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THE IMPACT OF THE INFLATION AND UNEMPLOYMENT VALUES FROM THE PREVIOUS PERIOD ON THE PHILLIPS CURVE

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Abstract. The article deals with formulation of a dynamical model which expresses the relationship between the rate of inflation and the rate of unemployment more accurately and which has become to be known as the Phillips curve. The original classic model of the relationship between the rate of inflation and the rate of unemployment which led to the Phillips curve has been replaced, in this article, by a new model expressing the real economic situation more precisely while respecting the influence of the history of the factors taken into account using so-called delay differential equations. The article also looks into the impact of the delay interval's length and the impact of inflation and unemployment values from the previous period.

Keywords: Phillips curves, dynamical model, delay differential equations, inflation, unemployment.

Jel classification: C02, C69, E24, E27, M21

1. Introduction

Unemployment, its causes, consequences and ways of dealing with it have been more or less intensively debated for more than 150 years both among specialists and among laymen. The main goal of most governments and politicians in economics is to reach a low rate of inflation as well as a low rate of unemployment. Historical experience, however, shows that such a goal is not achievable in the long run.

The classic economic model of the relationship between the rate of unemployment u and the rate of inflation π is denoted by a function whose graphic representation is so-called Phillips curve. It has become one of the tools which are still used to forecast the rate of inflation in the future. The model is limited to relations and mutual connections in time t only and eliminates the influence of factors from periods preceding time t. Mathematically, the model leads to a system of common differential equations.

The reasoning submitted in the article eliminates the simplifications by constructing a more realistic model, which takes into consideration the data from previous periods, and creates a new model expressed by a system of two delay differential equations.

2. Phillips curve

One of the tools which is still used to forecast the rate of inflation in the future is so-called Phillips curve. Its author is A.W. Phillips, a New Zealand born economist, wrote a paper in 1958 in which he describes how he observed an inverse relationship between hourly wage changes and unemployment in the British economy. In his article The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861−1957 (Phillips 1958), he analysed the relationship between the rate of unemployment and the hourly wage rate changes.

The curve describes a mutual inverse relationship between the rate of unemployment u and the rate of nominal hourly wage growth w. In other words, Phillips curve says that when the rate of unemployment is high, nominal wages grow more slowly, while if the rate of unemployment is low, nominal wages grow faster. The point where the Phillips curve intersects the x-axis is the so-called natural rate of unemployment.

Similar patterns were found in other countries and in 1960 Paul Samuelson a Robert Sollow „took" Phillips work, replaced the rate of wage changes with inflation and made the explicit link assuming that when inflation was high, unemployment was low, and vice-versa.. Samuelson a Solow (Samuelson and Solow 1960) showed the empirical validity of the relationship on US data. Moreover, they argued that there is a positive correlation between nominal hourly wage changes and inflation, which made it possible to interpret the original Phillips relationship as a relationship between price inflation and the rate of unemployment.

The inverse relationship between the rate of inflation and the rate of unemployment is graphed by the modified Phillips curve. This curve also intersects the x-axis, the axis of the unemployment rate, at the point corresponding with the so-called natural unemployment rate ū. In this case, the natural unemployment rate is achieved at the zero rate of inflation, i.e. at a stable price level.

Milton Friedman later modified the Phillips curve. Friedman did not deny the interchange ability of inflation and unemployment in the short run. However, he regarded such interchange ability as a consequence of so-called money illusion. Therefore Friedman claims that Phillips curve is vertical in the long run. In the long term, unemployment remains at the level of so-.called natural unemployment rate which corresponds with the economic output at the level of potential output. The ensuing economic development in the 1970s corresponded to Milton Friedman's theory of the natural unemployment rate (Friedman, 1968). The natural unemployment rate is often quoted in the literature as an indicator of NAIRU (non-accelerating inflation rate of unemployment) which indicates a balanced rate of unemployment still consistent with stable inflation. If this hypothesis correctly describes the time series properties of unemployment rates, deviations from the natural rate are short-lived and will die out eventually (Cuestas, Gil-Alana 2011).

