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STEPPED OPTIMIZATION ALGORITHM FOR LIGHT-WEIGHT ELASTIC-PLASTIC STRUCTURES CONSIDERING LOAD COMBINATIONS

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Abstract. The aim of the article is to present a developed algorithm of a stepped structural optimization method in the case of load combinations. The presentation of external loading via load combinations usually is employed in design codes. The pin-jointed steel structures as light-weight ones are widely used in actual engineering practice. The employment of dissipative features of material via elastic-plastic model ensures a significant reduction of structural carrying capacity reserves (essential economic effect) versus the employment of structural response only in the elastic range. The stability, stiffness, constructional requirements and actual functions for cross-sectional properties of standard profiles are employed in the optimization process aiming to obtain an optimal light-weight structure which deformed behavior is compatible with codified requirements for such class of engineering structures. FEM mathematical models realizing structural optimization method steps are developed.

Keywords: elastic-plastic pin-jointed structure, load combinations, stepped optimization, stability, stiffness and constructional constraints

1. Introduction

A demand of large span/volume storages hangars, sport halls, stadiums, as well as active spread of telecommunication services in Lithuania, East Europe during last decades stimulated the designing of large dimensioned or tall light-weight quickly erectable buildings. Steel structures actually match all these requirements. Ductility as an attractive quality of steel structure ensures the efficient employment of supplemental carrying capacity resources vs the brittle ones or those designed to response in an elastic way. Dissipation of steel structures, conditioned by yielding of structural members causes the redistribution of internal forces i.e. adapting of the structure to loading.

One can list many investigations of the analysis of structures, taking into account dissipative/elastic-plastic properties [1–6]. Among many investigations one can mention the ones, employing the mathematical programming theory combining with extreme energy principles and theory of duality [7–11]. Rigid-plastic structural optimization [7, 8, 11] yields the solution – optimal distribution of structural parameters according to plastic collapse conditions, resulting in zero carrying capacity reserve. But one must note that a structure, e.g. a steel one designed on the basis of such solution (introducing small safety factor) responses to loading by large displacements not admitted in actual engineering practice. The introduction of displacement constraints complicates the problem essentially realizing it numerically due to complementarity conditions. They are included in a mathematical model of the problem. The additional evaluation of stability requirements (often being a governing factor estimating the admissible behavior of a light-weight steel structure) causes supplemental numerical difficulties. As an alternative to a direct optimization method the stepped optimization method was proposed [12, 13].

One must note that generally the deformable response of an elastic-plastic structure up to the state of prior to plastic collapse depends on the actual loading history. That means that individual loading trajectory causes individual story of developing internal forces and displacements. If loading trajectories of a structure are not fixed (as in usual practice) only variation bounds of above residual values can be identified [14]. One can mention the investigations on optimization of structures considering residual displacement restrictions [15]. Numerical solution of problems is very complicated even in such formulation. Introduction constraints in respect of total displacement magnitudes should complicate it more.

The current investigation is assigned to developing of the stepped optimization method for elastic-plastic structures taking into account total displacement and stability constraints, both in concert with constructional ones. The non-uniqueness effect of the structural residual response values is evaluated in the following way: the structure stiffness conditions are formulated in respect of total displacement constraints for each individual load combination; the strength/stability conditions – in respect of extreme axial forces developed by all load combinations. Anyway, the obtained solution satisfies all introduced requirements vs all considered load combinations and can be considered as a tool for actual design compatible with principles employed in design codes.

The mathematical models employed in certain stages of stepped optimization are developed applying FEM techniques. Actual functional relations of cross-sectional properties are identified to employ them in a solution process. A numerical example illustrating the application of the developed method is presented by stepped optimization of a space pin-jointed tower subjected to load combinations.

