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# Uncertainty evaluation of proposed setup for the calibration of vertical angle measuring systems by using means for the flat angle calibration

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## ABSTRACT

Precision angle measurements performed by using total stations is a field showing a lot of potential. Horizontal angle measuring system calibration is a well documented and researched field however vertical angle measuring system calibration tends to lack proper investigation. The paper deals with the analysis of modern calibration methods of vertical angle measuring systems. New setup for the calibration of vertical angle measuring system of the total station using horizontal angle calibration means is presented in the paper. The principle of the method is explained and the uncertainty evaluation as well as preliminary results are given in the paper. The advantages and weaknesses of this new setup are discussed.

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## 1. Introduction

Total stations (TS) are electronic distance and angle measuring devices commonly used in geodetic measurements. Although these instruments are mostly used for outdoor measurements, good optics and precision angle measuring systems motivates scientists to use total stations in laboratory measurements. Since there are two angle encoders (horizontal and vertical) embedded in total stations it is very important to perform the calibration of these instruments in order to determine systematic errors.

As previous research showed, most of the methods in angle metrology deal with the flat angle calibration [8]. However, calibration of vertical angle measuring systems is a challenging task for scientists.

Analysis of means and methods for angle calibration led to a conclusion that angle calibration should be performed using standard means such as Moore's Precision Indexing

Table, polygon or other reference mirror and an autocollimator. For vertical angle calibration it is quite difficult to combine such instrumentation in a vertical plane. Therefore, an apparatus with the special frame which allows the horizontal position of the total station while performing the measurements was created at Korea Research Institute of Standards and Science. The novel approach for the calibration of vertical angle measuring systems by using the means for flat angle calibration is presented in this paper.

## 2. Vertical angle calibration methods

There are only a few comparators designed for vertical circle calibration. Such vertical circle comparator (VCC) was created as the standard in order to perform precise calibration of vertical angle measuring system of robotic total stations and laser trackers in ESRF, France. This comparator is designed to measure vertical angles in the range of  $90^\circ \pm 45^\circ$  and  $270^\circ \pm 45^\circ$  because this is the range mostly used while performing vertical angle measurements. An

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instrument to be calibrated is placed against the vertical circle comparator and the reflector is mounted on the adjustable part of the comparator. During the calibration procedure vertical circle readings are compared with the vertical displacements of its spherically mounted retro-reflector which are measured by the interferometer. The horizontal distance between comparator and the total station is measured by using distance meter calibration bench. While changing the position of the reflector different vertical angles can be measured. The VCC calibration procedure compares the total station's vertical circle readings with the angles determined based on vertical displacements of its reflector and the calibrated distance between the VCC and the device to be calibrated. The uncertainty of vertical angle measuring system calibration was determined to be  $1.4''$ . However, the drawback of this method is that this comparator is designed to calibrate vertical angle measuring systems of the robotic total stations, therefore, minimal distance is 2.5 m which reduces the vertical angle calibration range to  $\pm 23.75^\circ$ . The expanded uncertainty of vertical angle measuring system calibration using this vertical circle comparator is  $1.65''$  ( $k = 2$ ) [5–7].

Another method for vertical angle measurement system calibration developed by Leica, Switzerland is based on the mirror and autocollimator measurements. Practically, this principle was implemented by fixing a special mirror to the telescope of a total station which is mounted to special high precision theodolite testing machine on the top of the reference horizontal rotary encoder. This machine has two reference indexes (in horizontal and vertical planes) and a frame which enables autocollimator rotation around the TS to be calibrated in a vertical plane. TS's vertical encoder readings are compared to the reference vertical high precision indexing table readings. This method is implemented only by using special fully automatized Theodolite Testing Machine (TPM) which fulfills the condition of orthogonality of the axes. The standard deviation of  $0.058''$  for horizontal angles and  $0.091''$  is possible to achieve by this machine because of its unique structure and precision [3,4].

At National Metrology Center in Singapore the vertical angle calibration method where indexing table and a collimator is used was developed. A special fixture is used to ease the alignment process and minimize effect from compound angle. The expanded uncertainty obtained by this method is  $U_{95\%} = 2.0''$  ( $k = 2$ ) and the vertical angle calibration range of  $90 \pm 30^\circ$  is possible to achieve [10].