In paper (Cross 1987), R. O. Cross deals with the natural unemployment rate. In this paper, two models are used: in one the rate of inflation helps determine the natural rate of unemployment, as proposed by Friedman; in the other the process of moving from one inflation rate to another helps shape the natural rate, according to the hysteresis effect proposed by Phelps.

The natural unemployment rate hypothesis implies a fact that if inflation has become stable and does not decrease any longer, it should oscillate around its natural rate, which for example Gordon (Gordon 1989) identifies with the non-accelerating inflation rate of unemployment, i.e. NAIRU.

What should not be ignored, though, is the fact that mainly the European NAIRU increased considerably during the 1980s. Some economists claim, e.g. O. Blanchard (Blanchard 2002), that a long-term high rate of unemployment tends to increase the natural unemployment rate. The paper (McNelis 2003) applies neural network methodology to inflation forecasting in the Euro-area and the USA. McNelis observes in the paper that neural network methodology outperforms linear forecasting methods for the Euro Area at forecast horizons of one, three, and six month horizons, while the linear model is preferable for US data.

Although the Phillips curve remains to be seen as a controversial topic among economists, most of them now admit the concept of a short-term relationship between inflation and unemployment (Fisher *et al.* 2002). The relationship is commonly explained to arise from slow adaptation of prices in a short term. Politicians may benefit from such a situation by means of many tools being at their disposal and they are able to influence the combination of inflation and unemployment in the short run.

3. Hysteresis of unemployment in modern literature

Gordon (Gordon 1989) summarizes his explanation of the above mentioned phenomenon in two approaches − structural and hysteretic ones.

The structural approach saw and still sees the reason for growing NAIRU in specific obstacles on the supply side, while the hysteresis approach assumes that NAIRU follows the changes in current unemployment (Rotemberg 1982, (Layard *et al.* 2005, Sbdorne and Kumarov 2004, Gali and Gertler 2006).

In paper (Mihailov *et al.* 2009) we evaluate the relative influence of external versus domestic inflation drivers in the 12 new European Union (EU) member countries. This empirical analysis is based on the New Keynesian Phillips Curve (NKPC) derived in (Gali and Monacelli 2005) for small open economies (SOE).

Economists use the term „hysteresis"to denote the persisting influence of past economic events. The reason is that temporary failures of a system may result in a permanent change in its description. The current value of an endogenous variable then may depend on past rather than current values of some exogenous variables.

The hysteretic model of unemployment is based on the Phillips curve with adaptive expectations. The main consequence of hysteresis in unemployment is that any rate of unemployment is consistent with stable inflation whose rate only depends on the past development of inflation and unemployment (Steiger and Watson 2002). The difference between the hysteresis hypothesis and the natural unemployment rate hypothesis can be illustrated on the relationship between unemployment and the course of a business cycle. The natural unemployment rate expects the return to a balanced state in the long run. By contrast, the existence of hysteresis assumes that unemployment is continuously influenced by cyclical ups and downs.

Pérez-Alonzo (Pérez-Alonzo 2011) propose a new test for hysteresis based on a nonlinear unobserved components model. Observed unemployment rates are decomposed into a natural rate component and a cyclical component. A Monte Carlo simulation study shows the good performance of bootstrap algorithms. The bootstrap testing procedure is applied to data from Italy, France and the United States. Pérez-Alonzo finds evidence of hysteresis for all countries under study.

The existence of hysteresis and so-called Okun's law were also tested in paper (Dinu 2011), where, apart from other things, the authors tested the hypothesis of unemployment hysteresis in Romania during the period 1999–2008.

Fortin claims in (Fortin 1991) that understanding and reducing hysteresis and developing a consensus-based income policy that has a chance of alleviating the dismal trade-off should be top research and policy priorities. The paper presents a review of the two competing paradigms explaining inflation and unemployment fluctuations and singles out the Keynesian Phillips curve as the clear empirical winner over its classical competitors. Re-estimation of the Canadian Phillips curve for 1957–1990 identifies a sharp structural shift towards a high degree of unemployment hysteresis after 1972.