2. Stepped optimization algorithm in case of load combinations

A stepped optimization method is an efficient alternative [12, 13] of considered optimization problem solution vs the one solved in a direct way. It aims to avoid numerical difficulties (conditioned by necessary involvement of complementarity conditions) realizing direct optimization problem. The method ensures significant savings of computational efforts and resources. The algorithm of stepped optimization is described by performing certain optimization cycles. The cycle realizes the subsequent solutions of certain sub-problems – stages. The cyclic solution process is continued to the problem convergence. The main stages of a cycle of structural optimization are:

1. Determination of the values of elastic structural response, namely: displacements and axial forces due to all load combinations, creation of the envelope of axial forces.
2. Determination of the actual axial forces and displacements due to each load combination and those due to the envelope of axial forces. Certain analysis problems of elastic-plastic structure are solved for this purpose.
3. Determination of the optimal distribution of structural member areas, satisfying strength, stiffness and constructional requirements in respect of considered load combinations. The results of stage 2 are employed.

The stepped (cyclic) nature of problem solution is prescribed by a circumstance that elastic response values are conditioned by certain distribution of member areas, i.e. the optimization result of a cycle.

Let the considered load combination index be denoted via $\eta = 1, 2, \ldots, \omega$. Then the elastic solution of structure due to considered combination $\mathbf{F}^{\eta}$ is expressed by n-dimensional (where n is the number of members – finite elements) vector of axial forces and m-dimensional (where m is the number of global displacements) vector of displacements, reading:

$$\mathbf{N}_e^{\eta} = \left(N_{e,j}^{\eta}\right)^T = \left(N_{e,1}^{\eta}, N_{e,2}^{\eta}, \ldots, N_{e,j}^{\eta}, \ldots, N_{e,n}^{\eta}\right)^T,$$

$$\mathbf{u}_e^{\eta} = \left(u_{e,i}^{\eta}\right)^T = \left(u_{e,1}^{\eta}, u_{e,2}^{\eta}, \ldots, u_{e,i}^{\eta}, \ldots, u_{e,m}^{\eta}\right)^T.$$

They are obtained by:

$$\mathbf{N}_e^{\eta} = [K]\ \left[\bar{A}\right]^T [C]\ [K]^{-1}\mathbf{F}^{\eta}, \tag{1}$$

$$\mathbf{u}_e^{\eta} = [K]^{-1}\mathbf{F}^{\eta}. \tag{2}$$

The above expressions employ the following structure FEM values:

$[K]$ is $(n\times n)$-dimensional quasidiagonal structure stiffness matrix of elemental axial stiffnesses EA_j/l_j $(j = 1, 2, \ldots, n)$ in case of alike material, i.e. alike elasticity modulus E;

$[K] = [C]^T\left[\bar{K}\right]\ [C]$ is $(m\times m)$-dimensional stiffness matrix of a structure where $\left[\bar{K}\right]$ is $(6n\times 6n)$-dimensional quasidiagonal matrix of a structure obtained by assemblage of the elements stiffness matrices $\left[\bar{K}_j\right]$ in global coordinate system ([12]);

$[C]$ is $(6n\times m)$-dimensional configuration matrix of local $\bar{\mathbf{u}}$ and that of global $\mathbf{u}$ displacements;

$\left[\bar{A}\right]$ is $(6n\times n)$-dimensional fictitious matrix of structure elements equilibrium eqns $\left[\bar{A}\right]\mathbf{N}^{\eta} = \bar{\mathbf{F}}^{\eta}$ in global coordinate system, where vector $\bar{\mathbf{F}}^{\eta}$ couples element ends nodal forces of structure. Note that $[C]^T \bar{\mathbf{F}}^{\eta} = \mathbf{F}^{\eta}$.

The actual axial forces and displacements caused by load combination η are:

$$\mathbf{N}^{\eta} = \mathbf{N}_r^{\eta} + \mathbf{N}_e^{\eta} \quad (3)$$

and

$$\mathbf{u}^{\eta} = \mathbf{u}_r^{\eta} + \mathbf{u}_e^{\eta}. \quad (4)$$

The residual components of above values $\mathbf{N}_r^{\eta}$ and $\mathbf{u}_r^{\eta}$ are determined via the solution of the analysis problem for load combination η:

Find

$$\max \begin{pmatrix} 0.5\boldsymbol{\lambda}_1^{\eta^T}[G]\,\boldsymbol{\lambda}_1^{\eta} - 0.5\boldsymbol{\lambda}_1^{\eta^T}[G]\,\boldsymbol{\lambda}_2^{\eta} + 0.5\boldsymbol{\lambda}_2^{\eta^T}[G]\,\boldsymbol{\lambda}_2^{\eta} + \\ \boldsymbol{\lambda}_1^{\eta^T}\left(\mathbf{N}_e^{\eta} - \mathbf{N}_0\right) + \boldsymbol{\lambda}_2^{\eta^T}\left(-\mathbf{N}_e^{\eta} - \mathbf{N}_{cr}\right) \end{pmatrix} \quad (5)$$

subject to

$$\boldsymbol{\lambda}_1^{\eta} \geq \mathbf{0}, \quad \boldsymbol{\lambda}_2^{\eta} \geq \mathbf{0}, \quad (6)$$

where:

$\boldsymbol{\lambda}_1^{\eta} = \left(\lambda_{1,j}^{\eta}\right)^T$ is a vector of Lagrange multipliers of complementary slackness conditions for tensile members, reading $\boldsymbol{\lambda}_1^{\eta^T}\left(-\mathbf{N}_r^{\eta} - \mathbf{N}_e^{\eta} + \mathbf{N}_0\right) = 0$;

$\boldsymbol{\lambda}_2^{\eta} = \left(\lambda_{2,j}^{\eta}\right)^T$ is a vector of Lagrange multipliers of complementary slackness conditions for compressive members, reading $\boldsymbol{\lambda}_2^{\eta^T}\left(\mathbf{N}_r^{\eta} + \mathbf{N}_e^{\eta} + \mathbf{N}_{cr}\right) = 0$;

$\mathbf{N}_0 = \left(N_{0,1}, N_{0,2}, \ldots, N_{0,n_0}\right)^T$, $N_0 = A\sigma_y$, where A is member cross-sectional area and σ_y is material yield limit;

$\mathbf{N}_{cr} = \left(N_{cr,1}, N_{cr,2}, \ldots, N_{cr,n_0}\right)^T$, $N_{cr} = A\sigma_{cr}$, where A is member cross-sectional area and $\sigma_{cr} = \sigma_y \chi$ is member critical stress obtained employing buckling factor $\chi \leq 1$;

$$[H] = \left([C]^T[\bar{A}]\,[K]\,[\bar{A}]^T[C]\right)^{-1}[C]^T[\bar{A}]\,[K]; \quad (7a)$$

$$[G] = [K]\,[\bar{A}]^T[C]\,[H] - [K]. \quad (7b)$$

Then the required residual values are obtained by:

$$\mathbf{N}_r^{\eta} = [G]\left(\boldsymbol{\lambda}_1^{\eta} - \boldsymbol{\lambda}_2^{\eta}\right), \quad (8)$$

$$\mathbf{u}_r^{\eta} = [H]\left(\boldsymbol{\lambda}_1^{\eta} - \boldsymbol{\lambda}_2^{\eta}\right). \quad (9)$$

The structure minimum weight optimization problem (compatible with optimal distribution of its member areas) reads:

find

$$\rho\left\{\sum_{k=1}^{n_0} A_k \sum_{r=1}^{n_k} L_r\right\} \to \min, \quad (10)$$

subject to

$$-\sigma_y A_{k,j} \leq -N_j^{+}, \quad -\sigma_y\left(\chi_j A_{k,j}\right) \leq N_j^{-}, \quad (11)$$

$$u_{t,adm}^{-} \leq u_t^{\eta} \leq u_{t,adm}^{+},$$
$$\eta = 1, 2, \ldots, \omega; \quad t = 1, 2, \ldots, m_t, \quad (12)$$

$$A_k \geq A_k^{\min}, \quad k = 1, 2, \ldots, n_0, \quad (13)$$

here:

ρ is steel density;

A_k is member cross-sectional area corresponding to k-th group of equal area members;

n_k is a number of members corresponding to k-th group;

L_r is the total length of members corresponding to k-th group;

$n_1 + n_2 + \ldots + n_{n_0} = n$; n_0 is the number of groups;

u_t^{η} is actual displacement of considered load combination, being constrained in direction t;

m_t is the number of constrained displacements; $u_{t,adm}^{+} > 0$, $u_{t,adm}^{-} < 0$ are the upper and the lower admissible bounds for displacement to be constrained;

$A_{k,min}$ is the lower bound of cross-sectional area A_k magnitude conditioned by minimal slenderness magnitudes for certain classes of members fixed in design codes.