### 3. Proposed setup

#### 3.1. Principle and design of the system

The proposed angle measuring system is based on well-known and reliable angle measuring technique described in standards, latest angle metrology related papers and official reports of famous metrology institutes. The main principle of angle calibration is comparison of reference angle and measured angle. The reference angle can be obtained by using various techniques and instrumentation. One of the main techniques used for horizontal angle mea-

suring system calibration of total station is comparing measured angle with the reference angle created by the indexing table. 1440 Precision Index is considered a principal angle standard and is based on the circle division into 1440 even parts with the pitch of  $15'$  [9]. The instrument is guaranteed to be accurate to within  $\pm 0.1''$  at any of the 1440 indexed positions. Such accuracy is attained by controlling all the components of the rotary table. The serrated – tooth divider is the crucial component of Moore's 1440 Precision Indexing Table. This divider employs two face gears of identical shape and spacing of teeth. One member is displaced axially to disengage the teeth and then rotated radially to the desired angle. When two opposed faces of the gears are brought into forced engagement, they become locked in place, preventing rotation or side movement [11,12]. Since indexing tables are commonly calibrated by using autocollimators and reference mirrors, it was decided to use this principle for calibration of vertical angle measuring system of total station.

Because of the complicated design of total stations it is quite challenging to perform high quality calibration of vertical angle measuring systems. This new set up is based on principles of horizontal angle calibration.

The novel component of this angle measuring system is a special apparatus fixed on the top of Moore's 1440 Precision Indexing Table. This apparatus has a weight balanced structure and special frame which is designed to fit a total station in horizontal position. The special mount for fixing total station's tribrach is installed in this vertical angle measuring system as well as six adjustment screws, three on each side, of the frame to support and level upper part of a total station (Fig. 1).

Leveled total station is fixed to the apparatus horizontally. To perform autocollimator measurements there is a need to fix the mirror on the TS's telescope. Therefore, the special mirror mount was designed to fit the total station's telescope (Fig. 2).

This mirror mount has four adjustment screws as well as screws for fixing the mount to the telescope of the total station. The mirror is fixed to a special recess designed for it. The adjustment screws are in the plane parallel to the mirror located in four points around the mirror in order



Fig. 1. Components of the vertical angle measuring system.

to adjust the mirror in all positions. There are two spaces (1 mm each) designed to adjust the mirror position within this range. The use of such a mirror mount for the experiment is beneficial for better alignment of the devices. After fixing the mirror mount an electronic autocollimator is pointed to the center of the mirror which is adjusted to align both parts of the measurement system.

Autocollimators are used in angle metrology for non-contact small angle measurements, alignment of the devices and calibration of angle measuring systems. These optical instruments measure the deviation of the light beam reflected from the mirrored target. The working principle of an autocollimator is shown in Fig. 3 [13].

The object reticle is illuminated and through the beam splitter projected to infinity by the collimator objective. When the light beam reaches the mirror it is reflected in different path through the collimator objective to the image plane or the digital camera with the sensors (Charged Coupled Device) in electronic autocollimators. The difference between initial and latter positions of the light beam is measured. The angle at which the reflective target is tilted can be expressed as shown in (1):

$$\alpha = \frac{s}{2f} \quad (1)$$

where  $\alpha$  – tilted angle;  $s$  – shift of the light beam;  $f$  – focal length of an autocollimator.

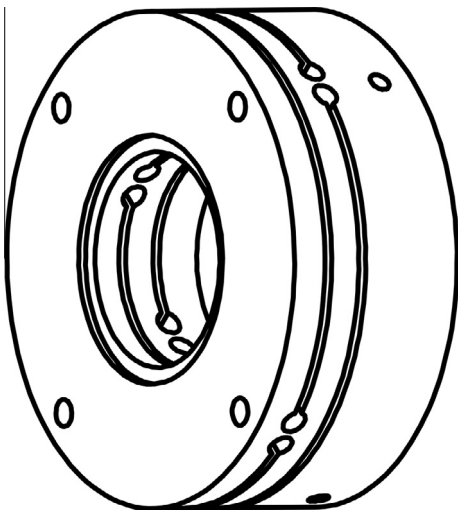


Fig. 2. The design of the mirror mount.

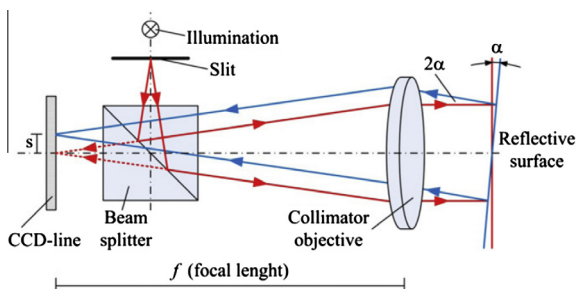


Fig. 3. The principle of the autocollimator.

