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THE VOLATILITY PUZZLE OF BONDS

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Abstract. In this financial engineering research we newly derive that a market price volatility of a typical coupon bond is not always deterministically decreasing during its life (investors' common concept); but we also identify a non-typical volatility development style which is characterized by a deterministic increase of volatility during its life to maturity. In addition: we also numerically calculate value of the "switching" points between these two styles with respect to the economic interest rates level and parameters of adequate bond. The purpose of this study is also to simplify for practitioners a complicated theoretical background of this portfolio management issue. The results of this research are also applicable to a bond portfolio behaviour at a certain point of time.

Keywords: bond volatility style, bond portfolio, maturity, yield to maturity, price sensitivity.

JEL classification: G1, G12.

1. Introduction

In this financial engineering research we study a bond clean price volatility development style of a typical fixed coupon rate bond without an embedded option (denoted as "bond" in the following text) with respect to the interest rate level in economy. This study is based on many interesting works regarding the general behavior of a bond (Fabozzi 1993, 1995, 2010; Smit, Swart 2006; Málek, Radová, Štěrba 2007), bonds volatility (Litterman, Scheinkman, Weiss 1991), volatility determinants (Fuller, Settle 1984), bond market price development at a time (Chance, Jordan 1966; Kang, Chen 2002; Tvaronavičienė, Michailova 2006; Křepelová, Jablonský 2013) or a portfolio of bonds behavioral (Dzikevičius, Vetrov 2013).

The main contribution of this research is to define more styles of bond volatility development with respect to the interest rates level in economy, find the "switching" value of interest rate between these styles, investigate on which parameters this value depend, find the minimum value and finally discuss an economic impact of this property of bonds on a practical portfolio management.

This research can be economically applied to one certain bond life during which its term to maturity is decreasing and the price volatility is changing. The question is then: "How does the volatility development style depend on the time to maturity, coupon rate and on the level of interest rate (yield to maturity)? " The other application may be to a certain bond portfolio with different maturities at a certain point of time.

Investors' common concept of the bond clean price (in percentage of its face value) volatility behavior is usually connected to a deterministically higher volatility while its term to maturity is also higher or in other words generally a long-term bond is more volatile than a short-term one. This common concept has been already mentioned by Fuller, Settle, 1984. According to this common notion also the volatility of a typical bond is decreasing during its life as its term to maturity is also decreasing – "typical" volatility development according to this research. But we will recognize that with respect to the interest rates level the behavior of the bond volatility may change from "typical" volatility development to "non-typical" one which is characterized by a volatility which is not always decreasing during its life to maturity.

This study is the basic research on the area of the bond price volatility and it has also certain practical importance for the portfolio managers.

2. Volatility determinants

The clean price volatility of a bond is closely connected to the changes of yield to maturity or its expectation and also to the other market factors (Litterman, Scheinkman, 1991; Steeley, 2006; Meng, Gwilym, Varas, 2009) according to models describing general market price development. Changes of dirty price with respect to the yield to maturity changes may be derived from the equation (1), where we use simple interest inside the first coupon period and compounded yield in the rest of the periods (this style of calculation is also used by US Treasury Convention, Moosmüller or Brass/Fangmayer yield):

$$P = \frac{1}{\left(1+\frac{d}{T}i\right)}\left[c+\frac{c}{(1+i)}+\frac{c}{(1+i)^2}+...+\frac{c+100}{(1+i)^{n-1}}\right], \quad (1)$$

where P is the dirty price of a bond in the percentage of its face value on purchasing day, c is the coupon rate per the coupon period, i is the yield to maturity per the coupon period, d is the number of days between the first coupon payment and the purchasing day, n is the number of coupon payments till the maturity and T is the number of days inside the coupon period.