To obtain displacement expression u_t^{η} via optimized parameters (i.e. areas) for expression (12) a virtual principle is employed. The displacement expression finally reads:

$$u_t^{\eta} = \sum_{j=1}^{n}\left(\mathbf{u}_{t,j}^{\eta}\right)^T\left[\bar{K}_j\right]\bar{\mathbf{u}}_{t,j}^{\eta}\,\frac{A_{\nu-1}}{A_{\nu}}, \quad (14)$$

where:

$\mathbf{u}_{t,j}^{\eta}$ is the vector of actual displacements of j-th finite element ends determined by elastic plastic analysis of the structure due to considered load combination;

$\bar{\mathbf{u}}_{t,j}^{\eta}$ is the vector of virtual displacements of j-th finite element ends obtained applying a unit load along direction to the structure being in the state prior the plastic collapse due to considered load combination;

ν is a cycle counter of stepped optimization.

Strength conditions (11) employ the actual magnitudes of axial forces. They are obtained having solved the analysis problem (5)–(6) in respect of the envelope of axial forces $\mathbf{N}_e^-$ and $\mathbf{N}_e^+$ created from all considered load combinations. Then the solution of the analysis problem yields the field of residual forces $\mathbf{N}_r^{tot}$. The actual forces under above strength conditions then are as follow:

$$N_j^+ = N_{r,j}^{tot} + N_{e,j}^+ ,\quad N_j^- = N_{r,j}^{tot} + N_{e,j}^- .$$

To relate the member values employed in optimization procedures the relationship of radius of gyration i vs cross-sectional area A must be identified. The above values can be approximated with sufficient accuracy (omitting for simplicity the member index):

$$\mathrm{i} = a_1 A^{b1}. \tag{15}$$

Find that the strength condition for a compressive member (see in (11)) is the nonlinear one, as buckling factor χ is functionally related via member area A. The slenderness of pinned member is $\lambda = \frac{l_b}{\mathrm{i}} = \frac{l}{\sqrt{I/A}}$, where I is moment of inertia. The Euler's slenderness is $\lambda_E = \sqrt{\pi^2 E/\sigma_y}$. Then combining the expression for member buckling factor χ according to EC3 [16] with (15) the left part of strength condition for compressive member (expressing the member critical force $N_{cr} = A\sigma_y\chi$) having performed the certain transformations finally reads:

$$N_{cr} = \frac{\sigma_y A}{\Psi + \sqrt{\Psi^2 - \left(\frac{\Omega}{A^{b1}}\right)^2}} , \tag{16}$$

$$\Psi = 0.5\left(1 + \alpha\left(\frac{\Omega}{A^{b1}} - 0.2\right) + \left(\frac{\Omega}{A^{b1}}\right)^2\right), \quad \Omega = \frac{L}{a_1 \lambda_E}, \tag{17}$$

where α is a variance coefficient depending on member profile manufacturing conditions.

The problem (10)–(13) is the nonlinear programming problem due to nonlinear strength and stiffness conditions. Its solution (i.e. determination of optimal distribution of member areas) can be analyzed in respect of conditions to be activated (satisfied as equalities). If the structural design model is not constrained too much (due to introduced constructional requirements and/or by its topology combined with the number of optimized parameters) to develop extreme displacements, usually the stiffness constraints (at least one of them) is activated ([12, 13]). But one can meet cases when strength conditions predominate (due to above described reasons) in optimal structure, i.e. displacement constraints are not activated in the optimal solution of the problem.

Efficient numerical realization of the optimization problem requires fixing of the upper bounds of optimized parameters. The latter is obtained from rigid-plastic optimization, which is also nonlinear due to nonlinear strength conditions for compressive members.

3. Numerical example

The stepped optimization method techniques and optimal solution analysis is performed for a 4-storey pin-jointed steel tower with 20 nodes (see Fig 1a). The structure consists of 56 members, its DOF = 48. The structural members are designed of 4 different cross-section types: A_1, A_2, A_3, A_4, constructed from cold formation pipes (variance coefficient α = 0.34). Steel properties are: yield limit σ_y = 235 Mpa, elasticity modulus E = 210 GPa, density ρ = 7850 kg/m^3.

