"Environmental Engineering" 10th International Conference
Vilnius Gediminas Technical University
Lithuania, 27–28 April 2017
http://enviro.vgtu.lt

eISSN 2029-7092 / eISBN 978-609-476-044-0

Article ID: enviro.2017.100
DOI: https://doi.org/10.3846/enviro.2017.100

Methodology of the Polyoptimization for Spatial Processes

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Abstract. Optimization is a complex activity that aims to find the best solution for a given activity, considering all existing limitations. The best variant possible in the set of acceptable variants is sought-out. In particular, in urban areas, optimization of land use function as the beginning of a decision-making process requires performing a great number of tasks, which minimize the risk of spatial conflicts, set at the stage of studies and analyses. Polyoptimization is optimization with a vector objective function. The aim of polyoptimization is to find the best solution, concurrently applying several criteria which, due to their limitations, are conflicting as a general rule. It leads to finding compromise solutions (polyoptimum variants in the set of acceptable variants).
In the paper the following ideas will be presented – the idea of spatial processes polyoptimization, the methods for determining the collection and selection of compromise solutions, the methodology for determining polioptimum states of the space use, the possibility of using polyoptimization methods that are regarded as supporting decision-making tools in the planning and management of space with the use of GIS tools. The Authors will show the benefits of using the polyoptimization. The methods of formulating and solving problems which are related to selection of optimum way use of land will be delivered.

Keywords: optimization, polyoptimization, spatial processes.

Conference topic: Sustainable urban development.

Introduction

Contemporary human activity is characterized by planned appropriation of space, which is related predominantly with social and economic growth. Distribution of urban space use functions is a resultant of activities performed by different entities. The proper choice and arrangement of urban areas by different forms of use is of great significance for the fulfilment of the social, economic, ecological, and functional needs of a city. The concept of land allotment greatly affects the spatial structure of each city, and the change of land use function depends on the approach taken. Social expectations usually differ from land owners' expectations as regards prospective profits, as well as from the environmental protection perspective.

The optimum state of land use may be construed as a function of the needs of people and nature, i.e. the sum of environmental and anthropogenic values which results in the highest land value. As environmental conditions and environmental possibilities change, a conflict arises between the need to satisfy human needs and the need to change a form of use.

Since space optimization is connected with its limitation, space should be developed rationally. An optimum solution with regard to only one criterion (e.g. cost) can rarely be found. The analyzed problems most often require the simultaneous consideration of many action evaluation criteria in search of the optimum solution. One of the tools in determining the optimum land use function is procedure optimizationi and polyoptimization of spatial processes.

The aim of this analysis is the optimum selection of variants, taking into account different criteria with a crucial effect on the implementation and functioning of a given solution which, in this case, concerns the selection of the optimum or polioptimum land use function.

The study was prepared as a result of implementation of research project No. UMO-2014/13/B/HS4/00171 financed from the funds of the National Science Centre.

Methodology of the research

Distribution of the functions of space use is a resultant of various entities' activities; however, it is not random. There are general rules governing the process of the formation of this structure. The proper selection and distribution of areas, particularly in a city, to be used in a variety of ways is essential to meeting the economic, functional, and planning needs of the city. In the field of spatial economy, the term "optimization" is used in verifying the best (optimum) land uses. This verification primarily relates to the issue of optimum spatial location of economic operators, and very often refers to the areas of cities and their surroundings (Bajerowski *et al.* 2003).

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Optimization is a complex activity that aims to find the best solution for a given activity, considering all existing limitations. The best variant possible in the set of acceptable variants is sought-out (Tarnowski 2011). According to the most popular definition of optimum land use, it is a use that contributes to the highest land value out of all physically possible use forms permitted by law, i.e. compliant with their function (Kinzy 1992). In particular, in urban areas, optimization of land use function as the beginning of a decision-making process requires performing a great number of tasks, which minimize the risk of spatial conflicts, set at the stage of studies and analyses.

