

Prediction of Hydraulic Conductivity of Fly Ash Built-in Mineral Sealing Layers

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Abstract. Mineral barrier protects the groundwater and soil from contamination by insulating the leakage of harmful substances from landfill. One of the most important parameters, which decides about usefulness of material to built-in sealing layers, is hydraulic conductivity. Researchers have conducted investigations with the possibility of utilising fly ash as a mineral sealing layer material, which is justified by its low permeability and other properties. It is known that laboratory tests of hydraulic conductivity are often long-term and require expensive equipment. Therefore, to avoid this, researchers trying to assess permeability of tested material with empirical or semi empirical formulas.

The aim of the paper is to compare the results of hydraulic conductivity of fly ash obtained from the laboratory tests and from estimation using different empirical formulas. Fly ash was compacted by the Standard Proctor compaction method at the optimum moisture content.

The results obtained from empirical equations were variable. It was observed that the Kozeny-Carman formula and other, based on a few physical parameters of the soil, gave better results in prediction of hydraulic conductivity of fly ash than equations based on only one parameter.

Keywords: fly ash, hydraulic conductivity, empirical equations.

Conference topic: Environmental protection.

Introduction

Using of combustion by products, instead of natural soils, brings profits to natural environment, because of decrease the value of the landfills. It used to be more common nowadays, that combustion by-products are conducted to be reused. Mineral sealing layers are very important component of municipal and industrial landfill. Their major function is to prevent the seepage of harmful substances to the ground water.

Flow of water through the soil has been described by Darcy's law according to equation (1):

$$v = k \cdot i, \quad (1)$$

where: k is coefficient of permeability (or hydraulic conductivity) as proposed by Darcy, i – hydraulic gradient $i = \frac{\Delta H}{l}$, where ΔH is the difference in elevation of water levels in the piezometers and l is the length of the flow path.

The volume flow rate of a liquid which is a measure of the volume of fluid that pass through an area per unit time is expressed by equation (2):

$$Q = k \cdot i \cdot A, \quad (2)$$

where: A is the cross-section area.

Saturation of the soil is described by the change in pore pressure Δu associated with various changes in total stress, which is expressed by the pore-pressure equation given by Skempton (1954):

$$\Delta u = B \left[\Delta \sigma_3 + A (\Delta \sigma_1 - \sigma_3) \right], \quad (3)$$

where: A and B are pore-pressure coefficients, $\Delta \sigma_1$ and $\Delta \sigma_3$ are changes in the principal stresses. A and B coefficients are measured experimentally in the undrained triaxial test. Saturation of the soil is assessed using pore pressure parameter B :

$$B = \frac{\Delta u}{\Delta \sigma}, \quad (4)$$

where Δu is change in pore water pressure corresponding to increase of total stress $\Delta \sigma$. In saturated soils (zero air voids) $B = 1$, while for dry soils $B = 0$.

In the literature k is described as hydraulic conductivity. The value of hydraulic conductivity depends on features and properties of the material and the fluid, which flows through the material. The parameters like: density and viscosity of the fluid, grain-size distribution and porosity of the soil (Cartwright, Hensel 1997) should be determine.

To determine the usefulness of fly ash to mineral layers, hydraulic conductivity has to be tested. Many predictive equations has been given for porous media so far. Formulas subject the fluid flow in the soil to porous space, for example the size of the pores, tortuosity and connections beetwen pores.

The paper attempts to assess the hydraulic conductivity of quasi-saturated fly ash, which were compacted at the optimum moisture content using the Standard Proctor test (PN-88/B-04481).

In saturated materials all of the void spaces are usually filled with water. In the case of some soil types it is claimed, that full saturation of the sample is given, when Skempton's parameter B obtains value near 1. This condition is considered to be unjustified (Shahu *et al.* 1999). For fly ash it is determined, that $B = 0.8$ can be considered as full saturation of the sample (Zabielska-Adamska 2006).

The values derived from the equations were compared to the ones obtained from laboratory tests. The test of hydraulic conductivity were conducted on previously consolidated fly ash samples.