For the special case when we purchase a bond on the day with zero accrued interest (could be for example an ex-coupon day) and the clean price equals to the dirty price we can use the formula (2) for the approximation of the required clean price development.

$$P_{clean} = \frac{c}{(1+i)} + \frac{c}{(1+i)^2} + ... + \frac{c+100}{(1+i)^n} \ . \quad (2)$$

Based on Taylor's theorem where *f(a)* is a function at point a and h is an increment value:

$$f(a+h) = f(a) + f'(a)\,h + \frac{f''(a)\,h^2}{2!} + \frac{f'''(a)\,h^3}{3!} + ... + R \, , \quad (3)$$

we obtain:

$$\Delta P_{clean} = P'_{clean}(i)\,\Delta i + \frac{P''_{clean}(i)\,\Delta i^2}{2!} + \frac{P'''_{clean}(i)\,\Delta i^3}{3!} + ... + R \ . \quad (4)$$

We use formula for ΔP_{clean} (as percentage of its face value) as the general measure of volatility:

$$\Delta P_{clean} \cong -Mac_{dur}\frac{P_{clean}}{(1+i)}\Delta i + \frac{1}{2}P_{clean}\cdot Conv \cdot \Delta i^2 \ . \quad (5)$$

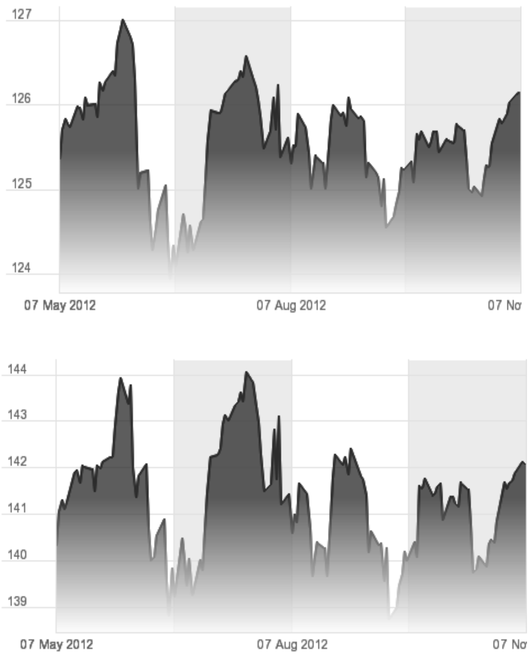
The clean price development with respect to the discrete time (only for the days with zero accrued interest) can be expressed by the Eqn. (7).

where Mac_{dur} is Macaulay's duration and $Conv$ is convexity.

$$Mac_D = \frac{\sum_{k=1}^{n}\frac{k\cdot c}{(1+i)^k} + \frac{n\cdot 100}{(1+i)^n}}{P_{clean}} \ ,$$

$$Conv = \frac{P''_{clean}(i)}{P_{clean}} \ . \quad (6)$$

From the formula (5) it is evident that the Macaulay's duration is not the only determinant of volatility which is often commonly accepted but ΔP depends also on the interest rate level.



a

b

Fig. 1. Volatility comparing of 5 (a) and 10-years (b) futures contracts (Euro-Bobl, Euro-Bund Futures) (source: EUREX)

Figure 1 supports the mentioned commonly accepted idea where two times higher Macaulay's duration causes an approximately two times higher ΔP. Such a situation is supported by formula (5) if the price and the yield to maturity are of similar values. This feature is observable in today's low levels of interest rates. What may happen if the interest rate level is increasing will be discussed in the following text.

3. Bond volatility development during its life

$$P_{clean} = c\cdot\frac{1-(1+i)^{-(n-t))}}{i} + \frac{100}{(1+i\ )^{(n-t)}} \ . \quad (7)$$

The first and the second derivatives with respect to the discrete time (equations 8, 9) describe the shape of the curve.

$$P'_{clean} = \left(100 - \frac{c}{i}\right) \cdot (1+i)^{t-n} \cdot \ln(1+i) \;; \quad (8)$$

$$P''_{clean} = \left(100 - \frac{c}{i}\right) \cdot (1+i)^{t-n} \cdot \ln^2(1+i) \;. \quad (9)$$

There is, by the way of an example, the clean price development in the figure 2 of 20 years bond, fixed coupon rate 10% p.a., coupon frequency equals one year and we assume the constant yield to maturity 3% p.a. over the whole time till maturity (Stádník, 2012). The clean price development is denoted in the figure as "constant yield to maturity curve" for yield to maturity equals 3% p.a. The time period in the picture is from the purchase day up to 14 years.