The essence of optimization is that for every part of a space, at any given moment, there is a possibility for obtaining the optimum state of use. However, the occurrence of appropriate features within a particular part of the space does not "force" the space to accept a proper state of use. It must only be noted that the probability of the transformation of the current state of use towards the optimum state is the highest (maximum). This will be due to chance that a particular state of use will actually be obtained (Bajerowski *et al*. 2003, as cited in: Zukav 1995). Each state of land use is, at the same time, a function of demand for such use of a space. Due to the demand, areas with features appropriate for a particular state are chosen, or a compulsion arises to transform these features in order to obtain the optimum state of use. Therefore, the state of land use may alternatively be referred to as a function of the area (Bajerowski *et al.* 2003).

The optimization method is otherwise called the method for land valuation, and it has originated in Poland as the "Warsaw optimization method" (Leśniak 1985). The method was developed in the years 1961–1963 as a tool supporting the planning of urban development after the Second World War. The purpose of the method was reasonable location of investment projects in a city. The main adopted principles were the minimisation of costs of acquiring land as well as of investment and operating costs. When possible land uses were determined, and combinations of particular variants were developed, a computer model helped evaluate the investment and operating costs. The optimization method was applied during the preparation of area development plans in the years 1961–1978 for several cities, namely Warszawa, Trójmiasto, Kraków, Łódź, Poznań, and Skopje (Broniewski, Suchorzewski 1979).

The optimization of spatial processes (space functions) aims to verify the most mismatched land functions and propose their replacement with functions best matched to the existing natural and anthropogenic features, as well as social, economic and ecological needs. Adaptation of problem areas – generating what are known as spatial conflicts, should be based on the opinion of the city's residents, reflecting their current needs, which can be called social optiization, and on so-called economic calculation – economic optimization, or assuming the minimization of envronmental effects – ecological optimization (Biłozor 2013; Biłozor, Renigier-Biłozor 2015).

The correct land use optimization procedure allows the reduction of uncertainty in the spatial planning process. The proposed system can be used on a different scale and at different levels of spatial analyses detail, and for so-called "spatial monitoring", which is used to analyze and verify particular land development forms (Biłozor 2013). The basic criterion of optimization is the so-called objective function, so, in this case:

- minimisation of modification costs (where: $f(x) = x_{cost} \rightarrow$ minimum),
- maximisation of social expectations (where: $f(x) = x_{social} \rightarrow$ maximum),
- maximisation of economic incomes (where: $f(x) = x_{economic} \rightarrow$ maximum),
- or maximisation of ecological values (where: $f(x) = x_{ecological} \rightarrow$ maximum) (Biłozor, Renigier-Biłozor 2015).

Optimization is a process in which the best solution meeting all requirements is searched for. According to Tarnowski (2011), optimization is a mathematical operation which needs to be formally (mathematically) defined in the form of a model. The procedure for solving the optimization task comprises four basic steps:

1. formulation of a mathematical optimization model – the determination of the object and scope of optimization, decision variables, parameters, and limitations;
2. selection of an optimization method;
3. verification of substantive assumptions of the model – analysis of limitations, check on the objective function for decision variables;
4. carrying out the process of optimization.

The model of an optimization task precedes the decision. Where the decision maker has doubts, a better solution is multi-criteria optimization, or polyoptimization.

Polyoptimization is optimization with a vector objective function. The aim of polyoptimization is to find the best solution, concurrently applying several criteria which, due to their limitations, are conflicting as a general rule. It leads to finding compromise solutions (polyoptimum variants in the set of acceptable variants) (Tarnowski 2011).

Polyoptimization is widely applied in the analysis of a set of acceptable variants as well as in decision making processes, e.g. in planning and organizing, constructing and designing of objects, process control, formulation of detailed principles of selection, organizing complex mathematical models, teaching neural networks, etc. (Tarnowski 2011). Polyoptimization, in contrast to optimization (which searches for the best solution in terms of the objective function in a set of decisions), searches for a subset of compromise variants due to multiple criteria (a set of compromise variants satisfying all assumed limitations is obtained). The task of polyoptimization in a simplified form most often comprises nine basic steps:

1. determination of the polyoptymization object and task;
2. determination of a polyoptimization criterion;

3. determination of decision variables;
4. definition of limitations;
5. development of a mathematical object model;
6. determination of assessment criteria;
7. creation of a polyoptimization model – objective function, function of limitations, and their verification;
8. analysis of the subset of compromise variants;
9. polyoptimization.