Empirical formulas for k value calculation

The methods for predicting k value are considering soil properties, such as: porosity n (or void ratio e), grain-size distribution curve and Atterberg limits. The hydraulic conductivity depends on the size of the pores, their distribution and connectivity. A well known relationship is the one proposed by Kozeny and improved by Carman, called as the Kozeny-Carman equation. These two authors, although they never published together, were interested in permeability of industrial powders to determine their specific surface. At a time when the determination of specific surface was time-consuming and gave inaccurate results, a permeability test required less time and allowed to determine quality of an industrial powders (Chapuis, Aubertin 2003b). The Kozeny-Carman equation has taken several forms, for example (Chapuis 2012):

$$k = C \frac{g}{\mu_w \rho_w} \frac{e^3}{S_s^2 G_s^2 (1+e)}, \quad (5)$$

where: C is a constant which depends on the shape and tortuosity of porous space, $C = 0.2 - 0.5$, g is the gravitational constant (m/s^2), μ_w is the dynamic viscosity of water ($Pa \cdot s$), ρ_w is the density of water (kg/m^3), G_s is the specific gravity of solids ($G_s = \rho_s / \rho_w$), S_s is the specific surface (m^2/kg) and e is the void ratio.

Carrier (2003) presented the Kozeny-Carman equation in following form:

$$k = \frac{\gamma}{\mu} \frac{1}{C_{K-C}} \frac{1}{S_0^2} \frac{e^3}{(1+e)}, \quad (6)$$

where: γ is unit weight of permeant, μ is viscosity of permeant, C_{K-C} is the Kozeny-Carman empirical coefficient, S_0 is specific surface area per unit volume of particles ($1/cm$), e is void ratio. In case, when the fluid is water at $20^\circ C$, $\gamma/\mu = 9.93 \cdot 10^4$ $1/cm \cdot s$. According to Carman's arrangements, Carrier took the coefficient value $C_{K-C} = 5$, for uniform spheres and received simplified form of the equation (4):

$$k = 1.99 \cdot 10^4 \frac{1}{S_0^2} \frac{e^3}{(1+e)}. \quad (7)$$

In practice the Kozeny-Carman formula isn't commonly used because of difficulties in determining the specific surface of material. Chapuis and Aubertin (2003b) found, that the (5) equation is valid for sands, but is inadequate for clays. Carman (1939) found that experimental value of hydraulic conductivity is not constant, but it depends on the porosity n . He ascribed divergence to a thin water layer that would be present at the surface of clayey particles. The equation dose not take into consideration the interaction between soil and fluid in clayey particles. Taylor (1948) confirmed Carman's conclusion adding, that there is no method to check the water film thickness.

A ccording to equation (5), Chapuis and Aubertin (2003a) proposed the relationship which could be valid for both – plastic and non plastic soils:

$$\log(k) = A + \log\left(\frac{e^3}{S_s^2 G_s^2 (1+e)}\right), \quad (8)$$

where: A is equal to 0.29–0.51 for a C value between 0.2 and 0.5.

Mbonimpa *et al.* (2002) gave different formulas for clayey materials (9) and granular materials (10), which use, similar to the Kozeny-Carman equation, soil physical properties:

$$k = C_p \frac{\gamma_w}{\mu_w} \frac{e^{3+x}}{1+e} \frac{1}{\rho_s^2 w_L^2 \chi}; \quad (9)$$

$$k = C_G \frac{\gamma_w}{\mu_w} \frac{e^{3+x}}{1+e} C_U^{1/3} D_{10}^2, \quad (10)$$

where: C_G is constant proposed for granular materials, can be fixed at 0.1 – it is based on tests results; x is parameter introduced in the void ratio function to take into account the effect of tortuosity of the porous material ($x = 2$); C_U is uniformity coefficient (D_{60}/D_{10}), D_{10} is effective diameter, corresponding to 10% passing on the cumulative grain-size distribution curve (cm); C_p is constant proposed for plastic/cohesive soils, $C_p = 5.6 \text{ g}^2/\text{m}^4$; χ is material parameter, $\chi = 1.5$, x is parameter introduced the void ratio function for plastic and cohesive soils, $x = 7.7w_L^{-0.15} - 3$. The values derived from equation (10) were different than value derived from research of clayey soils on about half order of magnitude.

