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COMPARISON OF MERTON´S MODEL, BLACK AND COX MODEL AND KMV MODEL

Mária Mišanková[1], Katarína Kočišová[2], Tomáš Klieštik[3]

[1, 2, 3] University of Žilina, Faculty of Operation and Economics of Transport and Communications, Department of Economics, Univerzitná 8215/1, 01026 Žilina, Slovakia
Email: maria.misankova@fpedas.uniza.sk; katarina.kocisova@fpedas.uniza.sk; tomas.kliestik@fpedas.uniza.sk

Abstract. Prediction of the default of the company is an important part of the interest of investors, creditors and companies. Default is considered as an event after which the company couldn´t manage to fulfill its commitment and this result in financial losses of security holders. Identification of the probability of the default can be made by several different models. The article is dedicated to the comparison of models used for the prediction of default, namely: Merton´s model, Black and Cox model and KMV model. These models are further specified with their advantages and disadvantages as well as with their use in companies.

Keywords: Merton's model, Black and Cox model, KMV model, credit risk, default.

JEL classification: C53.

1. Introduction

Companies in today´s competitive environment and global market conditions are challenging different types of risks. They have to deal with these risks and try to predict the probability of occurance. The risk can be defined in few different ways. Basically the risk is defined as the uncertainty in which we are capable by using different, mainly mathematical and statistical methods to quantify the likelihood of diversion of the actual conditions from anticipated. (Cisko *et al*. 2013) Risk is the risk of formation of certain damage or loss associated with the risk and is always connected with some negative rating. This understanding of risk is closely related to the likelihood of negative events. Companies in a modern dynamic market environment must undergo different types of risks such as economic, financial, technical, political, business or production risk. (Lehutova 2011).

Financial risk can be defined as the potential financial loss of the company, so not existing or not realized financial loss, but the potential loss in the future resulting from the financial or commodity instrument or from the financial or commodity portfolio. Credit risk is an unseperate part of financial risk and it has become a big issue in the last few decades. (Brigho, Tarenghi 2004)

The credit risk of the company is often referred as the default risk of the company and indeed both terms are interchangeable in this paper. Default of the company is usually associated with the bankruptcy of the company. However, this is just one among several credit events. We are interested in the credit event that the company fail to meet its repayment of the debt. Although the default of the company is a rare event, once it happens, it will have signicant losses and indeed there is no way to discriminate unambiguously between that will default and those that will not prior to the default event. (Schoutens 2006) Consequently, modeling of credit risk to forecast the time is paid closed attention by many individuals and companies. Many credit rating agencies such as Standard and Poor, Fitch and Moody's were born in such a case. The main functions of these rating agencies are similar, evaluating the credit risk outlook for individual companies and assign credit ratings.

2. The basic overwiev of models used for the prediction of default

Prediction of default of the company has become an extensive topic today, not onlz because of the innovation of the credit derrivates and companies debt products. Default can be predicted, but always only with a certain degree of probability. Probability of bankruptcy can be very small, but is never equal to zero. However, if default event occurs, it often causes the lender financial losses so the identification of the likelihood of default is an important issue. (Lando 2004)

People and companies have been forecasting default for decades. Default models concentrate on the default process modeling by using the market data. These models can be divided into two groups – structural models and reduced – form models. There also exist some hybrid models that try to integrate both, the structural and the reduced–form approach. Pricing models see the debt as a defaultable zero–coupon bond or as some structure build from it. Hence the main issue is how to price a defaultable zero–coupon bond. Roots of structural models go back to the work of Black and Scholes (1973) and Merton (1974). Merton showed that stock could be considered as a call option of the company with the strike price equal to the face value of a single payment debt issue. Geske (1977, 1979) extended Merton´s analysis by showing that multiple default options for coupons, sinking funds, junior debt, safety covenants, or other payment obligations could be treated as compound options.