Polyoptimization may also be defined as optimisation of a certain set of independent quality criteria $Q_1, Q_2, \ldots, Q_k$, which are determined by a certain number of control variables $a_1, a_2, \ldots a_r$. The aim of polyoptimization is to make use of all relationships between the quality criteria and control variables. A set determined on this basis within the area of control variables or quality criteria provides a basis for the determination of a compromise solution (Peschel, Riedel 1979).

Generally speaking, the concept of polyoptimization is to divide solutions into useful ones and bad ones. What is also applicable here is the basic assumptions of both the static compromise theory (choice theory), where the decision maker seeks to achieve the objective function while searching for the balance (compromise) between the benefits resulting from changes to a space and the costs of their implementation (Kubiak 2012). Similarly to the static compromise theory, where benefits resulting from the effect (the achieved objective function) are compared, derogations from the target structure can be analyzed, while looking at the changes taking place in the space from a dynamic perspective, by comparing the benefits from achieving the objective with the costs associated with the changes.

The algorithm of a change to land use, developed in earlier studies, (Biłozor, Jędrzejowska 2012; Biłozor 2013, 2014) which is an instrument of optimization of a planning space, is, in this case, an element which facilitates making the right decision concerning the use of an analyzed area. This ordered set of operations, by which we will obtain a solution of a specific task, namely the determination of the optimum land use, was modified and adjusted to the specificity of spatial economy. The procedure for carrying out of the decision process for the choice of optimum land use should be followed in accordance with the following steps:

1. spatial monitoring – selection of an area in which a change of the function is either possible or necessary,
2. determination of social conditions for an area optimization – development of principles and methods for the performance of sample research into the state of spatial development, and interpretation of the obtained results,
3. determination of economic conditions of the optimality of the functions of the area – an analysis of the process of land transformations as regards the functionality, costs, and profits.

   3.1 development of principles of the economic optimization of income – an analysis of transaction prices of real estate from the local real estate market, identification and analysis of the impact of the elements of economic calculation in the process of land transformation, determination of an optimum function generating the greatest possible profit, an analysis of technical and legal capacities,

   3.2 development of principles of the economic optimization of costs – identification of both anthropogenic and natural characteristics of the space determining the current state of land use, carrying out an analysis of geoinformation necessary in the process of spatial optimization, determination of an optimum function using a matrix of characteristics causing the optimum land use,
4. determination of ecological conditions for optimization of the area,
5. analysis of social, economic, and ecological prerequisite conditions for the area under study – analysis of the reasonableness of the change of the function of the area,
6. determination of a polyoptimization criterion as well as assessment criteria, and defining limitations,
7. analysis of the set of compromise variants,
8. optimization of the area destination.

The criteria applied in the polyoptimization process concern the need to meet specific social, economic and ecological conditions by a given function. Formulating the polyoptimization task, it is necessary to find such functions and parametersthat give partial criteria with extreme values for specific intervals of variables while fulfilling specific limitations. Corresponding weights that express the importance assigned to particular functions are assigned to objective functions.

Polyoptimization for spatial processes – case study

The optimum land development selection procedure was carried out in the area of the Town of Grudziądz. In the process of so-called "spatial management" 10 areas with possible or necessary change of function were selected. These are areas located in different parts of the town, developed and used in an improper way, and thus causing a number of spatial conflicts. The location of areas designated for optimization is presented in Fig. 1.

