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PARTIAL INDEX TRACKING: SATISFYING DIFFERENT INVESTMENT PROFILES WITH THE SAME SUBSET OF STOCKS

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Abstract. Pasive portfolio management aims to minimize the unsystematic risk of the portfolios by imitating the behaviour of a stock index. Partial index tracking enables passive portfolio management by only considering a subset of the stocks included in the index obtaining a substantial cost reduction compared with full index tracking. In the literature, three criteria are usually employed to undertake partial index tracking: tracking error variance, portfolio variance and expected return. We propose an additional criterion to be considered, the frontier curvature. The main advantage when using this new criterion for portfolio selection is that a manager can satisfy different investment profiles with the same subset of stocks.

Keywords: portfolio selection, index tracking, passive portfolio management, frontier curvature, tracking error variance.

Jel classification: G11, G23, G32

1. Introduction

The large number of academic papers published indicates that the analysis of the efficiency of investment funds remains a major area of research in the field of portfolio theory. The valuation of funds is still subject to analysis and comparison because of their crucial role in financial markets.

Jensen (1968) was among the first to point out the need to critically evaluate the performance of investment funds. The high number of research papers in this area has evolved in parallel with the growth in the number of managed funds and assets. Comparisons are often made between active and passive management funds (Elton *et al*. 1993; Malkiel 1995, 2003; Gruber 1996; Carhart 1997; Edelen 1999; Davis 2001).

These studies demonstrate how difficult it is for investment funds to outperform a benchmark. This is probably due to the difficulty to model and predict the evolution of the stock markets (Jarret, Schilling 2008; Teresiene 2009; Aktan *et al.* 2010)

The questionable success of many actively managed investment funds in outperforming the benchmark explains that index tracking is currently among the most popular techniques used by investment fund managers. (Frino, Gallagher 2001; Malkiel, Radisich 2001; Coleman *et al*. 2006) This technique has become even more popular after the appearance of exchange traded funds (ETFs).

Index tracking seeks to minimize the unsystematic risk component by imitating the movements of a reference benchmark – a stock index. Faced with active management techniques that endeavor to beat the underlying index, tracking portfolios in general and tracking indices in particular, are configured as a powerful passive strategy. Following this strategy, the manager does not necessarily pursue efficiency in the sense of mean-variance, but instead replicates the behavior of the market from a more conservative approach. So he/she seeks to minimize the unsystematic risk component, limiting the risk to the systematic component. This is an important difference respect to the mean-variance approach, were both systematic and unsystematic risk components are minimized. Mixed approaches that search for consensus solutions between the two approaches can also be found in the literature (Burmeister *et al*. 2005). Index Tracking can be full or partial depending on the number of stocks that are considered.

In the case of full tracking, the portfolio includes the same stocks as the index, and an exact tracking is produced if these stocks are weighted in the same proportion as the index. It is also possible to generate other combinations of risk and return by varying the weights of the stocks in the tracking portfolio. However, in this case, the imitation of the stock index is not accurate, and it does not necessarily outperform the index in the mean-variance sense; while the greater or lesser required returns may lead to an increase or decrease in the proportional risk of the position (Roll 1992).

In partial tracking, which is the subject of this paper, a manager builds a portfolio from a subset of stocks contained in the underlying index and this process removes some of the drawbacks listed above. The counterpart is that an exact tracking of the index cannot be built. However, this does not necessarily imply a decline in the risk-return relationship.

Usually partial tracking portfolio models have attempted to obtain a single portfolio that will only satisfy those investors whose profile is perfectly aligned with the configuration chosen by the portfolio manager. It is noteworthy how this analysis has not pursued a parallel strategy to that followed in Markowitz's classical mean-variance model of 1952, which enables the generation of the so-called efficient frontier rather than the identification of a specific portfolio with a fixed risk and return.