Four load combinations combining dead and temporary loads are considered. The dead loading, consisting of 4 vertical forces $F = 90$ kN is included in all combinations. The lateral loads of 1-st, 2-nd, 3-rd and 4-th combinations are presented in Fig 1 b), Fig 1 c), Fig 1 d) Fig 1e), respectively. Find that lateral loads are applied in each storey of tower.

To obtain the required coefficients for relation (15) a functional analysis of i (in cm) versus A (in cm^2) was performed. The obtained coefficient magnitudes of a_1 and b_1 for cold formation pipes of different pipe thicknesses are presented in Table 1.

When analyzing the curves of relation (15) in respect to different thickness pipes, one can find them to be close to linear.

The limitation $\lambda \le \lambda_{cr} = 150$ for member slenderness was introduced. The basic optimization was performed for pipes of 3 mm thickness. It yielded the minimum constructional areas to be (in cm^2): $A_1^{\min} = 8.0770$, $A_2^{\min} =$ 10.7700, $A_3^{\min} = 11.4230$ and $A_4^{\min} = 13.4620$.

The rigid-plastic solution (required to identify the limit lower bounds for design parameters) yielded the final minimal constructional magnitudes to be (in cm^2):

$$A_{1,pl}^{\min} = 8.0770,\quad A_{2,pl}^{\min} = 35.8102,\quad A_{3,pl}^{\min} = 11.4230$$

and $A_{4,pl}^{\min} = 20.6429$, resulting the structure weight of 5645.2 kg.