An electronic autocollimator shows the tilted angles in the display in two planes ( $x$  and  $y$ ) which in this case helps us to determine the position of the mirror mounted on the TS's telescope.

After aligning all the system the initial position of both devices (electronic autocollimator and total station) is set. Then the apparatus is rotated by desired angle ( $\theta_I$ ) along with the indexing table and fixed with its handle. The telescope of the total station is turned back to previous position based on the display readings of the total station ( $\theta_{TS}$ ). The principle of the method is shown in Fig. 4.

### 3.2. Uncertainty

Uncertainty of measurement in the International Vocabulary of Metrology (VIM) is defined as a non negative parameter characterizing the dispersion of the quantity values being attributed to a measurand based on the informations used [1,2]. All uncertainty components are arising from random or systematic effects (as errors) and can be evaluated by both uncertainty evaluation types. Type A evaluation of standard uncertainty is performed by statistical methods using series of observations. Type B evaluation of standard uncertainty is performed using other means when series of observations are not available. It might be specifications or other documentation as well as evaluation based on previous experience and scientific knowledge.

The components of combined uncertainty as well as their impact on measurement results have to be determined when methods with complex instrumentation are analyzed.

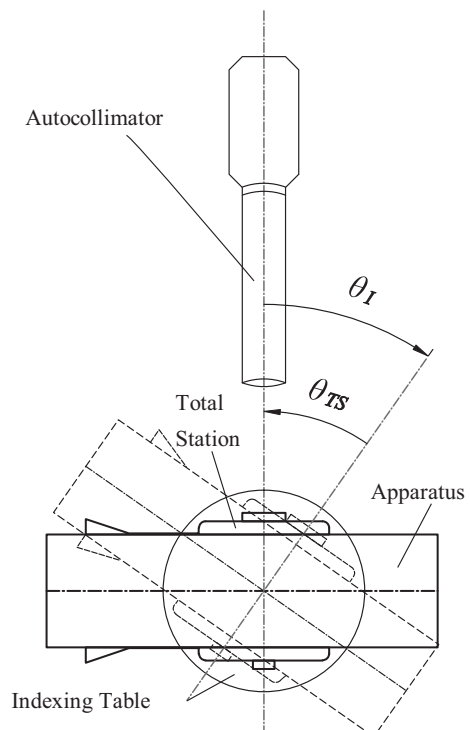


Fig. 4. The measurement principle of the new setup.

The measurement function of such measurements is expressed as it is shown in (2). The correction value is added algebraically to the uncorrected result of a measurement to compensate for systematic error.

The correction has to be evaluated considering all the instrumentation used for the measurements. The rotated angle of total station is expressed:

$$\theta_{TS} = \theta_{TSr} + B \quad (2)$$

where  $\theta_{TS}$  – rotated angle of the total station;  $\theta_{TSr}$  – vertical angle reading of the total station;  $B$  – correction value for the vertical angle readings of total station.

The reading of an autocollimator depends on both indexing table and total station rotated angles. Therefore, the reading of an autocollimator can be expressed as follows:

$$\theta_{AC} = (\theta_0 + \theta_{I,C}) - (\theta_{TSr} + B) \quad (3)$$

where  $\theta_{AC}$  – the reading of the autocollimator;  $\theta_0$  – nominal angle;  $\theta_{I,C}$  – correction value for the indexing table.

Thus the correction  $B$  can be obtained as (4)

$$B = (\theta_0 + \theta_{I,C}) - (\theta_{TSr} + \theta_{AC}) \quad (4)$$

In this method the total station, indexing table and the autocollimator are three main error sources influencing the magnitude of the uncertainty. The combined uncertainty of the correction value (5) can be expressed as the sum of squares of the uncertainty due to the indexing table  $u^2(\theta_{I,C})$ , uncertainty due to the total station reading  $u^2(\theta_{TSr})$  and uncertainty due to the autocollimator  $u^2(\theta_{AC})$  multiplied by their sensitivity coefficient squares ( $c_i^2$ ).

$$u_c^2(B) = c_{I,C}^2 u^2(\theta_{I,C}) + c_{TS}^2 u^2(\theta_{TSr}) + c_{AC}^2 u^2(\theta_{AC}) \quad (5)$$

Uncertainties due to total station and autocollimator readings have two more components each. Therefore, uncertainties due to resolution and repeatability of both instruments have to be included in whole uncertainty budget.