The paper (Kula 2010) examines the empirical validity of the hysteresis hypothesis in unemployment rates by education level in 17 OECD countries. These empirical findings provide evidence favourable to the non-stationarity of unemployment rates according to levels of primary and secondary education attainment in total unemployment, and therefore the existence of hysteresis for these levels of education. There is no evidence, however, of hysteresis for unemployment rates by tertiary education.

4. Hysteretic model of the relationship between unemployment and inflation

Samuelson and Nordhaus in (Samuelson and Nordhaus 2009) gave the definition of a model as a formal framework for representing the basic features of a complex system by a few central relationships. Models take the form of graphs, mathematical equation, and computer programs.

Begg, Fischer, and Dornbusch (Begg *et al.* 2000) found that a model or theory makes a series of simplification from which it deduces how people will behave. It is a deliberace simplification of reality.

The aim of every model simplification is to enable easier and deeper understanding of basic principles of real processes. The inability to carry out a controlled experiment and to achieve objective data suitable to assess the models proposed, and other possible limiting factors and drawbacks of available real economic data often lead to insufficiently precise, if not incorrect economic interpretation.

A number of applications from various scientific disciplines as well as findings of theoretical mathematics have shown in recent years that mathematical models may describe a very complicated behaviour of state values precisely (Barnes and Fulford 2002, Craigwell *et al.* 2011*)*.

The hysteretic model of the relationship between unemployment and inflation contains a simple version of the natural unemployment rate hypothesis in time t (denoted ūt), which connects inflation in time t (denoted πt) and unemployment rate in time t (denoted ut,). Further, the model considers the influence of unemployment rate and inflation rate in the period preceding time t by one time unit for simplicity, i.e. it also considers the influence of unemployment rate in time t-1, i.e.ut-1, and the inflation rate in time t-1, i.e. πt-1. This reasoning results in the following:

$$\pi_t = \alpha\pi_{t-1} + \beta(u_t - \overline{u}_t)\,, \qquad (1)$$

where:

α – denotes the inertia of the expected inflation development.

This version of a simple Phillips curve (Gordon 1989) can be placed in the context of adaptive expectations. If α = 1, then NAIRU (balanced unemployment ut) corresponds to a steady state in which πt = πt−1. Even values lower than one are consistent with the natural unemployment rate just because rationally acting agents may form their expectations with regard to a inflation rate drop. Allowing for the existence of hysteresis, we may define a rule which guides the development of a balanced unemployment rate ūt (represented by the degree of NAIRU):

$$\overline{u_t} = \eta u_{t-1} + z_t. \qquad (2)$$

Hysteresis occurs when ūt depends on the delay value of the unemployment rate ut−1 and on microeconomic determinants represented by variable zt. They can be identified with those listed by Friedmann in his natural unemployment rate hypothesis and we will assume that zt is more or less invariable in time. By putting the relations together we get:

$$\pi_t = \alpha\pi_{t-1} + \beta(u_t(u_{t-1} - z_t)), \quad (3)$$

The ensuing transformation leads to the equation:

$$\pi_t = \alpha\pi_{t-1} + \beta(1-)u_t + \beta(u_t - u_{t-1}) - \beta z_t\,, \quad (4)$$

Note the theoretical aspects and implications brought about by the hypothesis of unemployment's hysteretic character. It is apparent that for η=1 there is"full hysteresis ". In such a case the unique ūt will no longer exist and the balanced unemployment will be a completely variable quantity having no steady state value. Inflation in this case will not depend on the current unemployment rate but only on a change in unemployment. This, however, contradicts the natural unemployment rate hypothesis, which would tally with η=0.

Unemployment inertia corresponds to the example where 0 <η< 1, and it is in this case that NAIRU movement aims at a steady state, while the speed of this adjustment depends on parameter η (inertia rate).

The closer parameter η is to 1, the slower NAIRU's adjustment to the steady state will be and the lower the "inflation costs "will be of a demand-driven economic policy aiming at reducing the unemployment rate.