Chapuis (2004) analysing the Kozeny-Carman formula and related to Hazen's equation $k(\text{cm/s}) = D_{10}^2 (\text{mm}^2)$, gave another, different relationship for hydraulic conductivity of sand and gravel:

$$k = 2.4622 \left(\frac{D_{10}^2 e^3}{1+e} \right)^{0.7825}. \quad (11)$$

Assessment of empirical formulas from the available technical literature

Steiakakis *et al.* (2012) have evaluated the reliability of the Kozeny-Carman equation for compacted clayey soils. The authors compared results from the laboratory test to the ones, derived from the equation. Research were conducted on the sand-kaolin mixtures, compacted at 2% wet of optimum moisture content, using the Standard Proctor method. Hydraulic conductivity of mixtures of sand with addition of 70, 50, 30 and 15% of kaolin were tested. It was observed, that the differences between values obtained from the laboratory tests and predicted using the Kozeny-Carman equation has been exposed for clayey mixtures, in which the kaolin content was 30% or more. The researchers concluded the divergens between predicted and measured values of the hydraulic conductivity may be attributed to incomplete sample saturation or to theoretical limitations of the equation. They stated that the Kozeny-Carman formula gave good estimation of hydraulic conductivity.

Salarashayeri and Siosemarde (2012) tried to determine relationship between hydraulic conductivity and grain size distribution curve by using in equations soil particle diameter (mm) that 10, 50 and 60% of all soil particles are finer by weight – d_{10} , d_{50} and d_{60} . The authors based the formulas on the set of 25 fully saturated sand samples. Comparing results received from the laboratory tests and from the equations allowed to get the R^2 from 0.09 to 0.52.

Taylor (1948) performed hydraulic conductivity tests on fine grained soil samples in two types of apparatus and illustrated the relationship between e i $\log k$. He used the Kozeny-Carman equation, but not complete – without specific surface and constant, C parameter. The observed scattering of the points on the graph was described as probable difference due to water saturation of samples and internal structure of compacted clays. In spite of this scattering, it can be seen the linear relationship on the wide range of the more than tree cycles of the permeability scale (Taylor 1948).

Chapuis and Aubertin (2003b) have listed laboratory tests results of various types of soils including sand, clay, bentonite, kaolin, from many publications. The authors have predicted the hydraulic conductivity of given soils using the Kozeny-Carman formula and its modifications. It has been concluded, that the equation may be used to estimate k value of various types of soils in cases, when it is possible do evaluate properly the specific surface and when the laboratory tests of hydraulic conductivity are performed accurately.

Chapuis (2004) have collected laboratory tests results of hydraulic conductivity of sands and gravels from various publications and values calculated for this soils using equation (9). He obtained $R^2 = 0.91$, which indicates that the equation suits well for those types of soil.

Szymkiewicz and Kryczalfo (2011) summarized various equations using to assessment the hydraulic conductivity of sands and gravels. Based on common features, authors grouped formulas. The k values for two representative grain-size distribution curves – medium sand and gravel – has been calculated. The authors concluded, that the Kozeny-Carman formula is one of the most universal, due to fact, that equation taking into account characteristics of water flowing through the porous media and soil properties. However, the authors pointed out the necessity of checking the influence of particle size heterogeneity on the values of parameters in the equation.

Properties of tested fly ash

Tests of hydraulic conductivity were carried out on fly ash and bottom ash mixture from from hard-coal combustion at the Bialystok Power Plant, stored in dry disposal site, where bottom ash is up to 10% of the stored wastes. Hence,

due to the low content of slag in the mixture, in the paper, the mix is referred to as fly ash. Grain-size distribution of fly ash corresponds to sandy silt (saSi) which was shown in Fig. 1. Chosen physical parameters of material, like specific surface obtained according to Polish Standards (PN-88/B-04481), have been presented in the Table 1. The specific surface area tests were performed using the methylene blue dye adsorption method. Method is based on the assumption, that the adsorbed particles form a monomolecular layer. Specific surface area of solids is defined as total surface area per unit of mass of soil, expressed in units of m^2/g . Specific surface area determination using equations from Chapuis and Aubertin's (2003a) publication or using Polish Standards method (PN-88/B-04481) is only an estimation based on grain-size and soil particle density and porosity.