The final equation for the combined uncertainty of the correction value describing uncertainty budget is:

$$u_c^2(B) = c_{I,C}^2 u^2(\theta_{I,C}) + c_{TSres}^2 u^2(\theta_{TSres}) + c_{ACcal}^2 u^2(\theta_{ACcal}) + c_{ACrep}^2 u^2(\theta_{ACrep}) + c_{ACres}^2 u^2(\theta_{ACres}) \quad (6)$$

where  $u(\theta_{I,C})$  – uncertainty due to the indexing table taken from the calibration certificate,  $u(\theta_{TSres})$  – uncertainty due to the limited display resolution of the total station,  $u(\theta_{ACcal})$  – uncertainty due to electronic autocollimator taken from the calibration certificate,  $u(\theta_{ACrep})$  – uncertainty due to repeatability of the autocollimator's readings,  $u(\theta_{ACres})$  – uncertainty due to limited display resolution of the autocollimator.

The uncertainty due to repeatability of the total station ( $u(\theta_{TSrep})$ ) in this case is excluded because the total station every time was rotated to the position where exactly the same angle reading was shown on TS's display.

An estimate of the standard uncertainty attributed to the repeatability of the measurement is the experimental standard deviation of the mean expressed in (7):

$$u^2 = s^2(\bar{q}) = \frac{s^2(q_k)}{n} \quad (7)$$

where  $s(q_k)$  – experimental standard deviation;  $n$  – number of independent observations.

Uncertainty due to repeatability of the autocollimator was evaluated by calculating standard uncertainties of every data set of every measured angular position. The best way to evaluate the uncertainty when all individual data sets have their own uncertainties is to determine pooled standard uncertainty as shown in (8):

$$s_p = \sqrt{\frac{\sum_{i=1}^N v_i s_i^2}{\sum_{i=1}^N v_i}} \quad (8)$$

where  $s_i$  – standard uncertainty of every data set;  $N$  – number of data sets,  $v_i$  – degrees of freedom of the  $i$ th data set.

Then the standard uncertainty due to repeatability ( $u(\theta_{ACrep})$ ) of the autocollimator can be evaluated as follows:

$$u = \frac{s_p}{\sqrt{n}} \quad (9)$$

where  $s_p$  – pooled standard uncertainty;  $n$  – number of observations in a data set.

After evaluating the standard uncertainties the sensitivity coefficients must be evaluated as the partial derivatives. Sensitivity coefficients describe how the output estimates vary with changes in the values of the input estimates and can be calculated as shown in (10):

$$c_i = \frac{\partial f}{\partial x_i} \quad (10)$$

where  $c_i$  – sensitivity coefficient;  $\frac{\partial f}{\partial x_i}$  – the partial derivative of a function  $f$  with respect to the variable  $x_i$ .

Uncertainties due to the limited display resolution of the total station ( $u(\theta_{TSres})$ ) and due to the limited display resolution of the autocollimator ( $u(\theta_{ACres})$ ) have rectangular distributions because all the readings have the same probability to be displayed according to the device's rounding system. Such type of uncertainty was evaluated by the difference of upper and lower resolution limits of the device divided by 2 times square root of 3 as expressed in (11):

$$u(\theta_{res}) = \frac{R}{2\sqrt{3}} \quad (11)$$

where  $R$  – display resolution of the device.

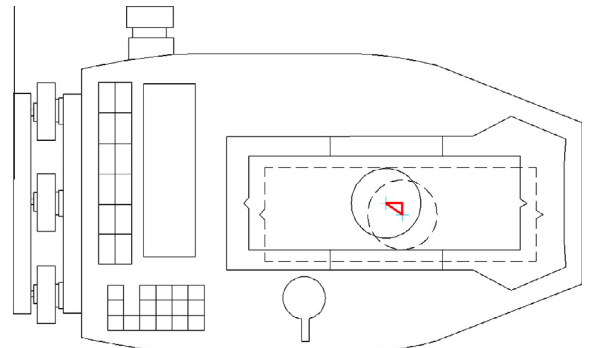
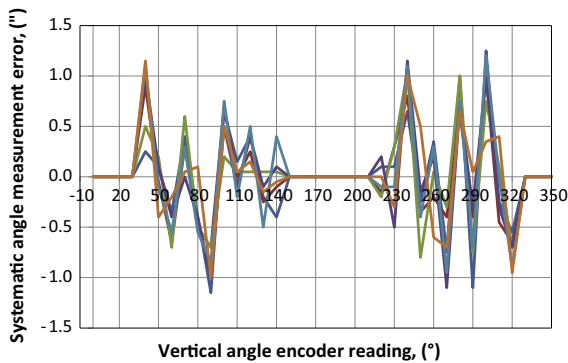


Fig. 5. The possible shift of TS telescope.