The basic Merton´s model is considered as a fundamental model of structural approach to credit risk modeling and it has been extended in many ways. Later it was extended by Black and Cox (1976), which allows subordination arrangements and limits on refinancing, by Turnbull (1979) who includes corporate taxes and bankruptcy costs. Far along Kim, Ramaswamy, and Sundaresan (1993) allow the riskless interest rate to follow a square root process which is correlated with the company´s value. The model showed that default risk is not particularly sensitive to the volatility of interest rates but it is sensitive to interest rate expectations. Longstaff and Schwartz (1995) also have stochastic interest rates correlated with the company process, an exogenous early default and an exogenous recovery rate. Leland (1994) and Leland and Toft (1996) proved the bankruptcy decision while accounting for taxes and bankruptcy costs. Nowadays Eom, Helwege, and Huang (2003) test the five structural models using bond prices and have found that all the structural models exhibit pricing errors, but report the options to refinance and continue.

Reduced – form models are based on an assumption that default is a rare event or posiso process. Mason and Bhattachayra (1981) allowed the company to follow a discontinuous poisson process with more complex boundary conditions, while Jarrow and Turnbull (1995) model default as a poisson event when pricing derivatives with credit risk. On the other side Duffie and Singleton (1997) consider the term structure of defaultable bonds or swaps and model the default event as an inaccessible stopping time, such as a poisson arrival. (Delianedis, Geske 2003) They proved that this is appropriate because when default occurs they are rarely anticipated even a short time before the event.

One of the most widespread approaches integrating both approaches together is the incomplete information approach started with Duffie and Lando (2001). A nice short introduction to pricing models is given in Giesecke (2004). All these pricing models try to explain spreads of defaultable zero–coupon bonds.

3. Merton´s model

In 1974, Merton proposed a model, which is based on the option pricing theory of the Black – Scholes due to the observable variables of the final function, to assess the credit risk of a company. The model links the credit risk to the capital structure of the company. This model is perhaps the most significant contribution to the area of the qualitative credit risk research. Relying on the some implicit assumption, the model assumes that equity is a call option on the value of assets of the company. From this insight, the value of debt can be derived from the equity value. By the use of Merton´s model can be calculated the probability of default event based on the proportions of the capital of the company and its liabilities. (Vašanič, Cisko 2012)

Merton suggested looking at actions or owning capital of the company as a European call option whose underlying asset is the market value of assets with an exercise price corresponding to the value of zero-coupon bonds, which represents all foreign financial sources of the company. The probability of default event of the borrower is affected mainly by the value of the underlying assets of the company and their volatility.

The probability of default can be viewed from two perspectives, an exogenous and an endogenous variable. If the probability of failure is analyzed from the exogenous view are monitored possible outside influences which can have impact on the possibility of default, such as market price changes. (Shumway, Bharath 2004)

Merton´s default model examines default as an endogenous variable. The likelihood of business failure is therefore determined by the structure of its fundamental variables and the model takes into account also the market value. Probability of default is then a key variable of the model and can be quantified as the probability of the situation that the value of the assets in the time interval falls below the default barrier. The default barrier is the nominal value of all company's obligations. The model is than based on several assumptions (Cisko, Klieštik 2013):

1. Validity of Modigiliani - Miller theorem, the value of the company is not dependent on the capital structure.
2. There are no transaction costs and taxes, company assets are infinitely divisible and all market participants are perfectly informed.
3. There is a possibility of short selling.
4. The liabilities of the company consist of one zero-coupon bond.
5. Debt structure is static, it does not change.
6. The riskiness of the investment will not be influenced by the fact how close the company is to the default.
7. Constant risk-free rate.
8. Rates for renting and lending capital are equal.
9. The dynamics of the assets value development is described by Brownian motion.
10. Dividends are not paid.
11. Development of the company assets value has lognormal distribution, it cannot be negative.
12. Absolute priority of creditors, costs associated with bankruptcy are equal to zero.
13. The absence of arbitrage in the market.
14. A company may only be declared bankrupt at the end of the time period T, in the maturity time of zero-coupon bonds.