Parameters of such models are usually estimated by using classic estimation techniques of a Bayesian analysis or statistic filters (for example (Kim, Moh 2010). Apel (Apel, Jansson 1999) used a method combining statistic approaches with economic information – the multidimensional Hodrick-Prescott filter where residues of econometric equations capturing economic relationships related to a filtered variable are added in a special-purpose function which contains deviations of a real time series from a smoothed one, and changes in dynamics of smoothed data. Another option is to use the Kalman filter which, apart from direct incorporating of economic relationships, allows estimate or expert calibration of parameters. On the other hand, great flexibility in parameter setting and setting of statistic attributes of variables has drawbacks in the form of starting parameters and high optimization demands.

Another way to capture the dynamics of processes in a model is to describe a dynamical model by means of differential equations. Using this option we must understand time as a continuous variable. Formally, the dynamical system is described by a set of differential or difference equations.

Using principles of dynamical systems in economics has a long tradition and one of the definitions of an economic dynamic system comes from Paul Samuelson and R. Frisch (Gandalfo 1971) says that:" A system is dynamical if behaviour over time is determined by functional equations in which variables at different points of time are involved in an essential way."

The term "dynamical economic models" is to be understood as such economic-mathematical models which include the behaviour of the analyzed system in time in their structures.

5. Dynamical model of the relationship between unemployment and inflation with a delay argument

If we introduce a new variable p which denotes the expected inflation rate and if we admit the hypothesis of adaptive expectations (adaptive formation of expected inflation unwinds from the changes in inflation in the past), then, instead of explaining the absolute value of π we can describe the changes in the course of time by means of the equation:

$$\frac{d\pi}{dt} = j\left(\mathrm{p_t} - \pi_\mathrm{t}\right), \left(0 < j < 1\right). \quad (5)$$

In the set of these two equations there are three variables, one of which, however, must be regarded as exogenous. Variables π and p shall be regarded as endogenous and variable u shall be regarded as exogenous. In order to explain the third variable u we shall add another equation in which the fact that inflation may impact unemployment will be taken into account.

If we consider the possibility of the impact of changes in monetary policy on the inflation rate, and if m shall denote the growth rate of money supply, then the change in the inflation rate can be defined:

$$\frac{du}{dt} = -k(m - p_t), \ \ (k > 0), \quad (6)$$

In this relationship the change in the inflation rate is negatively dependant on the growth rate of money supply.

By substituting (4) in (5) and rearranging we obtain a system of two linear differential equations with delay argument where $x_1(t)=\pi_t\ (=\pi(t))$, $x_2(t)= u_t\ (=u(t))$ and $t\in[0,T]$:

$$\begin{aligned} \frac{dx(t)}{dt} &= a_{11}x_1(t)+a_{12}x_2(t)+ \\ & b_{11}x_1(t-()+b_{12}x_2(t-()+q_1 \\ \frac{dx_2(t)}{dt} &= a_{21}x_1(t)+a_{22}x_2(t)+ \\ & b_{21}x_1(t-\Delta)+b_{22}x_2(t-\Delta)q_2. \end{aligned} \quad (7)$$

where:

$$a_{11}=-j, a_{12}=\beta j, a_{21}=0, a_{22}=k\beta,$$
$$b_{11}=0, b_{12}=-j\beta\eta, b_{21}=0, b_{22}=-k\beta j,$$
$$q_1=-j\left(\beta z+\alpha p_{t-1}\right), q_2=-k\left(m+\beta z\right).$$

The system can be briefly put down as:

$$\frac{dx(t)}{dt}=Ax(t)+Bx(t)+q, \quad (8)$$

where:

$$A=\begin{pmatrix} -j & j\beta \\ 0 & k\beta \end{pmatrix},\ B=\begin{pmatrix} 0 & -j\beta\eta \\ 0 & -k\beta \end{pmatrix},$$
$$q=\begin{pmatrix} -j(\beta z+\alpha p_{t-1}) \\ -k\left(m+\beta z\right) \end{pmatrix}.$$
$$x(t)=\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, t\in[0,T]$$

In order to explore economic relationships in the above model, we have to know conditions for the coefficient of system (7) which ensures the existence, or possibly unique existence of a solution to the system crossing the solution space in question at random point, i.e. meeting random so-called starting conditions.