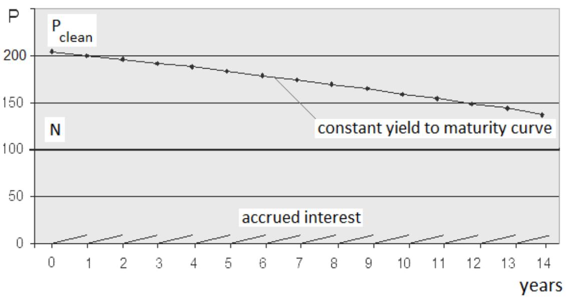


Fig. 2. Clean price development in case of a constant yield to maturity (source: Stádník 2012).

We can observe the Pull to Par effect there.

The development according to the figure 2 is unrealistic because in reality the yield to maturity is changing at time and differs from the purchase yield. For example in the figures 4, 5 the initial purchase clean price starts at certain point P_p. If the yield to maturity is not changing till maturity the chart will be the smooth line starting from P_p ("constant yield to maturity curve"). When the value of the yield to maturity is changing the clean price deflects from the "constant yield to maturity curve". We expect the volatility to be higher with the higher clean price/yield sensitivity and lower with the sensitivity decreasing.

If we consider an example (according to the figure 3) of the bond with coupon rate of 5 %, maturity 90 years and expected yield to maturity to be changing between 2 % and the value of "switching" point (the point will be exactly defined later) then the volatility development at the time is symbolically figured in the figure 4. The feature of this development style can be derived from the figure 3 in the following way.

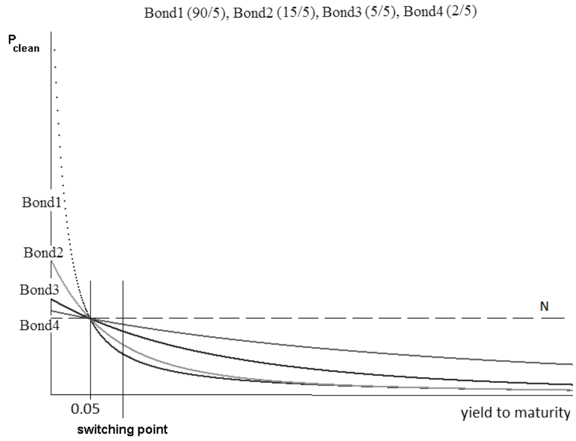


Fig. 3. Clean price with respect to yield to maturity

From the figure 3 it is visually evident (without any mathematical approach) that if the yield to maturity is lower than "switching" point, ΔP is increasing with respect to the higher term to maturity while Δi remains the same. Our 90 years bond has 90 years to maturity on the purchase day and its clean price/yield characteristic is given by the curve for "Bond1" in the figure. After 75 years, when the bond has 15 years to maturity, its price/yield characteristic is given by the curve for "Bond2" and while decreasing the time till maturity the characteristic is given analogically by the "Bond3" and "Bond4" curves. From the picture it is evident that the clean price/yield sensitivity is the highest for "Bond4" (the slope of the curve is the highest according to the figure 3) then for "Bond3", "Bond2" and the lowest is for "Bond1". Such sensitivity changes during the bond life with its impact on volatility are shown in the Figure 4 – "typical" volatility development. In the figure we assume that yield to maturity varies between 2% and the value of "switching" point.