Indeed, all the papers in the passive portfolio literature are characterized by the search for a single portfolio, characterized up to three possible parameters (Chow 1995): tracking error variance, excess return and volatility of returns. The stocks in the tracking portfolio are identified during this process and the given weighting complies with the constraints imposed on those parameters.

Those approaches mean that different values of TEV, return or risk necessarily imply to construct different portfolios, with different weights in the stocks and, what is even more important, with different selected stocks. So, if the fund manager seeks to satisfy different client profiles he/she will be forced to invest in most of the stocks included in the index; so renouncing to the main advantages of partial tracking. In short, a global and wider perspective is required. Enabling the minimum selection of stocks and covering the wider client profiles must be necessarily considered.

Our proposal adds a new parameter to be analyzed: the curvature of the mean-variance frontier. This criterion is not defined for a given portfolio, but for the set of portfolios that define the tracking frontier. The main advantage is that a fund manager can satisfy different investment profiles using the same subset of stocks – with all the portfolios on the frontier containing the same stocks and so reducing transaction costs–, and can also simultaneously consider different criteria in the tracking index problem.

The rest of the paper is structured as follows. The second section analytically presents the three key concepts for tracking indices: tracking error variance, excess return, and portfolio variance. The following section introduces a new criterion, the curvature of the tracking frontier, and discusses the benefits that arise from adding the concept of gradient to the previous ones. A summary of the main conclusions is presented in the final section.

2. Parameters in the tracking portfolio problem: tracking error variance, excess return, and portfolio variance

Tracking error is defined as the absolute difference between tracking portfolio returns and the returns produced by the tracked index. Since the aim is for both portfolios to maintain a parallel evolution over time, the problem is posed as a minimization of the volatility in the tracking error. A reduction in the volatility of the tracking error means minimizing the variance in returns between the tracking portfolio and the stock index (Roll 1992). In this way, a clear parallel with the mean-variance model (Markowitz 1952) is established. However, with the difference that instead of looking for the portfolio with the least volatility for a given return, managers try to obtain the portfolio with the minimum tracking error variance for a given level of return in excess of the index. These are the foundations of the TEV (Tracking Error Variance) criterion: (1) minimize the TEV; (2) assume a certain TE (Tracking Error). Both objectives are inherently conflicting, so the manager should look for consensus solutions.

The TEV is given by the expression (1):

$$TEV = x^t \mathrm{V}_x \,, \qquad (1)$$

where:

x – vector of dimension $N \times 1$, contains the weightings difference of the N stocks between the tracking portfolio and the index;

That is, $x = x_p - x_b$, where x_p is the vector of weightings in the tracking portfolio and x_b is the weighting vector in the index (subscript b for benchmark). A full tracking is obtained if all elements of x are zero, while non-zero deviations can take risk-return positions that differ from the index. In the partial tracking, the vector x_p will have the same number of non-zero elements as there are stocks included in the tracking, n, and the remaining weights will be left with a value of zero.

V = variance-covariance matrix for the stocks returns.

The excess return G on the index is obtained as the difference between the returns of the tracking portfolio and the index (2):

$$G = x^t R = x_p^t R - x_b^t R = R_p - R_b \,, \qquad (2)$$

where:

R – vector of returns of N stocks.

$R_p(R_b)$ – returns of the tracking portfolio (index).

Unlike other models, in the tracking portfolio the return in excess *G* is obtained by subtracting the index return, and not the return of the risk-free asset. The full tracking can be easily resolved by using a quadratic mathematical model (3):

$$Min = x^t V_x$$
$$s.t. \quad x^t R = G \tag{3}$$

where:

1 – vector of dimension N×1 with all the elements 1.

Note the need to explicitly include the constraint on *G*, since the profitability of the tracking portfolio and the index can differ by a constant, and the value of the TEV could paradoxically be zero. The second constraint ensures that the total investment in the tracking portfolio is the same as the index – and so the sum of positive and negative deviations is compensated. If the intention is to implement a partial tracking then an additional constraint should be added to *n*, although mathematical programming algorithms do not ensure the global optimum.