$$x(t_0)=c_{0,} \quad (9)$$

where:

$$t_0\in[0,T], c_0=\begin{pmatrix} c_{01} \\ c_{02} \end{pmatrix}, c_{01}, c_{02}\in R,$$

Next conditions for a non-negative solution, i.e.

$$x(t)\geq 0\ pro\, t\in[0,T]. \quad (10)$$

Conditions which guarantee the solution starting in values x(0) to the same values in time T, i.e.

$$x(T)=x(0), \quad (11)$$

(so-called periodic boundary condition).

6. Solution of a delay dynamic model

As equations (7) and (8) are equations with delay argument, they must be completed in both constituents by values in time preceding the values in time $t\in[0,T]$. In this case (the delay is given by a time unit $t=\Delta$) they are values of constituents of solution in the interval $[-\Delta,0]$.

In accordance with the usual symbolism of differential equations with delay argument we shall denote them $\varphi(t)=\begin{pmatrix} \varphi_1(t) \\ \varphi_2(t) \end{pmatrix},\ t\in[-,0]$.

The solution of system (7) or (8´) meeting some of conditions 9-11 shall be completed by a condition of the behaviour of solution in the interval $[-\Delta,0]$, i.e.

$$x(t)=\varphi(t),\ t\in[-,0]\ , \quad (12)$$

The conditions (9-12) must be taken per constituents.

All these questions can be answered by a modern theory of functional differential equations, a special part of which covers a theory of linear differential equations with delay arguments.

Monograph (Kiguradze, Půža 2003) contains a general theory allowing solutions to not only the above mentioned questions but also many others; the application of the theory to the above mentioned types of differential equations with delay argument, including the description of the construction of a required solution is dealt with in. (Kuchyňková and Maňásek 2006) and all the sources quoted in it.

7. Phillips Curve in the conditions of the Czech Republic

7.1. Model parameters

In this part of our article we apply the aforementioned model in the conditions of the Czech Republic. Values of parameters which need to be known to set up the model have been taken from the article (Němec, Vašíček 2010). This article deals with the analysis of the non-accelerating inflation rate of unemployment (NAIRU) and the corresponding unemployment gap.

Estimating the parameters of a hysteretic model of the Phillips curve we have relied on quarterly macroeconomic data of the Czech Republic in the period from the 2nd quarter 1995 to the 1st quarter 2007. In other words, we have used season-cleared data of the net year-to-date inflation and unemployment. The data source was the Czech Statistical Office and the Czech National Bank. As the estimate approach we have chosen the Gibbs sampler, for this model has been conceived as a normal linear regressive one with a priori normal gamma density.

$$\alpha = 0.7409,\ \beta = -1.0928,$$
$$\eta = 0.7713,\ Z = 2.1975$$

The size of the η parameter shows a strong unemployment hysteresis (although based on the data the probability of an extreme case of full hysteresis is only 15 %). Appropriate economic policy (even of expansive character) is thus able to reduce the unemployment rate without any negative inflation consequences on a long term basis. The relatively high value of the η parameter estimate implies strong adaptivity of expectations of economic agents, which means that inflation in the Czech Republic has not been developing in a jumping and irregular way (it got reduced step by step).

Parameters j and k have been set in the model taking into account the conditions of the Czech Republic and the ability to adapt in local conditions (j indicator) and the degree of money illusion (k indicator). For demonstration we have chosen the following values of these parameters: j=0.6, k=0.4, m=0.03.