Merton´s model assumes a public limited company, which is traded on an exchange and has issued only shares. The market value of the stock at time t is equal to the S_t. Liabilities of the company are expressed by one zero – coupon bond with maturity T and with the current market value $D(t,T)$. The nominal value of the bond is K. After issuing the bond the stock company will be funded by capital structure composed of the bond and of the value of company´s shares. This is expressed in the following Figure 1.

Assets	*Liabilities*
Market value of assets A_t	Market value of securities S_T
	Market Value of bond $D(t, T)$

Fig. 1. Balance sheet of the Merton´s model: assets (left side) and liabilities (right side) (source: Merton, R.C.: On the Pricing of Corporate Debt: The Risk Structure of Interest Rates)

From the figure one where is shown balance sheet of the Merton´s model we can assume that:

$$A_t = S_T + D(t,T). \qquad (1)$$

This equation is conditioned by the assumption that $0 \le t \le T$. There are also further assumptions: there are no bankruptcy charges, meaning the liquidation value equals the company value, the debt and equity are frictionless trade able assets.

In case of a default, the equity is useless and the remaining value of assets goes to the creditor, otherwise the debt is repaid in full amount K and the amount $A_t - K$ belongs to the stockholders. Therefore, the pay-off of the defaultable zero–coupon bond at maturity is:

$$\min\left(A_t, K\right) = K - \left(K - A_t, 0\right)^{+} \qquad (2)$$

and the pay–off of the equity is:

$$\left(A_t - K\right)^{+}. \qquad (3)$$

One can see that the bond's pay–off at maturity is the face value of the bond lowered by the pay–off of the put option on the company´s values with strike K and the pay–off of equity is the pay–off of the call option on the company´s value. This approach is often called the option theoretic approach or the company value approach. (Bielicki, Crépey, Jeanblanc, Rutkowski, 2008)

The probability of default is the probability that A_t will be below K:

$$DP = P\left[A_t < K\right] = P\left[A_0 \exp(X_t) < K\right] = P\left[X_t < \log\frac{K}{A_0}\right], \qquad (4)$$

which is equal to the cumulative distribution function Fx_t of X_t if Fx_t is continuous in the point log *(K/A₀)*. Otherwise the left limit is chosen as its value. The expected loss on the loan computed at time 0 is equal to the expected pay–off of the put option on the company´s value with respect to a real world probability measure P:

$$EL = E\left[\left(K - A_t\right)^{+}\right]. \qquad (5)$$

Recall that we assume the dynamics $A_t = A_0 \exp(X_t)$. If the Lebesgue density f of X_t exists, the expected loss is:

$$EL = \int_{-\infty}^{\infty} \left(K - A_O e^{x}\right)^{+} f\left(x\right) dx \qquad (6)$$

and the expected return of the bond is K – EL. From a pricing point of view we are interested in a fair present value of the defaultable zero–coupon bond using risk–neutral pricing techniques. So we conclude that the value of the defaultable bond with face value K at time 0 is:

$$D(0,\mathrm{t}) = K\,B(0,\mathrm{t}) - E_p * \left[\frac{1}{B_t}(K - A_t)^+\right]. \quad (7)$$

There remain a couple of difficulties to solve. First of all we need to find a risk neutral measure P*. Under the no arbitrage condition there exists exactly one risk neutral measure in a complete market. Unfortunately a complete market is rather an exception when we use a general Lévy process X_t. In an incomplete market there exist infinitely many risk neutral measures.