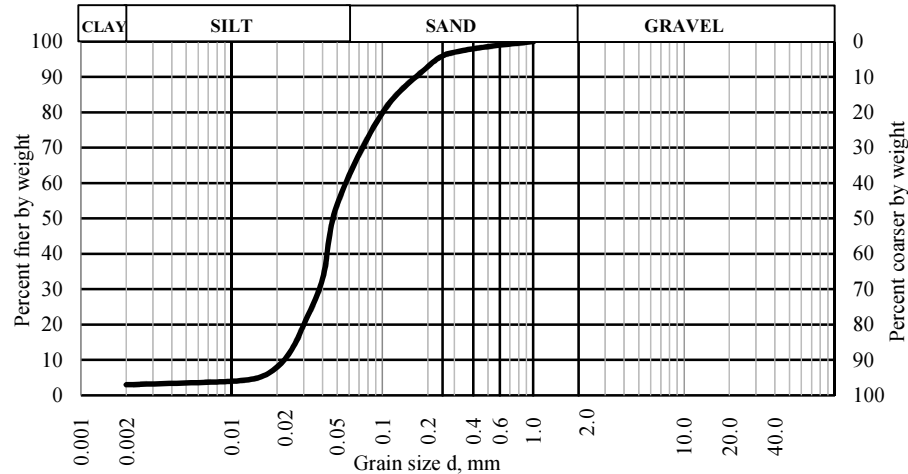


Fig. 1. Grain-size distribution curve of the fly ash (source: own elaboration)

These methods are less accurate and they are not taking into account specific structure of fly ash particles, which are predominantly spherical in shape and hollow.

Figure 2 presents micrograph taken using Scanning Electron Microscope in 1,600 magnification. It can be seen, that spherical grains of various sizes are predominant in the sample.

Table 1. Geotechnical parameters of the mixture determined using on Polish Standards (PN-88/B-04481) (source: own elaboration)

D_{50} (mm)	C_u (-)	S_s (m^2/g)	C_c (-)	ρ_s (g/cm^3)	Compaction – Standard Proctor Test	
					w_{opt} (%)	ρ_{dmax} (g/cm^3)
0.045	2.17	2.47	1.22	2.12	44.00	0.996

Hydraulic conductivity laboratory tests were performed in consolidation cell (the Rowe type). Samples, with 15 cm in diameter and 5 cm height, have been saturated using back pressure method until Skempton's parameter B reaches value equal or higher than 0.8. More specific details of laboratory tests and description of Rowe cell is located in earlier publication (Wasil 2012). Water flow tests were conducted at effective stress σ' : 25, 50, 100 i 200 kPa and with hydraulic gradient $i = 12$.

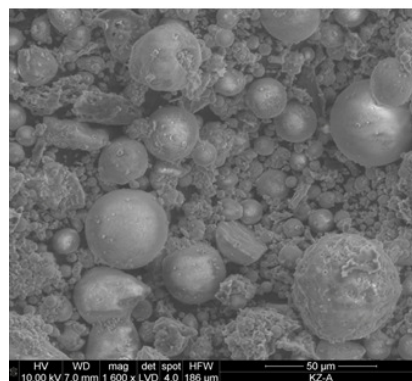


Fig. 2. Scanning electron micrograph of fly ash from Bialystok Power Plant in 1,600 magnification (source: own elaboration)

Assesment of hydraulic conductivity of fly ash using empirical equations

The Kozeny-Carman equation predicts (Chapuis 2012), that for given soil should exist linear relationship between k and $e^3/(1+e)$. To check that statement, graph shown in Fig. 3 has been plotted. A linear relationship with high value of $R^2 = 0.9531$ has been attained. Therefore, the attempt to calculate hydraulic conductivity using the Kozeny-Carman equation in commonly used form (5) and equation (8) was made. Received results have been compared to ones obtained in laboratory tests. Figure 4 plots values of hydraulic conductivity based on equation (5). Chapuis and Aubertin (2003a) posted, that constant C for sands should have value between 0.2 and 0.5. Fly ash is material with spherical particles of various sizes. In addition, grains may be crushed during the compaction process (Zabielska-Adamska 2008). Therefore, to calculation the k value (see Fig. 4), constants $C = 0.5$ and $C = 0.85$ have been applied. It allowed to gain the most similar values in comparing to results obtained in the laboratory, however diverging from the line plotted according to equation (5).