**Table 1**  
Uncertainty budget.

Source of uncertainty	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i$	Uncertainty contribution $u(x_i)  c_i $	Probability distribution	Degrees of freedom $\nu$	Effective degrees of freedom $\nu_{eff}$
Uncertainty due to the indexing table $u(\theta_{I,C})$	0.05"	1	0.05"	Normal	$\infty$	
Uncertainty due to the limited display resolution of the total station $u(\theta_{TSres})$	0.029"	1	0.029"	Rectangular	$\infty$	
Uncertainty due to autocollimator $u(\theta_{ACcal})$	0.25"	1	0.25"	Normal	$\infty$	
Uncertainty due to repeatability of the autocollimator $u(\theta_{ACrep})$	0.049"	1	0.049"	t-(Student's)	5	
Uncertainty due to limited display resolution of the autocollimator $u(\theta_{ACres})$	0.014"	1	0.014"	Rectangular	$\infty$	
Uncertainty of the correction value $u_c(B)$	0.262"			Normal		$\infty$



**Fig. 6.** Angle errors of vertical encoder of the total station.

Total station is a device designed to be used in vertical orientation. In this proposed method it is mounted to the apparatus horizontally therefore, uncertainty due to the possible telescope shift while in horizontal position  $u(\theta_{TSshift})$  needs to be analyzed deeper.

In horizontal position there could be a possible biaxial shift of the telescope as shown in Fig. 5. Upon a query, manufacturer of the total station affirmed the presents of the special mechanisms embedded in the total station for compensation of such influence and the measurement errors due to position of the total station are negligible. Such kind of uncertainty source should be investigated in further research to prove the stability of inner total station components while it is in horizontal position. However, in this uncertainty evaluation it was not estimated.

#### 4. Results

The electronic autocollimator Möller-Wedel, total station Leica TC 2003 and the apparatus mounted on the top of the Moore's special Index were used for the experiment. The indexing table was rotated six times in clockwise and six times in counterclockwise directions at 10° pitch. After performing the measurements primary results were obtained. Uncertainty budget is displayed in Table 1.

The expanded uncertainty of this setup is  $U_{95\%} = 0.52''$  ( $k = 2$ ).

The resolution of the total station Leica TC 2003 is  $R = 0.1''$  and resolution of the electronic autocollimator (Möller-Wedel) is  $R = 0.05''$ . Uncertainties due to the index-

ing table ( $u(\theta_{I,Cal})$ ) and due to the autocollimator ( $u(\theta_{ACcal})$ ) were evaluated using type B uncertainty evaluation method.

The measurement results of 6 data sets are shown in Fig. 6.

As it is shown in Fig. 6 there is a slight difference between errors of the measurements performed in the range of 40–140° (Side I) and 220–320° (Side II) of the total station vertical encoder. The average deviation from the mean of Side I is 0.22" while that of Side II is 0.33".

Although there is a visual symmetry in the distribution of standard deviation values, the bigger error is noticeable in Side II measurements. Such a difference can be a result caused by an influence of vibration or the slight tilt of the mirror mounted on the telescope of the total station. The possibility that this error of vertical angle measuring system of the total station can be caused due to the shift while the device was in a horizontal position cannot be absolutely ignored, however it needs further investigations. Fig. 6 also shows the range in which angle calibration of vertical angle measuring system of the total station can be done. Comparing to other previously analyzed vertical angle calibration methods, this enlarges the measurement range up to  $90^\circ \pm 50^\circ$  and  $270^\circ \pm 50^\circ$  and it is one of the main advantages of this setup.

#### 5. Conclusions

The new setup for vertical angle calibration using flat angle calibration means is presented in the paper. Although this method is not automated it can be easily performed by an operator, however the alignment might be time consuming. The measurement range of this setup was enlarged to  $90^\circ \pm 50^\circ$  and  $270^\circ \pm 50^\circ$ . After performing measurements, primary results were processed and the expanded uncertainty of  $U_{95\%} = 0.52''$  ( $k = 2$ ) was determined. The uncertainty due to electronic autocollimator  $u(\theta_{ACcal}) = 0.25''$  has significant impact on measurement results.

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