A widely–used approach to choose the risk neutral measure is the Esscher transform. In the case when the interest rate is a constant r and the risk neutral measure $P^* = P_{\square}$ is chosen by the Esscher transform we have:

$$\begin{aligned} &Ep * \left[\frac{1}{B_t}(K - A_t)^+\right] = \\ &E\left[\frac{\exp(\theta X_t)}{E\exp(\theta X_t)} e^{-rt}(K - A_0 \exp(X_t))^+\right], \end{aligned} \quad (8)$$

where Θ is chosen such that the discounted company value process A_t is a martingale with respect to the measure P_Θ.

In the original paper of Merton (1974), the Lévy process X_t is assumed to be:

$$X_t = \left(\mu - \sigma^2/2\right)t + \sigma W_t, \quad (9)$$

where $\mu \in \theta, \sigma > 0$, and W_t is a standard Brownian motion. Since W_t is normally distributed with expected value 0 and variance t we have equality in distribution:

$$W_t \stackrel{d}{=} \sqrt{t}Y, \quad (10)$$

where Y is a standard normally distributed variable. In that setting the default probability DP is:

$$\begin{aligned} DP &= P\left[X_t < \log\frac{K}{A_0}\right] \\ &= P\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}Y < \log\frac{K}{A_0}\right] \\ &= P\left[Y < \frac{\log\frac{K}{A_0} - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\right], \\ &= \Phi(d) \end{aligned} \quad (11)$$

where Φ is the cumulative distribution function of a standard normal distribution and:

$$d = \frac{\log\frac{K}{A_0} - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}. \quad (12)$$

The value d is often called the distance to the default. The expected loss is then:

$$\begin{aligned} EL &= \int_{-\infty}^{\infty}\left(K - A_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{tx}}\right)^+ \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}dx \\ &= K\int_{-\infty}^{d}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \\ &\quad - A_0\int_{-\infty}^{d} e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{tx}} \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx \\ &= K\Phi(d) - A_0 e^{\mu t}\Phi\left(d - \sigma\sqrt{t}\right) \end{aligned} \quad (13)$$

Under a risk neutral measure P* the company value process evolves also as a geometric Brownian motion but with a different drift equal to the risk neutral interest rate, which is the result of Black and Scholes (1973). If we assume a constant interest rate r, the process X_t is:

$$X_t = \left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t \quad (14)$$

with respect to the measure P*. Hence using equation (14), the expected loss of the bond with respect to measure P* is:

$$Ep*L = K\Phi\left(d\right) - A_0 e^{rt}\Phi\left(d - \sigma\sqrt{t}\right), \quad (15)$$

where:

$$d = \frac{\log\frac{K}{A_0} - \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}. \quad (16)$$

The price of the defaultable zero–coupon bond is the price of the risk free zero–coupon bond minus the discounted expected loss with respect to risk neutral measure P*. Therefore:

$$\begin{aligned} D\left(0,t\right) &= e^{-rt}K - e^{-rt}\left(K\Phi\left(d\right) - A_0 e^{rt}\Phi\left(d - \sigma\sqrt{t}\right)\right) \\ &= e^{-rt}K\Phi\left(-d\right) - A_0\Phi\left(d - \sigma\sqrt{t}\right) \end{aligned} \quad (17)$$

In presented case we assumed one company that is financed by equity and a bond is very simple. If we assume more companies we can easily compute the expected loss of the portfolio since the expected loss of portfolio is simply the sum of the expected losses of the particular loans. However, if we are interested in computing the unexpected loss, VAR or CVAR, of the portfolio we need to consider dependencies between the company´s value processes. Also in practice, the financial structure of the company is much more complicated than just an equity and bonds. In that case we have to take into account which liabilities of the company have higher priority and include it in the model. Determining of the loss distribution is often done by simulation. (Ammann, 2001)

4. Black and Cox model

Black and Cox in 1976 modified the Black and Scholes and Merton framework to allow for default before time T. Classic BSM framework is based on the assumption that default can only occur at maturity time and this constitutes a major limitation of this framework. Original idea of Black and Cox model focused on the possibility of a borrower violating its safety covenants; however this framework can be extended to any situation where a default barrier is hit before the maturity date. (Lehutová, Križanová, Kliеštik, 2013).