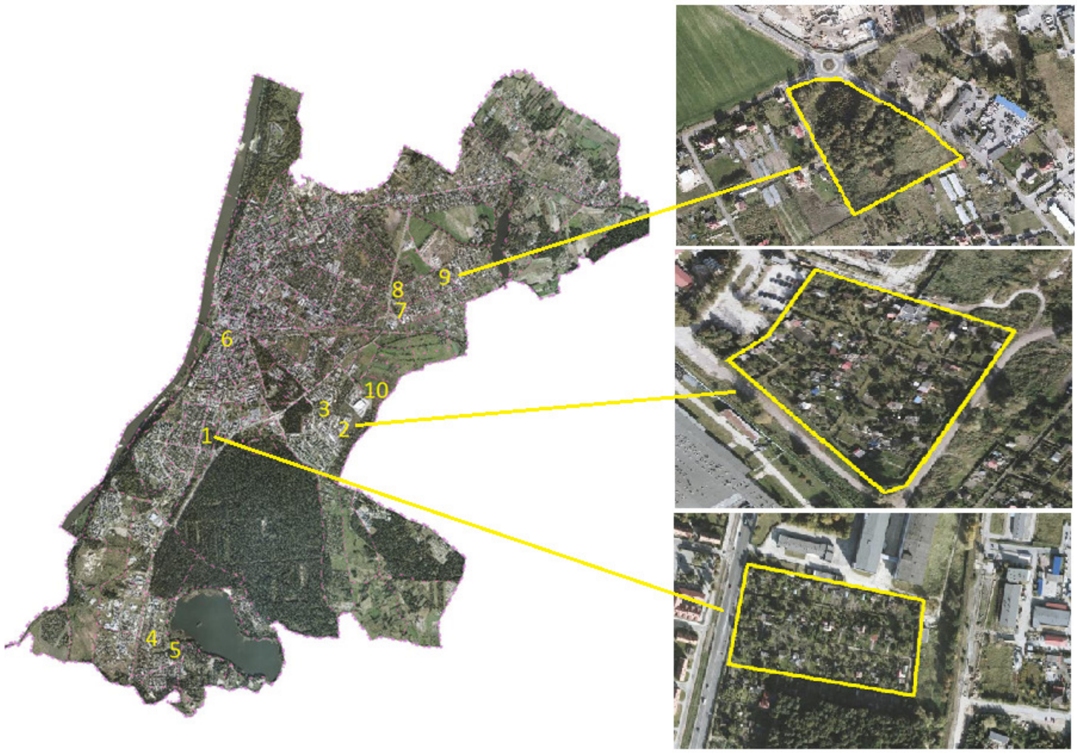

Fig. 1. The areas selected for conducting the optimization process
Source: own study with the use of geoportal web page.

Results of the survey indicate which functions, and to what extent, are most useful for social optimization. In the survey, the inhabitants' needs and requirements were determined as regards the most and the least attractive places in the city, the availability of social, trade and services, and sports and recreational infrastructure, missing objects, and forms of area development. Respondents would most willingly designate conflict areas for:

- services – 31% of respondents,
- industry – 25%,
- areas under single-family and multi-family housing developments – 20%,
- green areas – 12%,
- areas for sports and recreation – 7%.

Economic optimization is focused on the financial approach to spatial planning, and involves the use, to the greatest extent possible, of the economic potential of the space in the city. The income approach in economic optimization is aimed at the maximization of income, determined on the basis of an analysis of transaction prices for the functions being the object of trade in the last two years. Averaged prices for the functions being the object of trade in the last year, obtained from the Municipal Council in Grudziądz, are presented below:

- service function –147.93 PLN/m^2;
- single-family residential function – 58.67 PLN/m^2;
- multi-family residential function – 62.50 PLN/m^2;
- industrial function – 76.18 PLN/m^2;
- recreational – 26.24 PLN/m^2;
- sports and recreational – 46.78 PLN/m^2;
- green area – 21.87 PLN/m^2.

In the process of economic cost optimization for all 10 areas, an optimum function was determined using a matrix of connections of urban space functions (land use). An example of such a matrix for the area No 1, 2 and 9 is presented in Tables 1, 2 and 3.

Ecological optimization which is primarily focused on the protection and preservation of the natural environment in the best possible condition. To this end, a number of available documents related to this sphere of urban space were analyzed, and ecological priorities for particular areas were determined.