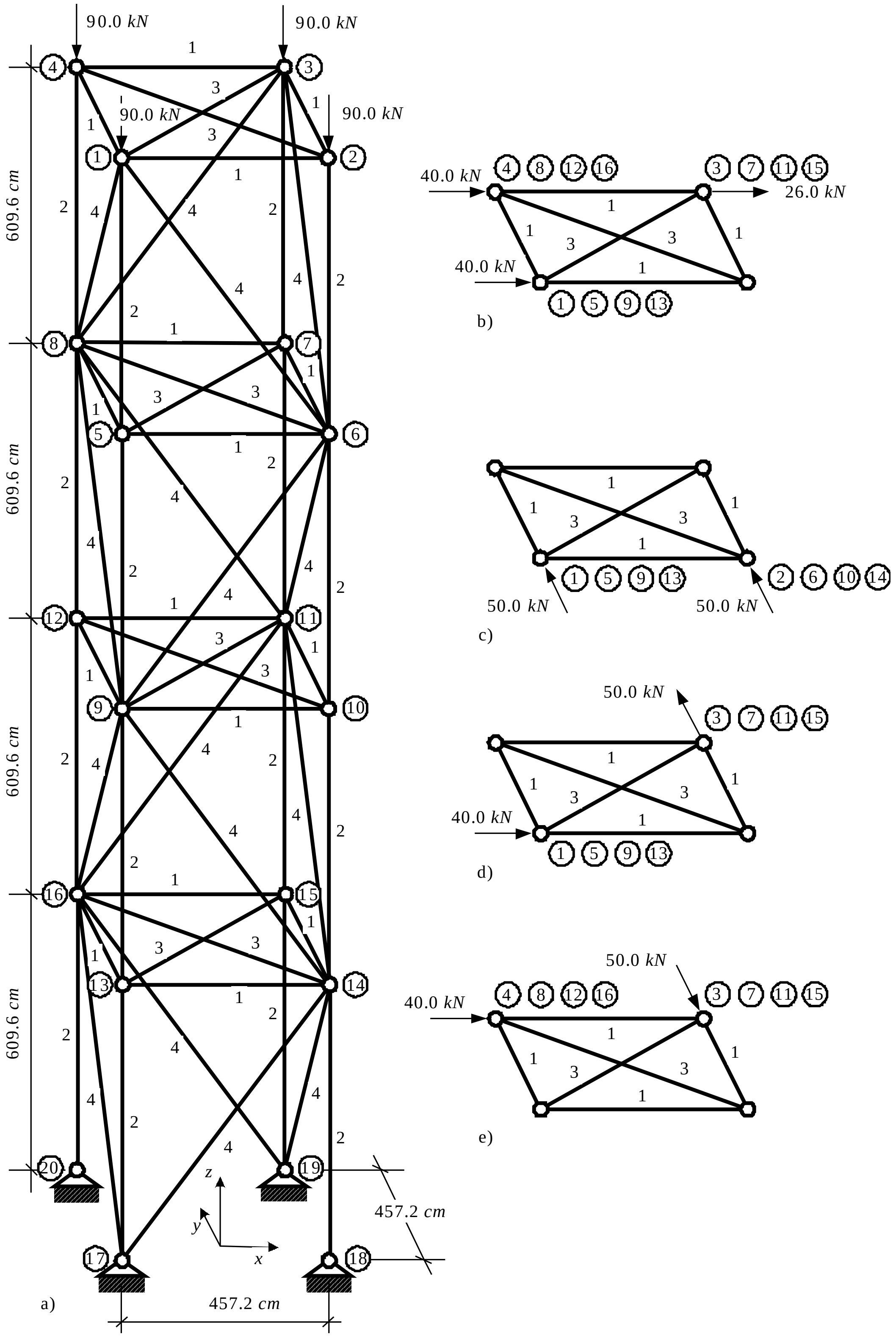

Fig 1. 56-member tower: a) general view, b) 1-st load combination, c) 2-nd load combination, d) 3-rd load combination, e) 4-th load combination

Table 1. Coefficients of relation (15)

Pipe thickness (in mm)	a_1	a_2
2	0.5659	0.9958
2.5	0.4534	0.9965
3	0.3784	0.9968
4	0.2834	0.9978
5	0.2267	0.9982
6	0.1892	0.9978
8	0.1420	0.9980
10	0.1135	0.9982
12	0.0945	0.9985
14	0.0810	0.9988

The stepped optimization dynamics is presented in Table 2 in case of the following admitted nodal displacement magnitudes: for horizontal ones – 10 cm, for vertical ones – 3 cm. The last column of Table 2 is assigned for horizontal (governing) nodal displacements as vertical ones develop far from introduced constraining limits. When analyzing the obtained solution (see the last row of Table 2), one can find that stiffness constraints are not conditioning ones for minimum weight structure responding to load combinations in elastic-plastic range.

Two types of areas, namely the 1-st and the 3-rd reach their minimal constructional magnitudes. The critical limit states are reached in members *14–18* and *16–19* in case of the first load combination. No yielding limit states are reached for all load combinations. The extreme displacements develop in:

direction x – in the 4-th node from the first load combination, namely $u_{4x} = 9.2368$ cm;

direction y – in the 2-d node from the second load combination, namely $u_{2y} = 8.4411$ cm;

direction z – in the 2-d node from the second load combination, namely $u_{2z} = -1.1681$ cm.

When testing the structure for the smaller admitted horizontal displacement bounds, it was found that stiffness constraints were activated only for displacement limit of 9.1 *cm*. The stepped optimization dynamics in case of this constraint is presented in Table 3.

Analyzing the obtained solution (see the last row of Table 3), two types of areas, namely the 1-st and the 3-rd reach their minimal constructional magnitudes. The critical limit state is reached in member *16–19* in case of the first load combination. No yielding limit states are reached for all load combinations. The displacement limit of 9.1 cm is reached in direction of the 4-th node from the first load combination. The other extreme displacements develop in:

direction y – in the 2-nd node from the second load combination, namely $u_{2y} = 8.3686$ cm;

direction z – in the 2-nd node from the second load combination, namely $u_{2z} = -1.3077$ cm.

Due to the obtained optimal distribution of areas (see the last row of Table 3) the actual profiles were chosen (see Table 4). The structural weight of this structure is 6958.35 *kg*.