Models of this type are called first-passage-time models since they cast the default problem as one of estimating the probability and timing of the first time that company´s assets pass through the default point *(K)*, even if this occurs before time *t*. This can be explained by the right of bondholders to exercise a safety covenant that allows them to liquidate the company if at any time its value drops below the specified threshold *K(t)*. Based on this assumption the default tie is given by:

$$\tau = \inf\{t > 0 \; : \; A_t < K(t)\} \quad (18)$$

For the choice of the time dependent barrier, observe that if *K(t)* > *K* then bondholders are always completely covered, which is certainly unrealistic. On the other hand, one should clearly have Kt≤K for a consistent definition of default. One natural, but certainly not the only, choice is to take an increasing time-dependent barrier:

$$K(t) = K_0 e^{kt}, K_0 \le Ke^{-kt}. \quad (19)$$

The first passage time to the default barrier can now be reduced to the first passage time for Brownian motion with drift. Observing that:

$$\begin{aligned} &\{A_t < K(t)\} = \\ &\{W_t + \sigma^{-1}(r - \sigma^2/2 - k)t \le \sigma^{-1}\log(K_0/A_0)\} \end{aligned} \quad (20)$$

We obtain that the risk neutral probability of default occurring before time $t \le T$ is then given by:

$$\begin{aligned} Q[0 \le \tau < t] &= Q\left[\min_{s\le t}(A_s / K(s)) \le 1\}\right] \\ &= Q\left[\min_{s\le t} X_s \le \sigma^{-1}\log\left(\frac{K_0}{A_0}\right)\right] \end{aligned}, \quad (21)$$

where:

$$X_t = W_t + mt, m = \sigma^{-1}(r - \sigma^2/2 - k) \quad (22)$$

This is a classic problem of probability, whose solution is given by:

$$\begin{aligned} &Q\left[\min_{s\le t} X_t \le d\right] = 1 - FP(-d;-m,t) \\ &FP(d;m,t) = N\left[\frac{d - mt}{\sqrt{t}}\right] - e^{2md}N\left[\frac{-d - mt}{\sqrt{t}}\right]. \\ &d \ge 0 \end{aligned} \quad (23)$$

Thus we obtain the formula:

$$Q[0 \le \tau < t] = 1 - FP(-d;-m,t) \quad (24)$$

with:

$$m = \sigma^{-1}(r - \sigma^2 / 2 - k) \quad (25)$$

and

$$d = \sigma^{-1}\log(K_0 / A_0) < 0 \quad (26)$$

The pay-off for equity holders at maturity is:

$$\begin{aligned}&(A_T - K)^+ 1_{\{\min_{s\le T} X_s > d\}} \\ &= (e^{kT} A_0 e^{\sigma X_T} - K)^+ 1_{\{\min_{s\le T} X_s > d\}}\end{aligned} . \quad (27)$$

This is equivalent to the payoff of a down-and-out call option, and can be priced by Black-Scholes type closed form expressions. The equity in the Black and Cox model is smaller than the share value obtained in the Merton model, and is not monotone in the volatility. (Saunders, Allen 2002).

In the event of default, the pay-off for debt holders is $A_\tau = K(\tau)$ at the time of default, and the fair recovery value can be computed by integrating K(s), discounted, with respect to the risk - neutral PDF for the time of default. The value of the bond at time t prior to default is a sum $D_t = D_t^b + D_t^m$ of the recovery value and the value of the payment at maturity. The recovery value is thus:

$$D_t^b = \int_t^T e^{r(t-s)} K(s)(-\partial_s FP(-d_t; -m, s-t))dx, \quad (28)$$

where

$$d_t = \sigma^{-1}\log(K(t) / A_t). \quad (29)$$

The remaining term can be written:

$$D_t^m = E^Q\left[e^{-r(T-t)}\left[A_T - (A_T - K)^+\right]1_{\{\tau > T\}} \middle| F_t\right], \quad (30)$$

which is a difference of barrier call options (one with zero strike).