It is also worth to underline that the portfolios obtained with strictly positive values of *G* do not necessarily beat the index. To outperform the index, in addition to having a better return ($G > 0$), it is necessary to obtain less volatility, something which is not guaranteed by model (3).

Some researchers (Canakgoz and Beasley 2008) impose a restriction on the alpha and beta of the tracking portfolio, as estimated from the market model (4):

$$R_p = \alpha + \beta R_b, \tag{4}$$

The exact imitation of the index supposes imposing restrictions (1) $\alpha = 0$, and (2) $\beta = 1$. The first restriction is equivalent to considering $G = 1$. The second restriction does not guarantee the achievement of efficient portfolios in the mean-variance sense. For proof of this statement, let us consider the decomposition of the total risk of a portfolio *p* in its systematic and unsystematic components (5):

$$\sigma_p^2 = \beta^2 \sigma_b^2 + \sigma_{e_p}^2, \tag{5}$$

where:

$\sigma_p^2(\sigma_b^2)$ – return variance for tracking portfolio (index),

with $\sigma_p^2 = x_p^t V x_p (\sigma_b^2 = x_b^t V x_b)$,

$\beta^2 \sigma_b^2$ = systematic risk of the tracking portfolio,

$\sigma_{e_p}^2$ = unsystematic risk of the tracking portfolio: variance of the regression residuals between the index returns and the tracking portfolio returns.

As necessarily $\sigma_{e_p}^2 \geq 0$, we have $\sigma_e^2 \geq \sigma_b^2$ in order to impose $\beta = 1$. That is, the tracking portfolio will offer the same return as the index ($\alpha = 0$), but also with at least the same risk, which means it cannot outperform the index in the mean-variance space.

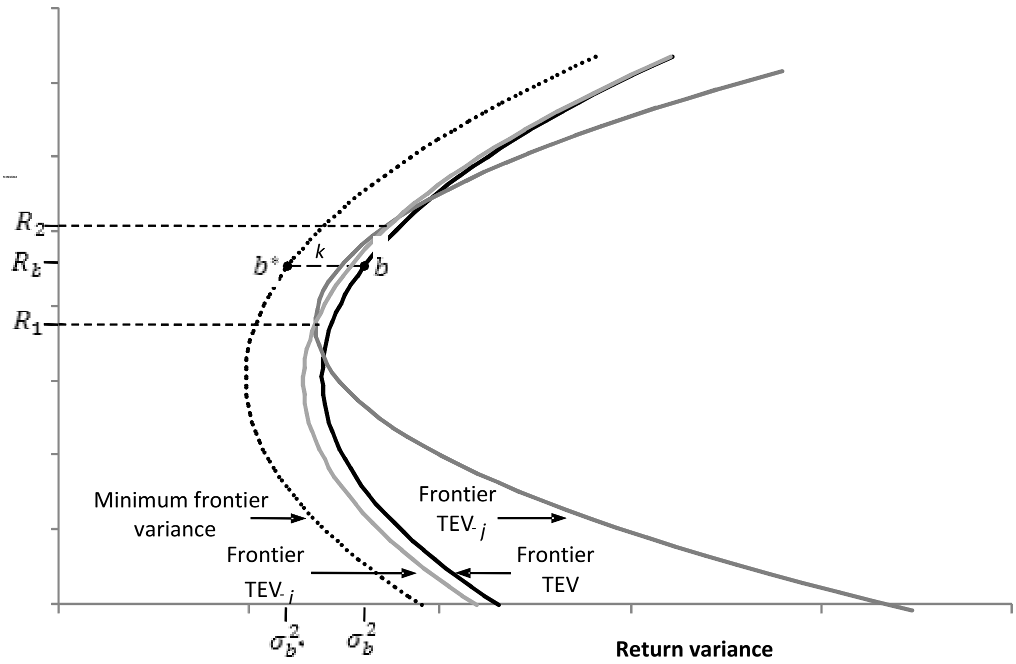
To overcome the agency problem that arises from this situation (Jorion 2003), it is necessary to constrain the total portfolio risk rather than the systematic risk, which facilitates the generation of tracking portfolios that can beat the index in the mean-variance space. Chow (1995) proposes a parametric model that in addition to considering the TEV and excess return, also considers the third criterion set out in this section: return variance in the tracking portfolio.