Having performed the analysis of structural response it

Table 2. The space truss iterative solution convergence for $u_{hor,adm} =$ 10 cm

Cycle number	A_1, cm^2	A_2, cm^2	A_3, cm^2	A_4, cm^2	Weight, kg	Extreme u_{hor}, cm
0	80.0000	80.0000	80.0000	80.0000	20211.19	3.8361
1	8.0770	36.4707	11.4230	25.9496	6203.61	9.2131
2	8.0770	36.4707	11.4230	25.7026	6179.98	9.2365
3	8.0770	36.4707	11.4230	25.6999	61180.04	9.2368
4	**8.0770**	**36.4707**	**11.4230**	**25.6999**	**61180.04**	**9.2368**

Table 3. The space truss iterative solution convergence for $u_{hor,adm} =$ 9.1 *cm*

Cycle number	A_1, cm^2	A_2, cm^2	A_3, cm^2	A_4, cm^2	Weight, kg	Extreme u_{hor}, cm
0	80.0000	80.0000	80.0000	80.0000	20211.19	3.8361
1	8.0770	36.7644	11.4230	25.9496	6226.10	9.0995
2	8.0770	36.9347	11.4230	25.6017	6205.85	9.1000
3	8.0770	36.9390	11.4230	25.5932	6205.36	9.1000
4	8.0770	36.9391	11.4230	25.5930	6205.35	9.1000
5	8.0770	36.9391	11.4230	25.5929	6205.35	9.1000
6	**8.0770**	**36.9391**	**11.4230**	**25.5929**	**6205.35**	**9.1000**

Table 4. Data of the chosen pipe types

	1-st	2-nd	3-rd	4-th
Diam., mm	88.9	323.9	139.7	244.5
Thickness	3	4	3	4
A, cm^2	8.10	40.20	12.88	30.22
i, cm	3.04	11.31	4.83	8.5
λ	150	54	134	90

was identified that now any strength condition is activated. The extreme displacements develop in:

direction x – in the 4-th node from the first load combination, namely $u_{4x} = 8.1983$ cm;

direction y – in the 2-d node from the second load combination, namely $u_{2y} = 7.5559$ cm;

direction z – in the 2-nd node from the second load combination, namely $u_{2z} = -1.0441$ cm.

The increment of total weight (reduction of extreme displacements per considered load combinations) of designed structure vs optimal one can be explained by the limited set of manufactured pipes available to choose.

4. Conclusions

An algorithm of stepped optimization method in the case of load combinations is developed. The proposed algorithm employs developed FEM mathematical models for the analysis and optimization in concert with the approximated relation of actually manufactured cross-sectional parameters required for optimization procedures. Numerical simulations proved the efficiency and good convergence of the developed algorithm. The results obtained via proposed techniques also allow to make certain conclusions regarding an influence of principle design scheme and constructional requirements on available decisions aiming to reach other optimality level of the structure.

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TAMPRIOS-PLASTINĖS LENGVOS LANKSTINĖS PLIENINĖS KONSTRUKCIJOS PAKOPINĖS OPTIMIZACIJOS ALGORITMAS APKROVŲ DERINIŲ ATVEJU

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S a n t r a u k a

Straipsnyje pateikiamas patobulintas pakopinės optimizacijos metodo algoritmas lengvai lankstinei plieno konstrukcijai (dažnai naudojamai inžinerinėje praktikoje), kurios išorinė apkrova projektavimo normose paprastai modeliuojama apkrovų deriniais. Disipacinių savybių įvertinimas, naudojant tamprios-plastinės medžiagos modelį, lemia reikšmingą konstrukcijos laikomosios galios rezervo sumažinimą (esminį ekonominį efektą) optimalios konstrukcijos, dirbančios tik tamprumo būklės, atžvilgiu. Standumo, stabilumo, konstrukciniai reikalavimai yra įvertinti. Optimizacijos procedūrų metu naudojami funkciniai ryšiai tarp skerspjūvio parametrų, nustatyti standartiniams profiliams. Visa tai leidžia suoptimizuoti konstrukciją, kurios atsakas apkrovimui yra adekvatus tokios klasės inžinerinių konstrukcijų normatyviniams reikalavimams. Pateikiami patobulinti BEM matematiniai modeliai, naudojami pakopinės optimizacijos stadijose.

Raktažodžiai: tampri-plastinė lankstinė konstrukcija, apkrovų deriniai, stabilumo, standumo ir konstrukciniai apribojimai.

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