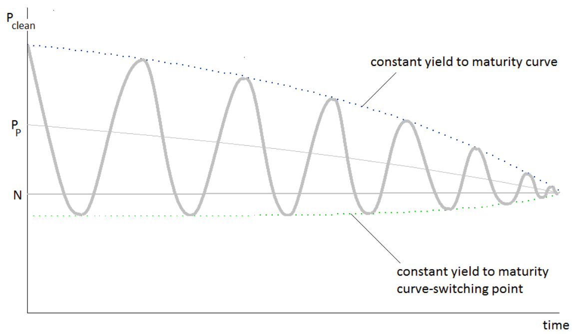


Fig. 4. Symbolic image of "typical" clean price volatility development

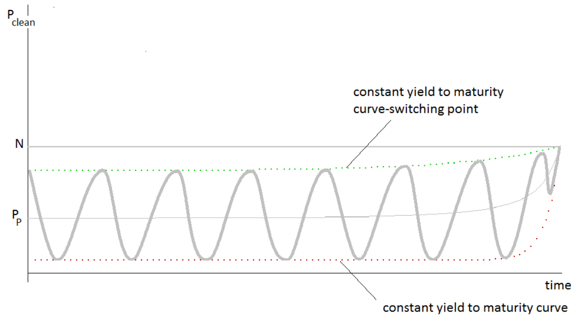


Fig. 5. Symbolic image of clean price "non-typical" volatility envelope

If we consider the level of interest rates to be higher than "switching" point during the bond life then clean price/yield sensitivity is evidently the highest for "Bond2" curve then for "Bond1", "Bond3", and the lowest is for "Bond4". Such situation is shown in the figure 5 - "non-typical" volatility development. In the figure we assume that yield to maturity varies between "switching" point and certain higher yield and thus creates "non-typical" volatility envelope. The volatility development is characterized by initial volatility increasing and by its decreasing at the end of the life of the bond.

4. "Switching" point of volatility styles definition

For the practical assessment we try to find the value of interest rate level where "typical" volatility development during a bond life is changing to a "non-typical". Based on figure 3, we feel there is some minimum value of interest rate level where "typical" volatility development regime switches to "non-typical" one. Let's define the minimum interest rate level of "switching" between "typical" and "non-typical" volatility development as the "switching". In other words we can state that if the interest rate level reaches from some lower value "switching" point the sensitivity of a bond clean price starts to increase while it was decreasing until this moment. We define the "switching" point as the lowest values due to the possible practical impact with respect to the real level of the interest rates in economy.

The "switching" point is given as the solution on the lowest realistic i from one or more inequalities from the set of inequalities (10) where on the left and right sides of the inequalities there are the first derivatives of the clean price according to i, so we use only the first term of the right side of the equation (4). For a one certain bond life we state $c_1=c_2$, n is the number of coupon periods till maturity and $n=m+1$.

1.

$$-\frac{1\cdot c_1}{(1+i)^2}-\frac{2\cdot c_1}{(1+i)^3}-\ldots-\frac{m\cdot(c_1+100)}{(1+i)^{m+1}}\leq$$
$$-\frac{1\cdot c_2}{(1+i)^2}-\frac{2\cdot c_2}{(1+i)^3}-\ldots-\frac{n\cdot(c_2+100)}{(1+i)^{n+1}}$$

2.

$$-\frac{1\cdot c_1}{(1+i)^2}-\frac{2\cdot c_1}{(1+i)^3}-\ldots-\frac{(m-1)(c_1+100)}{(1+i)^{(m-1)+1}}\leq$$
$$-\frac{1\cdot c_2}{(1+i)^2}-\frac{2\cdot c_2}{(1+i)^3}-\ldots-\frac{(n-1)(c_2+100)}{(1+i)^{(n-1)+1}}$$

3.

$$-\frac{1\cdot c_1}{(1+i)^2}-\frac{2\cdot c_1}{(1+i)^3}-\ldots-\frac{(m-2)(c_1+100)}{(1+i)^{(m-2)+1}}\leq$$
$$-\frac{1\cdot c_2}{(1+i)^2}-\frac{2\cdot c_2}{(1+i)^3}-\ldots-\frac{(n-2)(c_2+100)}{(1+i)^{(n-2)+1}}$$

. ...

. ...

. ...

n-1

$$-\frac{(c_1+100)}{(1+i)^2}\leq-\frac{1\cdot c_2}{(1+i)^2}-\frac{2\cdot(c_2+100)}{(1+i)^3}\quad. \qquad (10)$$

From the set of the inequalities (10) we are also able for example to state what is the minimum value of interest rates where the bond with higher maturity starts to be more volatile than a bond with lower maturity.