We can go further with the Black-Cox model and consider what happens if an additional bond is issued with face value \$1 (considered to be negligible), and maturity $T_1 < T$. In the event $\tau \le T_1$ the bond would pay the recovery fraction $R(\tau) = K(\tau)/K$, while in the event $\tau \ge T_1$ the bond pays the principal at maturity.

The implication of the Black and Cox model is that prediction of default under the more realistic assumptions will be higher than under Black and Scholes and Merton model. This make sense, since BSM model is a special case of Black and Cox model and under BC model defaults can happen under all the conditions of BSM model as well as in additional cases. (Zvaríková, 2012)

The nature of the recovery assumptions may sometimes result in counterintuitive changes in spreads relative to BSM model so the BC model results in lower spreads than BSM. If we assume that the creditor receives all the asset value at the company hits the default barrier without bankruptcy costs, then the creditor will be better off than under the BSM framework.

Difference between the BSM model and BC model is based on the European – option nature of the BSM model. The equity holder retains some option value related to the assets since default cannot occur until the maturity date of the debt. We can see higher spreads with the Black and Cox model than with a BSM framework.

5. KMV model

Model KMV was developed by Keaholfer, McQuown and Vasicek in 1974 and is based on Merton´s bond pricing model. Later in 2002 was bought by Moody´s. KMV is the name given to a successful practical implementation of structural credit modeling. They made some assumptions in order to produce commercially acceptable credit methods. The main difficulty, as in all structural models, is in assigning dynamics to the company value, which is an unobserved process. (Ammann 2001).

KMV model uses assumptions and conclusions of Merton´s work for quantification of credit risk, so the equity value of the company is also seen as a call option with the underlying asset corresponding with the value of the company and with strike price on the level of foreign sources of analyzed company. KMV model defines the failure of the analyzed company at a time when the market value of the business assets derived from the market price of the equity falls below the payable debt. (Cisko, Klieštik, 2013) For the quantification of credit risk then KMV model introduces a new variable – distance to default (DD), which indicates the number of standard deviations of the market value of the assets of the analyzed company at time t from the level of foreign capital payable at time T. On the probability of default is then applied distribution function of a standard normal distribution. (Lando 2004).

The main objective of KMV model is to predict the Expected Default Frequency which is a likelihood of the default determined by capital structure, the market value of assets and the volatility of these assets.

According to KMV model the company leads to bankruptcy, when the value of assets falls somewhere between the total value of liabilities and the value of short-term liabilities, that is the company does not have sufficient funds needed to cover liabilities with a close maturity. This point is called Default Point and in model KMV quantified as the volume of short-term debt increased by half volume of long term debt.

$$d^* = short - term\ debt + \frac{1}{2} \times long - term\ debt \quad (31)$$

Distance to default is then a number dependent on the volatility of revenues and on the distance from the median of the distribution of market prices of assets at the end of forecast horizon.

$$d_f = \frac{A_t - d^*}{\sigma_A}, \quad (32)$$

where A_t is the current market value of the company expected at the end of forecast horizon, d* is default point and σ is the annualized company´s value volatility. (Shumvay, Bharath, 2004)

By a strict structural interpretation, EDF, the expected default frequency, meaning the probability of observing the company to default within one year, ought to equal the normal probability $EDF_t = N(DD_t)$. KMV, however, breaks the model at this point and instead relies on its large database of historical defaults to map DD to EDF by a proprietary function EDF = f(DD). f(DD) is designed to give the actual fraction of all companies with the given DD that have been observed to default within one year. Studies indicate that the distance to default DD_t is a reasonable company-specific dynamic quantity that correlates strongly with credit spreads and observed historical default frequency. (Klieštik, 2008)

Assuming all requirements there can be summarized key features in KMV model:

1. Dynamics of EDF comes mostly from the *dynamics of the equity values.*
2. *Distance to default ratio* determines the level of default risk:
   - this key ratio compares the company´s net worth to its volatility,
   - the net worth is based on values from the equity market, so it is both timely and superior estimate of the company value.
3. Ability to adjust to the *credit cycle* and ability to quickly reflect any *deterioration in credit quality.*
4. Work best in highly *efficient liquid market conditions.*

6. Advantages, disadvantages and comparison of analyzed structural models

The comparison of choosen structural models is based on the definition of basic points, on comparison of the looks on the default, the time when default can occurs as well as advantages and disadvantages of each model. Comparison of these models is summarized in the table 1 in conclusion.

The Merton model assumption is that the company has a single issue of zero-coupon debt. That is unrealistic. Modeling multiple issues with different maturities and seniorities complicates default. In response some models have suggested that default occurs when the company´s assets hit a lower boundary. That boundary has a monotonic relation to the company´s total outstanding debt. The first passage time is when the value of the company´s assets crosses through the lower boundary. (Bielicki, Jeanblanc, Rutkowski, 2009)

The Merton model is only a starting point for studying credit risk, and is obviously far from realistic:

- The non-stationary structure of the debt that leads to the termination of operations on a fixed date, and default can only happen on that date. Geske extended the Merton model to the case of bonds of different maturities.
- It is incorrect to assume that the company´s value is tradeable. In fact, the company´s value and its parameters is not even directly observed.
- Interest rates should certainly be taken to be stochastic: this is not a serious drawback and its generalization was included in Merton's original paper.
- The short end of the yield spread curve in calibrated versions of the Merton model typically remains essentially zero for months, in strong contradiction with observations.

Black and Cox model is considered as a first passage model – bond indenture provisions often include safety covenants that give bond holders the right to reorganize the company if the value falls below a given barrier.

The first passage model defines the survival probability as p (t,T) that the distance to default does not reach zero at any date τ between t and T. The distance to default is often measured in terms of standard deviations.

In the figure 2 we can see the probability of default based on the classic Merton´s model who is a founder of structural models.

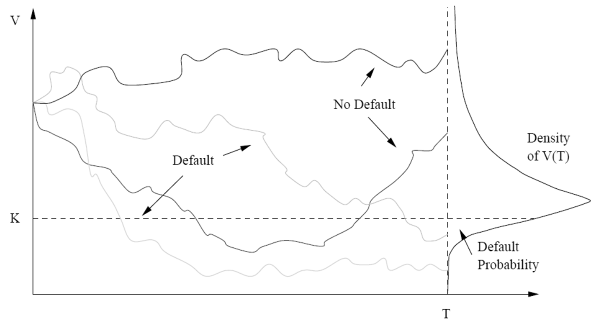

Fig. 2. Default in the classical Merton´s model (1974) (source: Merton, R.C.: On the Pricing of Corporate Debt: The Risk Structure of Interest Rates)

In the figure 3 is probability of default based on the Black and Cox model.

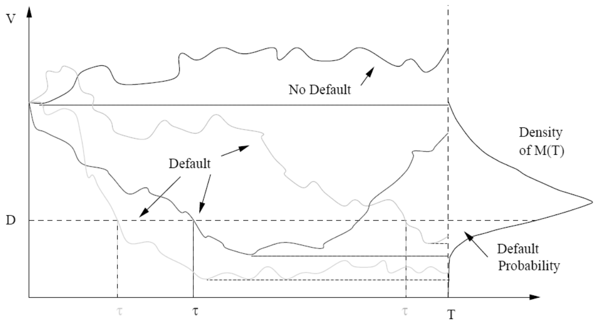

Fig. 3. Default in Black and Cox model (1976) (source: Black, F.; Cox, J.C.: Valuing corporate securities: Some effects of bond indenture provisions)

As we can see in figure 2 and figure 3 there is a difference between models in prediction of default. The main is that Black and Cox model consider probability of default in any time before maturity date while Merton´s only on maturity date. This situation is also on figures. We can see also that the density is umlimited for raise in high in Merton´s model but in Black and Cox is limited and raises until the line which divided default zone from no default zone.