7.2. System of differential equations

Putting down the parameters to equations 1, 2, and 3 we get:

$$\begin{aligned}\frac{d\pi}{dt} &= -0.656u(t) - 0.6\pi(t) \\ &\quad + 0.506\chi(t)u(t-1), \quad (13) \\ &\quad +0.506(1-\chi(t))\varphi_1(t) \\ &\quad + 0.445p(t-1) + 1.298 \\ \frac{du}{dt} &= -0.3278u(t) \\ &\quad +0.253\chi(t)u(t-1) \quad , (14) \\ &\quad +0.253(1-\chi(t))\varphi_1(t) \\ &\quad +0.225p(t-1) + 0.019\end{aligned}$$

As initial conditions we have chosen $\pi(0) = 5$, $u(0) = 7.7$, which corresponds to inflation and unemployment rates at the end of 2008. The initial functions of the iteration process were defined as constant for the sake of simplicity (they may be any continuous function). At the same time, we have accepted the natural requirement for continuous continuation of functions describing the history in the interval [-Δ,0] with solution in the interval [0,T], i.e. we have chosen $c_{01} = \varphi_1(0)$ a $c_{02} = \varphi_2(0)$.

We will solve the system using the Maple system. Maple system is a leading multipurpose mathematical software tool. It provides an advanced high-performance mathematical computation core with fully integrated numerical and symbolic systems (Maplesoft 2011).

As mathematical software, Maple is primarily used for its symbolic computation. It is similar to Mathematica and Maxima software programmes, which however offer much less functions. Maple can make analytical computation with formulas but it can very well create user-friendly environment providing a very extensive range of possibilities to use quantitative methods in practice, application tasks, and scientific computation for various fields, etc. For numerical solution of the envisioned task for the system of differential equations with a delayed argument we have used the approach derived in current studies regarding solution of boundary tasks for systems of „functional differential equations“ – see the list of references. This procedure allows us to solve the problem through a sequence of related systems of ordinary differential equations without delay. The solution of these systems lies in the sequence of gradual approximations of the solution of a delayed argument system we are looking for. Therefore, with a sufficiently large number of approximations we can get the solution we are seeking with a sufficient accuracy. Figure 1 depicts diagrams 1–10 of the solution approximation. The tenth approximation is further used as an approximate solution of the original task for other tasks.

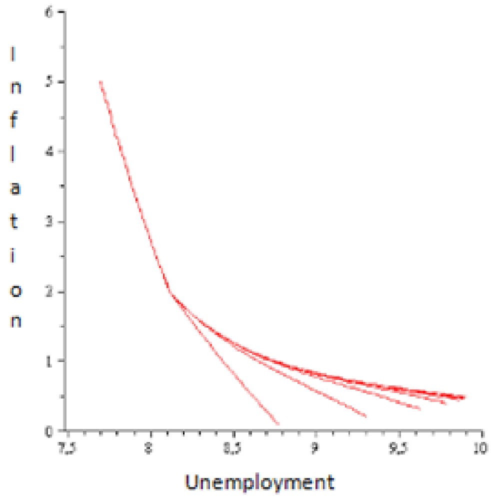

Fig. 1. Approximation 1–10

An approximate solution of the system is the curve depicted on picture 1. It shows the effect of history which causes a considerable break. Should the effect of data history be smaller, i.e. value Δ is lower, the break point will move to the left in the diagram.

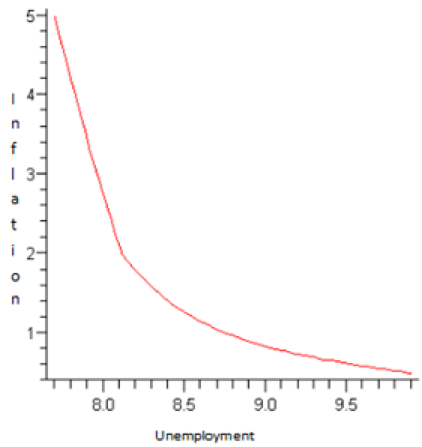

Fig. 2. Phillips curve

To be able to compare we have inserted the polynomic trend in the data measured in 2009–2011 using the regressive analysis method, which is described in detail in (Wonnacott, Wonnacott 1990). This method is used very often in economics (Doubravský, Doskočil 2011, Němec, Vašíček 2010, Cipra 2006). Data, relating to real economy, we obtained from the data bank from ČNB's (Czech Nationl Bank) website http://www.cnb.cz/. The regression equation is

$$y = 1.7826x^2 - 32,41x + 148,12\,, \qquad (15)$$

The determination indicator is $R^2 = 0.67$, confirming its good informative ability.