3. An additional parameter: the curvature of the TEV frontier

Model (3) enables to obtain different portfolios depending of the value of excess return *G*. These different portfolios are obtained by varying the weights of the stocks, and/or varying the stocks when the tracking is partial. Markowitz's minimum variance frontier and TEV frontier appear in Figure 1. For the case of the full tracking, Roll (1992) shows that the distance in the axis of the returns variance between the two frontiers is constant, *k*, for any value of return R_p. Therefore, the TEV frontier is a simple shift of Markowitz's frontier in the variance axis, and the inefficiency of the index *b* can be quantified as $k = \sigma_{b*}^2 - \sigma_b^2$, being constant for any portfolio on the tracking frontier.

The above property is not satisfied in the case of partial tracking. Figure 1 shows two TEV frontiers, each generated by removing a single stock from the tracking. The TEV frontier TEV_{-i} (TEV_{-j}) results from the exclusion of the tracking of the i-th stock (j-th). Generally, the removal of one or more stocks from the tracking means a greater TEV without necessarily reducing the efficiency of the portfolios. In the example in the figure, the TEV_{-i} frontier and the TEV_{-j} frontierpartially improve the efficiency of the original TEV in the mean-variance sense.

Specifically, both frontiers generate better risk-returns in portfolios nearer to the R_b index than the TEV frontier in the full tracking. If the TEV_{-i} and TEV_{-j} frontiers are compared then different results will again be reached according to the considered return. However, it must always be remembered that Figure 1 only reflects risk and return, and not TEV.



Key: - - - Minimum variance frontier; —TEV frontier; —TEV frontier excluding the j-th stock; —TEV frontier excluding the i-th stock; b: position of the index in the mean-variance plane; b^*: projection of the index on the minimum variance frontier; R_1: return of portfolio 1 (see Roll, 1992); R_2: return of portfolio 2 (see Roll 1992); R_b: index return; σ_b^2 : index return variance; $\sigma_{b^*}^2$: return variance of portfolio b^*.

Fig. 1. The minimum variance frontier and various TEV frontier (Source: Roll 1992)

Figure 1 shows the different curvature of the TEV_{-i} and TEV_{-j} frontiers. It is precisely this characteristic that can be very useful for the fund manager. The TEV_{-j} frontier provides a better risk-return combination than the TEV_{-i} frontier for portfolios with a return of $R_p \in [R_1, R_2]$. However, for returns outside this range, the TEV_{-i} frontier generates returns that are clearly better than the portfolios on the TEV_{-j} frontier. In this situation, the manager must consider which of the two frontiers can best satisfy client profiles. For conservative profiles that intend to simply mimic the index, the TEV_{-j} frontier is the most suitable, and so the j-th stock is removed from the tracking. But if a return in excess G is required, then the TEV_{-i} frontier would be the best option.

Therefore, not considering the curvature of the tracking portfolio frontier means that the proposed portfolios only satisfy specific values of risk, return and TEV, without considering the possibly varying risk profiles of the fund's clients. When choosing between two tracking frontiers for a given value of G and with the same levels of risk-return and TEV, the manager must select the frontier with less curvature – because this enables more efficient options to be offered to investors. Examining the curvature of the tracking portfolio enables the manager to make a more global analysis of the offer presented to his/her clients. To achieve this, we propose the entire TEV frontier to be necessarily examined and not just a specific point on it.