The most critical inputs to the KMV model are clearly the market value of equity, the face value of debt, and the volatility of equity. As the market value of equity declines, the probablity of default increases. This is both a strength and weakness of the model. For the model to work well, both the Merton model assumptions must be met and markets must be efficient and well informed. (Shumway, Barath, 2004)

The most important implication is that KMV uses the normal distribution to define the probability default. In fact, using the normal distribution is very poor choice to define the probability of default. Firstly, let's go back to the default point. In Merton approach, the default point is a constant, and equals to the debt. However, in KMV approach, the default point is a variable; it somehow links to the repurchase or issue of debts. In particular, the company often adjusts their liabilities as they near default. Secondly, the default time is not necessary equal to the maturity time of the debt obligation; it could be any time before or at the time horizon. Indeed, market data can be updated daily because of changes in default point. Finally, the asset returns are wider tails than the normal distribution. (Bielicki, Rutkowski, 2009)

The KMV approach does not distinguish between different types of debt (bonds that vary by seniority, collateral, covenants, convertibility, etc.)

The KMV model is static, so once the debt is in place the company does not change it. The default behavior of companies that manage their leverage positions is not captured.

Strength of KMV approach (Shumway, Barath, 2004):

- accurate and timely information from the *equity market* provides a *continuous* credit monitoring process that is difficult and expensive to duplicate using traditional credit analysis,
- changes in EDF tend to anticipate at lest one year earlier than the downgrading of the issuer by rating agencies like Moody´s and S & P´s,
- annual reviews and other traditional credit processes cannot maintain the same degree of vigilance that EDF´s calculated on a monthly or a daily basis can provide,
- EDF provides a cardinal rather than ordinal ranking of credit quality.

Weaknesses of KMV approach (Lando 2004):

- it requires some *subjective estimation* of the input parameters,
- it is difficult to construct theoretical EDF without the *assumption of normality* of asset returns,
- *private companies EDF´s* can only be constructed by using accounting data and other observable characteristics of the borrower,
- it does not *distinguish* among different types of long-term bonds according to their sen-

iority, collateral, covenants or convertibility.

7. Conclusions

Measuring credit risk and forecasting tho probability of default of the company has become an important topic nowadays but with this issue people has been dealing for years.

The article is dedicated to choosen structural models. Namely Merton´s model, Black and Cox model and KMV model. These models are used for the determination of the likelihood of the default of the company. Merton´s model is considered as a fundamental model of structural models designed for the prediction of default event. From this model are other models developed.

Table 1. Comparison of selected structural models (source: compiled by authors)

	Merton´s model	Black and Cox model	KMV model
Author	Merton	Black and Cox	Keaholfer, McQuown and Vasicek
Probability of default	Only on maturity date	Any time before maturity	Any time before maturity
Risk identification	Default probability, density of V(t)	Default probability, density of M(t)	Distance to default, default probability, expected default frequency
Input data	Market data, financial data of the company	Market data, financial data of the company	Market data – price and number of stocs of the company, financial data of the company, risk – free interest rate
Numerical approach	Analytic	Analytic	Analytic
Risk rate of	Company	Company	Company
Type of the company	Publicly traded	Publicly traded	Publicly traded

These models have specific assumptions as well as advantages of its use, but on the other hand also some restriction and disadvantages. The comparison of choosen structural models is based on the definition of fundamental assumptions, on comparison of the views on the default of the company and on the time when default can occurs.

These assumptions of analysed models are important components for capturing realistic credit spread dynamics as well as distinguishing the credit quality of companies that pay out significant dividends from those that do not.

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