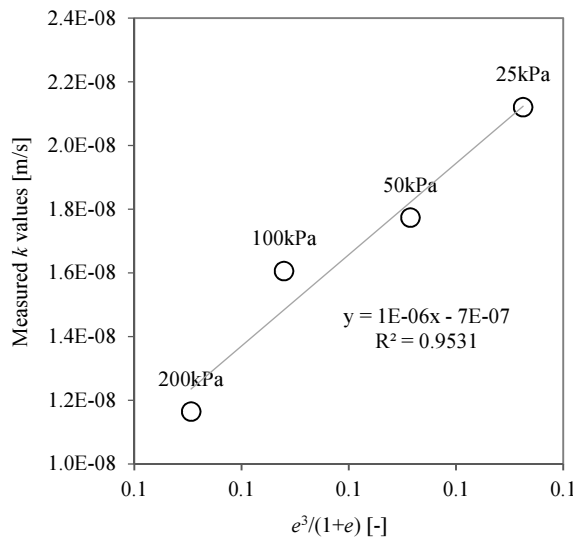


Fig. 3. Relationship between k , tested under various effective stress, and $e^3/(1+e)$ (source: own elaboration)

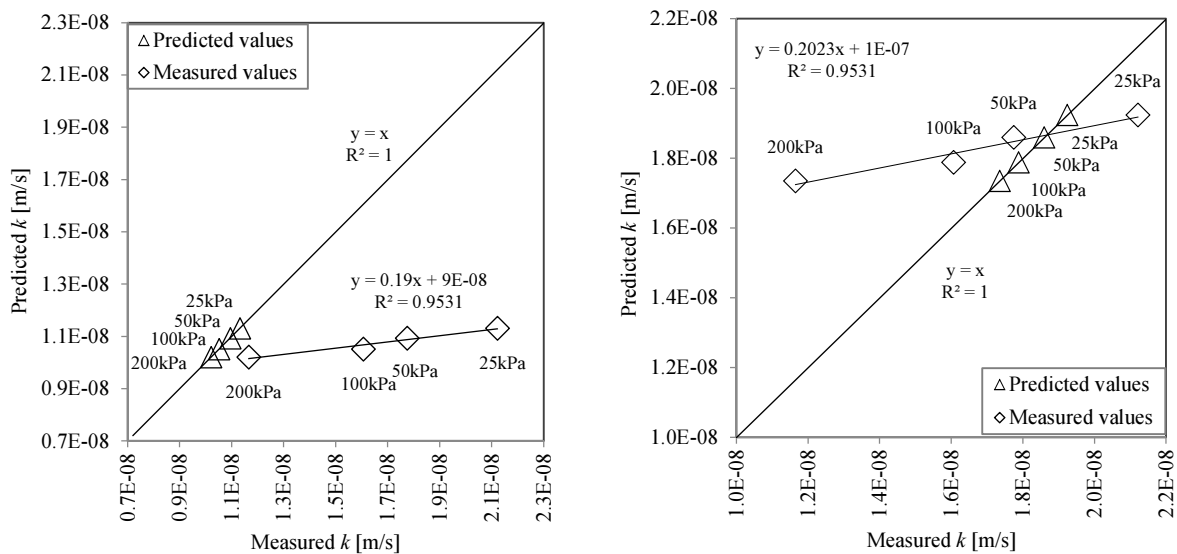


Fig. 4. Predicted versus measured values of hydraulic conductivity k for the samples, tested under various effective stress. Values predicted using equation (5): a) with constant $C = 0.5$; b) with constant $C = 0.85$ (source: own elaboration)

Using the equation (8) and proposed by Chapuis and Aubertin (2003a) value of parameter $A = 0.51$, allowed to obtain results of hydraulic conductivity presented in Fig. 5a. Values obtained from laboratory tests and calculated by means of empirical formulas differ from each other by not more than half an order of magnitude. Increased value of A parameter to value 0.74 allowed to obtain even more consistent results to ones received from laboratory tests (Fig. 5b).