We can conclude that the manager will have the following preferences when evaluating tracking portfolios for the criteria presented:

a. Criteria concerning the tracking portfolio
   - a.1 Return: portfolios with higher returns are preferred, *ceteris paribus*.

a.2 Returns variance: portfolios with less risk are preferred, *ceteris paribus*.

a.3 TEV: portfolios with less TEV are preferred, *ceteris paribus*.

b. Criteria concerning the TEV frontier

b.1 Curvature of the TEV frontier: TEV frontiers with less curvature are preferred, *ceteris paribus*.

Note that for the curvature of the TEV frontier, Figure 1 only shows the frontier in the mean-variance plane. We will assume that TEV frontiers are preferred with less curvature in the mean-variance and mean-TEV spaces.

For the joint consideration of these criteria we propose the use of multi-objective mathematical programming. In this way the solution can generate a new frontier as a consensus between the frontiers obtained by separately considering each criterion. Besides, the inclusion of the curvature of the tracking frontier as a new criterion enables us to contemplate a wider range of investment profiles. With this criterion, it is possible to go beyond the objective of building a single tracking portfolio and to aim for a more general goal: to obtain a tracking frontier that satisfies a larger number of investors by using the same subset of stocks.

4. A multi-objective approach to the problem of partial tracking portfolios

It is possible to consider the TEV frontier curvature, along with other criteria already referred to in the literature (excess return, return variance, and TEV) into the utility function (6):

$$U(p) = x_0 R_p = w_1\sigma_p^2 - w_2 TEV_p - w_3 k_f \quad ,(6)$$

where:

k_f = represents the curvature of the TEV frontier, of which portfolio p forms a part.

w_i = weights of each criteria, with i = 0.3.

Note that the curvature is defined on a frontier *f*, and not on a given portfolio *p*, since the curvature is the same for all portfolios on the frontier (the returns variance and the TEV are quadratic functions).

Given that in the tracking portfolios the manager fixes a value for the parameter *G*, all of the portfolios evaluated with utility function (6) obtain the same return $R_p = R_b + G$. In this way, (6) can be simplified as (7):

$$U(p) = w_1\sigma_p^2 - w_2 TEV_p - w_3 k_f \ , \qquad (7)$$

For convenience, the proposed model will be presented as a minimization problem (8):

$$\text{Max} U(p) \equiv \text{Min}(-U(p)) \equiv \text{Min } w_1\sigma_p^2 w_2 TEV_p + w_3\kappa_f \ ,(8)$$

The multi-objective mathematical programming model is (9):

$$\begin{aligned} &\text{Min} = w_1 x_p^t V x_p + w_2 x^t V x + w_3 \kappa_f \\ &s.t. \quad x^t R = G \\ &x^t 1 = 0 \\ &x_p = x_b + x \ , \qquad (9) \end{aligned}$$

where the only unknown element is the weightings vector *x*. Note that no restrictions are included on the cardinality of the tracking portfolio.

For the application of model (9) it is necessary to address three issues. The first relates to how to find a good solution within the exponential number of portfolios that can be formed and limiting to *n* the number of stocks in the tracking portfolio. The objective of model (9) is to make a comparison between these portfolios using the utility function, and not to generate a frontier. The second question to address is how to calculate k_f , the only parameter that has not yet been derived analytically. Finally, there remains the determination of the w_i weights in the utility function. Each of these questions is discussed separately in the following subsections.

4.1. Search for local optima

As mentioned in the introduction, the optimal solution to the problem of partially tracking portfolios is a hard problem from a computational point of view. All optimal local search methodologies in the literature are consistent with model (9), and it is not the aim of this paper to propose new heuristic strategies. The greatest computational burden when solving an instance of model (9) is calculating the curvature of the TEV frontier. To this end we recommend to use an adaptation of the algorithm proposed by Tabata and Takeda (1995). We chose this algorithm because it is simple to implement and generates good local optima.