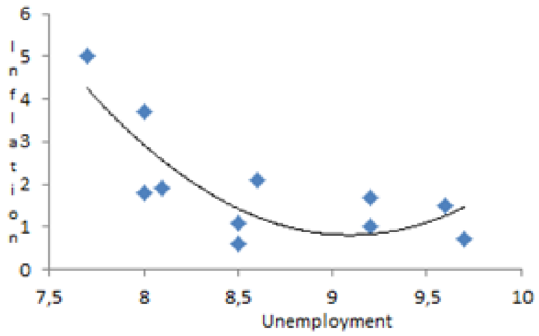

Fig. 3. Phillips curve

Comparing the values of the tenth approximation in our model with the ones obtained from the actual data we have discovered that the values are very similar nearly in the entire course monitored, and hence that the tenth approximation of the solution corresponds with the reality.

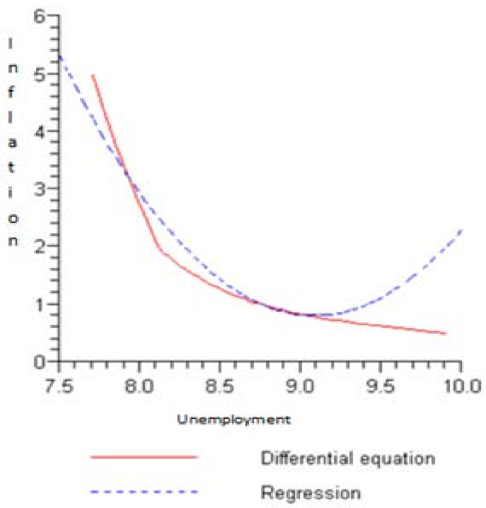

Fig. 4. Phillips curves

Presented comparisons show us that in intervals where real values were obtained, the solution for Phillips' Curve model − when delayed − differentiates minimally from values, obtained by statistical methods (see fig.4).

The model given may thus be undoubtedly used for more detailed analysis of relations between inflation and unemployment.

Advantage of the model suggested is that it allows for inspecting the influence of individual values of economical process on the shape of Phillips' Curve, including importance of influence of historical data in time periods of variable length.

8. Conclusions

The thesis in its first part composes of analysis of contemporary publications about correlations between the inflation rate and unemployment that produces the so called Phillips' Curve and corresponding mathematical model (two ordinary differential equations).

The second part of thesis is dedicated to more detail analysis of economic relations between those economic characteristics, furthermore conditioned by the influence of their conduct in previous periods; that is their hysteresis. Even though this approach is not new in the economic literature, hysteresis models of Phillips' Curve Publisher so far used results of statistical methods to estimate the parameters of the curve.

Instead, this publication employs new methods of mathematical analysis – differential equations systems with delayed arguments, that were published in the last ten years and allows for exact solutions of the corresponding models with numeric methods. Phillips' Curve, obtained by solving the delayed model, likely corresponds to real values, as evident from the previous chapter's conclusion.

When modelling complex economic issues we often have to face the fact trade-offs between variables are chase in time. The dynamic character can be captured by including delay exogenous and endogenous variables in specifying the structure of a model.

Another way to include dynamic processes in models is to see time as a continuous variable and to describe dynamic models by means of differential equations.

The original classic model of the relationship between the rate of inflation and the rate of unemployment resulting in the Phillips curve has been replaced, in this paper, by a new model capturing more accurately the real economic situation and respecting the influence of the past of the factors considered.

The new model can be solved (under various circumstances) and constructed even on the above mentioned more complex conditions and the impact of particular parameters of the model on its solution can be observed.

High and unstable inflation has a negative impact on economy; therefore it must be fought against. In order to fight efficiently, we must fight in time, i.e. we must observe inflation and try to forecast it accurately, which the model can considerably contribute to.

It is to be expected that the procedure for solving the dynamic economic model, described above, that uses contemporary mathematic methods of so-called "Differential Equations Theory" with delayed argument, can be successfully used for both modelling further concrete economic relations between inflation and unemployment and for economic models in general.

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