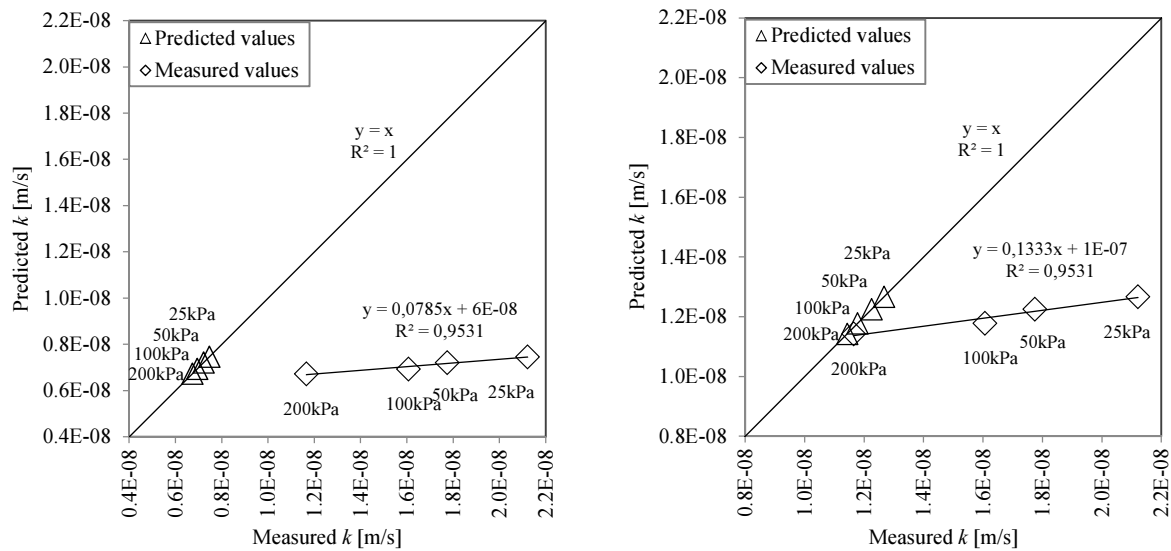


Fig. 5. Predicted versus measured values of hydraulic conductivity k for the samples, tested under various effective stress. Values predicted using equation (8): a) with constant $A = 0.51$; b) with constant $A = 0.74$ (source: own elaboration)

Generalization

Geotechnical properties of fly ash indicate to its similarity to natural fine-grained soil, so it may be used as antropogenic soil.

The Kozeny-Carman formula was generally created to assess specific surface of industrial powder using values of hydraulic conductivity. However, nowadays when there is more methods of testing specific surface area, empirical equations are mainly used to quickly and relativity easy estimate the hydraulic conductivity. Consideration should be given for equations presented in literature for different types of soil and to the values of coefficient and constant in this equations. For example, for the Kozeny-Carman formula, as proposed in (5) and (8), using of higher constant values, respectively constant $C = 0.85$ and parameter $A = 0.74$, provided to received similar results of hydraulic conductivity of fly ash to the ones obtained in laboratory tests. Although, even proposed values of the parameters ($C = 0.50$ and $A = 0.51$) allow to obtain satisfying results for fly ash samples.

The Kozeny-Carman and other equations, based on detailed descripton of physical parameters of the soil, determine much better value of the hydraulic conductivity than equations based only on grain-size distribution curve (effective sizes).

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Contribution

Authors are welcome to declare any involvement in writing a manuscript (e.g. conception and design of the work, acquisition of data, or analysis and interpretation of data, drafting the article or revising it critically for important intellectual content, etc.).

Disclosure statement

Author declare not to have any competing financial, professional, or personal interests from other parties.

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