In the table 1 we have numerically solved "switching" points. We consider the yield to maturity to be changing between 0 and 100% p.a. and coupon rates 1, 3, 5, 7, 9 % p.a.

We can see that for higher maturities the "switching" point is of lower value than in case of lower maturities which is in accordance with Fuller, Settle, 1984. We can state also that for higher maturities the "switching" point has a certain practical value.

Table 1. "Switching" points for different maturities and coupon rates

maturity	scwitching point [%]				
	c=1	**c=3**	**c=5**	**c=7**	**c=9**
1	-	-	-	-	-
2	-	-	-	-	-
3	51.6	54.5	57.5	60.5	63.5
4	34.7	37.4	40.1	42.7	45.4
5	26.5	28.8	31.3	33.8	36.3
6	21.3	23.6	26.1	28.4	30.8
7	18.1	20.2	22.5	24.9	27.2
8	15.5	17.8	20.0	22.3	24.6
9	13.7	15.9	18.2	20.4	22.7
10	12.3	14.5	16.7	19.3	21.2
11	11.1	13.3	15.5	17.7	19.9
12	10.1	12.4	14.6	16.8	19.0
13	9.1	11.6	13.9	16.0	18.2
14	8.8	11.0	13.1	15.3	17.4
15	8.3	10.4	12.6	14.7	16.8
20	6.4	8.5	10.6	12.7	14.8
25	5.3	7.3	9.4	11.5	13.6
30	4.5	6.6	8.7	10.7	12.8
35	4.0	6.1	8.1	10.2	12.3
40	3.6	5.7	7.7	9.8	11.8
45	3.3	5.4	7.4	9.5	11.5
50	3.1	5.2	7.2	9.2	11.3
55	2.9	5.0	7.0	9.0	11.1
60	2.8	4.8	6.8	8.9	10.9
65	2.6	4.7	6.7	8.7	10.8
70	2.5	4.5	6.6	8.6	10.6
75	2.4	4.4	6.5	8.5	10.5
80	2.3	4.4	6.4	8.4	10.4
85	2.3	4.3	6.3	8.3	10.3
90	2.2	4.2	6.2	8.3	10.3
95	2.1	4.1	6.2	8.2	10.2
100	2.1	4.1	6.1	8.1	10.2

5. Volatility regimes "switching" inside a bond portfolio

In the chapter 3 we were assuming one bond and its life through which its term to maturity is decreasing. The situations according to the figures 4, 5 can be also applied to a bond portfolio containing four bonds with maturity 90, 15, 5 and 2 years to maturity (figure 3). Within the meaning of the chapter 3 it is obvious that the result of comparing of mutual volatility depends on the level of interest rates.

If we consider a portfolio consisting of long and short-term bonds then the "switching" point (the volatility of a long-term bond starts to be lower than the volatility of a short-term bond with respect to the interest rate level) is given by table 2 and 3. In the tables we consider short-term bonds with maturities 1 and 2 years and fixed coupon rates 1% and 5% p.a. and long bonds with maturities 15-100 years and also perpetuity bond. The coupon rate of the long-term bond is 1% p.a. in the table 2 and 5 % p.a. in the table 3. We consider the yield to maturity to be changing between 0 and 100% p.a.

Table 2. "Switching" point for long and short bond portfolio

Marurity of long bond	Coupon rate of long bond	scwitching p. [%], mat of short bond =1		scwitching p [%], mat of short bond =2	
		c=1	**c=5**	**c=1**	**c=5**
15	**c=1**	23.5	23.1	18.4	17.8
16	**c=1**	22.5	22.2	17.7	17.1
17	**c=1**	21.6	21.3	17.0	16.5
18	**c=1**	20.8	20.5	16.4	15.9
19	**c=1**	20.1	19.8	15.9	15.4
20	**c=1**	19.6	19.2	15.4	15.0
30	**c=1**	15.3	15.1	12.1	11.8
40	**c=1**	13.3	13.1	10.5	10.2
50	**c=1**	12.1	12.0	9.5	9.2
60	**c=1**	11.6	11.4	8.8	8.6
70	**c=1**	11.3	11.1	8.4	8.2
80	**c=1**	11.1	11.0	8.2	8.0
90	**c=1**	11.1	10.8	8.0	7.8
100	**c=1**	11.1	10.8	8.0	7.7
PERP	**c=1**	11.1	10.8	7.9	7.6

