar{g}}} , \tag{24}$$

$$p_{m,i} = \frac{1}{\bar{g}} \int_{(i-1)(q_\gamma)_m}^{i(q_\gamma)_m} e^{-\frac{x}{\bar{g}}} dx = e^{-\frac{(i-1)(q_\gamma)_m}{\bar{g}}} - e^{-\frac{i(q_\gamma)_m}{\bar{g}}} . \tag{25}$$

If the lot sizes distributed according to the normal law, the probability of a random value q to be in the interval $\left[(q_\gamma)_{j-1}, (q_\gamma)_j\right]$ may be found in the following way:

$$p_j = P\left\{(q_\gamma)_{j-1} < g < (q_\gamma)_j\right\} = \Phi * \left[\frac{(q_\gamma)_j - \bar{g}}{\sigma_g}\right] - \left[\frac{(q_\gamma)_{j-1} - \bar{g}}{\sigma_g}\right], \tag{26}$$

here, σ_g – mean square deviation of the random value.

$$p_{m,i} = \begin{cases} \Phi * \left[\dfrac{(q_\gamma)_m - \bar{g}}{\sigma_g}\right] - \Phi * \left[\dfrac{(q_\gamma)_{m-1} - \bar{g}}{\sigma_g}\right], & i = 1; \\ \Phi * \left[\dfrac{i(q_\gamma)_m - \bar{g}}{\sigma_g}\right] - \Phi * \left[\dfrac{(i-1)(q_\gamma)_m - \bar{g}}{\sigma_g}\right], & i > 1. \end{cases} \tag{27}$$

In some cases, transporters and shippers relate the lot size of goods to cargo-carrying capacity of a vehicle. Then an average lot size of goods to be transported will be:

$$\bar{g} = \sum_{j=1}^{m-1} p_j (q_\gamma)_j + (q_\gamma)_m \sum_{i=1}^{\infty} i p_{m,i} , \tag{28}$$

here, $(q_\gamma)_j, (q_\gamma)_m$ – the largest vehicle capacities based on vehicle body capacity and the kind of transported goods.

An average lot size of goods carried in a haul:

$$\bar{g}_e = \sum_{j=1}^{m-1} p_j (q_\gamma)_j + (q_\gamma)_m \sum_{i=1}^{\infty} p_{m,i} . \tag{29}$$

An average vehicle cargo-carrying capacity calculated per haul:

$$\bar{q}_e = \sum_{j=1}^{m-1} p_j q_j + q_m \sum_{i=1}^{\infty} p_{m,i} . \tag{30}$$

An average value of the static coefficient of the utilized vehicle fleet capacity:

$$\gamma_{st} = \frac{\bar{g}_e}{\bar{q}_e} = \frac{\sum_{j=1}^{m-1} p_j (q_\gamma)_j + (q_\gamma)_m \sum_{i=1}^{\infty} p_{m,i}}{\sum_{j=1}^{m-1} p_j q_j + q_m \sum_{i=1}^{\infty} p_{m,i}} . \tag{31}$$

A number of hauls made by the vehicles of the fleet in a considered period:

$$n_e = \frac{P}{\bar{q}_e \bar{\gamma}_{st}}, \tag{32}$$

here, P – total volume of transported goods, tons.
A number of hauls made by j – type vehicles:

$$n_e = \frac{P}{\bar{q}_e \bar{\gamma}_{st}}, \tag{32}$$

and by the largest capacity vehicles:

$$n_{em} = n_e \sum_{i=1}^{\infty} p_{m,i} = n_e - \sum_{j=1}^{m-1} n_{ej}. \tag{34}$$

Total volume of goods carried by q_j capacity vehicles:

$$P_j = n_{ej} (q_\gamma)_j, \quad j = 1, 2, \ldots, m. \tag{35}$$

The required number of q_j capacity vehicles:

$$\bar{A}_j = \frac{P_j}{D\alpha_j P_{\text{par}.j}}, \quad j = 1, 2, \ldots, m, \tag{36}$$

here, $P_{\text{par}.j}$ – vehicle output per 24 hours.

$$P_{\text{par}.j} = \frac{\upsilon_{tj} \beta_j q_j \gamma_{stj} T_{nj}}{l_{g.ej} + \upsilon_{tj} \beta_j t_{npj}}. \tag{37}$$

The investigation has shown that the size of the lots of goods carried from the manufacturers' terminals to a distribution network distributed according to the exponential law (Fig. 1). The distribution density is as follows:

$$f(x) = 0{,}0675 e^{-0{,}0675x}.$$

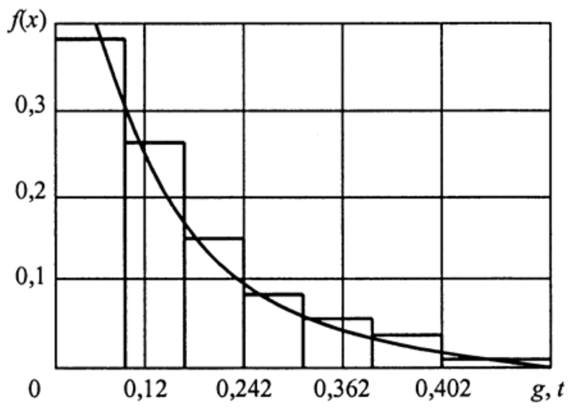

Figure 1. *Size distribution of lots of goods*

4. CONCLUSIONS

1. The assignment model offered for distributing freight flows at the transport terminal allows us to determine the dimensions and cargo-carrying capacity of a transport facility, delivery time and time resource limitations of a vehicle as well as overall costs.

2. In order to determine the optimal structure of the fleet of vehicles and the particular organizational form of transportation, the total volume and lots of the transported cargo should be analysed in terms of time. Since the demand for transportation and lot sizes are random values, mathematical statistical approaches are preferable for their analysis in time.

3. The methods for determining the optimal size of lots of goods and way of transportation based on general costs of their storage and carriage are suggested.

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