4.2. The TEV frontier curvature

As Roll (1992) demonstrated, the full tracking TEV frontier is a shift of Markowitz's minimum variance frontier, and the curvatures of both frontiers necessarily coincide (Fig. 1). This section sets out various propositions, including one that shows that the curvature of the TEV frontier generated from a subset of *n* stocks matches the curvature of the minimum variance frontier generated from the same *n* stocks.

The variance of a minimum variance portfolio *p* can be obtained by analytically solving Markowitz's mean-variance model (10).

$$\text{Min} = \frac{1}{2} x_p^t V x_p$$
$$s.a. \quad x_p^t R = R_p$$
$$x_p^t 1 = 1, \qquad (10)$$

Following Merton (1972), we propose using the Lagrangian (11) method on this model, deriving for the vector of portfolio weights x_p and multipliers λ_1 and λ_2, and equating to zero. The solution to the equation system appears in the expression (12).

$$L = \frac{1}{2} x_p^t V x_p + \lambda_1 (x_p^t R - R_p) + \lambda_2 (x_p^t 1 - 1) \qquad (11)$$

$$x_p = V^{-1} [R \ 1] A^{-1} \begin{bmatrix} R_p \\ 1 \end{bmatrix} \qquad (12)$$

where:

$$A = [R \ 1]^t V^{-1} [R \ 1] = \begin{pmatrix} a & b \\ b & c \end{pmatrix},$$
$$a = R^t V^{-1} R,$$
$$b = R^t V^{-1} 1,$$
$$c = 1^t V^{-1} 1.$$

We can express the variance of the *p* portfolio using (12) such as $\sigma_p^2 = x_p^t V x_p$, and developing its expression to arrive at a result which depends on *a*, *b* and *c*. The k_f curvature of the frontier of minimum variance is obtained as the second derivative of σ_p^2 with respect to R_p (13):

$$k_f = \frac{\partial^2 \sigma_p^2}{\partial R_p} = \frac{2c}{ac - b} \qquad ,(13)$$

This curvature matches the curvature of the TEV frontier if the tracking is full. If the tracking is partial, the curvature cannot be calculated using the expression (14), as the values of *a*, *b* and *c* are linked to the full set of stocks.

4.3. Calculating the weights of the criteria

The solution of the multi-objective programming model (9) depends on the w_i weights set for each of the three parameters considered in the objective function. This section proposes a solution for objectively quantifying these parameters:

Step 1. Apply Algorithm 1 with weights $w_1 = 1$ and $w_2 = w_3 = 0$. Use the resulting vector x_n^* to calculate the weight of the variance criteria of the tracking portfolio: $x_1^* = 1/VAR_{x_n^*}$, being $VAR_{x_n^*}$ the variance of the tracking portfolio defined by weight vector x_n^*.

Step 2. Apply Algorithm 1 with weights $w_2 = 1$ and $w_1 = w_3 = 0$. Use the resulting vector xn* to calculate the weights of the TEV criteria: $x_2^* = 1/TEV_{x_n^*}$.

Step 3. Apply Algorithm 1 with weights $w_3 = 1$ and $w_1 = w_2 = 0$. Use the x_n^* vector resulting to calculate the weights of the curvature criteria: $x_3^* = 1/k_{x_n^*}$, with $k_{x_n^*}$ being the curvature of the TEV frontier generated with the stocks in the tracking portfolio.

The weight of each parameter is fixed in a way that is inversely proportional to the solution – the ideal value – that is obtained when applying Algorithm 1 to the corresponding mono-objective problem.

5. Conclusions

The questionable success of many actively managed investment funds in outperforming the benchmark has triggered index tracking among the most popular techniques used by investment fund managers.

Researchers have made use of a limited number of parameters when building a tracking portfolio: Tracking error variance (TEV), excess return and volatility of returns. This paper considers a forth parameter to be used: Frontier curvature. This criterion is not defined for a given portfolio, but for the set of portfolios that define the tracking frontier. The main implication is that the manager can satisfy different investment profiles using the same subset of stocks, with all the portfolios containing the same stocks and so reducing transaction costs.

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