The calculations are based on the equation (11) where n is term to maturity of the first bond and m is term to maturity of the second bond, $n>m$. We find "switching" point as the minimum realistic i solving the inequality (11).

$$\frac{-\frac{1\cdot c_1}{(1+i)^2}-\frac{2\cdot c_1}{(1+i)^3}-\ldots-\frac{m\cdot(c_1+100)}{(1+i)^{m+1}}}{-\frac{1\cdot c_2}{(1+i)^2}-\frac{2\cdot c_2}{(1+i)^3}-\ldots-\frac{n\cdot(c_2+100)}{(1+i)^{n+1}}}\leq . \quad (11)$$

Table 3. "Switching" points for long-short bond portfolio

Marurity of long bond	Coupon rate of long bond	scwitching p. [%], mat of short bond =1		scwitching p. [%], mat of short bond =2	
		c=1	**c=5**	**c=1**	**c=5**
15	**c=5**	33.5	32.8	26.1	25.0
16	**c=5**	32.7	32.0	25.4	24.4
17	**c=5**	32.1	31.4	24.8	23.9
18	**c=5**	31.4	30.8	24.3	23.4
19	**c=5**	31.2	30.0	23.9	23.0
20	**c=5**	30.6	29.9	23.5	22.6
30	**c=5**	28.9	28.2	21.5	20.7
40	**c=5**	28.7	28.0	21.0	20.2
50	**c=5**	28.6	27.9	20.9	20.0
60	**c=5**	28.6	27.9	20.8	20.0
70	**c=5**	28.6	27.9	20.8	20.0
80	**c=5**	28.6	27.9	20.8	20.0
90	**c=5**	28.6	27.9	20.8	20.0
100	**c=5**	28.6	27.9	20.8	20.0
PERP	**c=5**	28.6	27.9	20.8	20.0

6. Economic impact of "switching" of volatility regimes (brief view)

Regimes "switching" point do have an impact on volatility development style at the time and also on many practical activities connected to this issue. For example, the higher price changes of 10-years futures price compared to 5-years (figure 1) become conversely lower if the interest rates level increases over the "switching" point. We can also assume that for example the value of futures market minimum price ticks, which are closely connected to the measured volatility, can be switched between short and long-term maturities with respect to the level of interest rates in economy. Also margin levels can be switched in the same way. Other financial market activities which are closely connected to sensitivity and volatility as for example hedging or speculations priorities should be significantly influenced by the reaching of "switching" point.

7. Conclusions

In this research we have defined different regimes of a common bond clean price volatility development style with respect to its practical usage. We have found the values of "switching" points between these regimes with respect to the interest rate level using numerical calculations, we have investigated on which parameters its value depends and finally briefly discussed an economic impact of this property of bonds and its practical value.

In the table 1 we have numerically solved "switching" points for maturities from 1 up to 100 years. We can see that the "switching" point (between regime of the "typical" development according to the figure 4 and other regimes) is of lower value for higher maturities, which is also in accordance with Fuller, Settle, 1984. We can also state that for higher maturities the "switching" point has its practical value within the meaning of today's interest rate level (Visokavičien 2008; Janda, Svárovská 2010; Žďárek 2009; Rutkauskas, Stasytytė, Maknickienė 2014). If the clean price of a bond is developing inside the volatility envelopes according to the figures 4, 5 its sensitivity (volatility) is increasing/decreasing according to the shape of the envelope in the figure.

Within the meaning of the chapter 5 it is obvious that the comparison of mutual volatility of bonds inside a certain portfolio also depends on the level of interest rates. If we consider a portfolio consisting of long (and also perpetuity) and short-term bonds then the "switching" point is given by table 2 and 3.

From the tables 1 and 2 we can also recognize that the values of "switching" points increase with the higher coupon rate.

This study is a basic research on the area of the bond price volatility but from the table 1 we can see that it can have a certain practical value for the portfolio managers. For example the "switching" point for the higher maturities and the lower coupon rates is very close to the realistic yields (2–3%).

References

Chance, D. M.; Jordan, J.V. 1966. Duration, convexity, and time as components of bond returns, *The Journal of Fixed Income*, 88–96.

Dzikevičius, A.; Vetrov, V. 2013. Investment Portfolio Management Using the Business Cycle Approach, *Bussiness: Theory and Practice/Verslas: Teorija ir Praktika* 14(1).

Fabozzi, F. J.; Fabozzi, T. D. 1995. *The Handbook of Fixed Income Securities*, Fourth Edition, Irwin Professional Publishing.

Fabozzi, F. J. 2010. *Bond Markets, Analysis and Strategies*, seventh edition, Pearson.

Fabozzi, F. J. 1993. *Fixed Income Mathematics*, Chicago: Probus, Publishing Company.

Fuller; Settle. 1985. Determinants of Duration and Bond Volatility, *Journal of Portfolio Management* 10(4): 66–72.

Janda, K.; Svárovská, B. 2010. Investing into microfinance, *Journal of Business Economics and Management* 11(3): 483–510. http://dx.doi.org/10.3846/jbem.2010.24

Kang, J. C.; Chen, A. H. 2002. Evidence on theta and convexity in treasury returns, *The Journal of Fixed Income* June: 41–50. http://dx.doi.org/10.3905/jfi.2002.319317

Křepelová, M.; Jablonský, J. 2013. Analysis of Government Bonds as an Indicator for Stock Market, *Politická ekonomie* 5: 509–543.

Litterman, R.; Scheinkman, J.;Weiss, L. 1991. Volatility and the yield curve, *The Journal of Fixed Income*, June: 49–53. http://dx.doi.org/10.3905/jfi.1991.692346

Litterman, R.; Scheinkman, J. 1991. Common factors affecting bond returns, *The Journal of Fixed Income*, June: 54–61. http://dx.doi.org/10.3905/jfi.1991.692347

Málek, J.; Radová, J.; Štěrba, F. 2007. Yield curve construction using government bonds in the Czech Republic, *Politická Ekonomie* 6(200): 792–808.

Meng, L. A; Gwilym, O. A. B.; Varas, J. C.2009. Volatility transmission among the CDS, equity, and bond markets, *Journal of Fixed Income* 18(3): 33–46. http://dx.doi.org/10.3905/JFI.2009.18.3.033

Rutkauskas, A. V.; Stasytytė, V., Maknickienėa, N. 2014. Government debt as the integral portfolio of assets and liabilities generated by debt, *Journal of Business Economics and Management* 15(1): 22–40. http://dx.doi.org/10.3846/16111699.2013.815129

Smit, L.; A, Swart, B. B. 2006. Calculating the price of bond convexity, *Journal of Portfolio Management* 32(2): 99–106. http://dx.doi.org/10.3905/jpm.2006.611809

Stádník, B. 2012. *Theory and Praxis of Bonds I.* Oeconomica, University of Economics, Prague.

Steeley, J. M. 2006. Volatility transmission between stock and bond markets, *Journal of International Financial Markets, Institutions and Money* 16(1): 71–86. Aston Business School, Aston University, Birmingham B4 7ET, United Kingdom.

Tvaronavičiene, M.; Michailova, J. 2006. Factors affecting securities prices: Theoretical versus practical approach, *Journal of Business Economics and Management* 7(4): 213–222.

Visokavičienė, B. 2008. Money Supply and Assets Value, *Bussiness: Theory and Practice/ Verslas: Teorija ir Praktika* 9(3): 210–214.

Žďárek, V. 2009. Moderní způsoby produkce a přímé zahraniční investice, *Politická ekonomie* 4: 509–543.