Table 1. Area 1 – matrix of connections of urban space functions (the land use function) with land features and with the present infrastructure (Source: own elaboration)

No	Urban space function / Land feature	MN	MW	U	US	UC	P	ZP	ZC	WS	K	IT
1	Electricity	8	8	9	8	8	10	0	5	3	4	10
3	Waterworks	9	8	9	7	9	9	3	6	3	2	9
4	Sewage system	7	8	6	3	5	9	–6	1	3	3	8
6	Easy access by road	7	7	8	2	6	7	1	5	4	6	4
9	Restaurants	3	4	7	2	3	–7	1	–6	1	6	–9
11	Multi-family blocks of flats	–7	10	–2	–6	5	–4	2	–9	–8	–3	–7
13	Public buildings	–3	1	8	–9	2	–4	4	–7	–8	0	–5
14	Clubs, pubs	–6	3	7	1	4	–4	–8	–7	3	2	–4
17	Access to education	4	5	2	–6	1	–2	–9	–7	–9	3	–1
18	Cinemas, theatres, cultural centres	–4	–3	6	–9	–5	–7	–7	–6	–8	3	1
19	Small floor space shops	4	4	10	3	–7	1	–4	2	–3	4	–1
20	Large format stores	–7	–5	1	–10	10	4	–10	–6	–9	9	5
21	Hard-surfaced roads	6	7	8	2	9	10	–3	9	–2	10	6
23	Religious buildings	–1	3	–5	–3	–6	–2	4	9	–8	4	–6
29	Groups of trees, groves	1	–6	–9	6	–3	–1	7	3	6	–8	–1
30	Single trees	–2	–3	–3	4	–1	0	9	7	6	–6	3
31	Bush belts, hedges	3	3	5	–2	–1	0	10	7	6	3	2
34	Western exposure	3	2	–4	5	–6	–5	3	3	3	–10	–4
36	Small land slope	6	1	6	–3	5	7	5	7	4	8	8
Sum		31	57	**69**	–5	38	21	2	16	–13	40	18

Table 2. Area 2 – matrix of connections of urban space functions (the land use function) with land features and with the present infrastructure (Source: own elaboration)

No	Urban space function / Land feature	MN	MW	U	US	UC	P	ZP	ZC	WS	K	IT
1	Electricity	8	8	9	8	8	10	0	5	3	4	10
3	Waterworks	9	8	9	7	9	9	3	6	3	2	9
4	Sewage system	7	8	6	3	5	9	–6	1	3	3	8
11	Multi-family blocks of flats	–7	10	–2	–6	5	–4	2	–9	–8	–3	–7
21	Hard-surfaced roads	6	7	8	2	9	10	–3	9	–2	10	6
29	Groups of trees, groves	1	–6	–9	6	–3	–1	7	3	6	–8	–1
30	Single trees	–2	–3	–3	4	–1	0	9	7	6	–6	3
31	Bush belts, hedges	3	3	5	–2	–1	0	10	7	6	3	2
34	Western exposure	3	2	–4	5	–6	–5	3	3	3	–10	–4
35	No land slope	6	3	–3	–6	10	10	–3	6	–2	9	9
Sum		34	**40**	16	21	35	38	22	38	18	4	35

Table 3. Area 9 – matrix of connections of urban space functions (the land use function) with land features and with the present infrastructure (Source: own elaboration)

No	Urban space function / Land feature	MN	MW	U	US	UC	P	ZP	ZC	WS	K	IT
6	Easy access by road	7	7	8	2	6	7	1	5	4	6	4
12	Single-family houses	10	–8	–3	–3	–3	–9	5	–8	–9	–6	–8
19	Small floor space shops	4	4	10	3	–7	1	–4	2	–3	4	–1
21	Hard-surfaced roads	6	7	8	2	9	10	–3	9	–2	10	6
29	Groups of trees, groves	1	–6	–9	6	–3	–1	7	3	6	–8	–1
34	Western exposure	3	2	–4	5	–6	–5	3	3	3	–10	–4
36	Small land slope	6	1	6	–3	5	7	5	7	4	8	8
Sum		**37**	7	16	12	1	10	14	21	3	4	4

Notes: MN – Residential areas with single-family homes, MW– Residential areas with multi-family homes, U – Areas of retail-service buildings, US – Sport and recreation areas, UC – Areas of large format stores, P – Areas of productive facilities, depots and stores, ZP – Green areas (parks), ZC – Cemeteries, ZL – Forest, WS – Areas of inland surface water, K – Areas of transport, IT – Areas of technical infrastructure.

The proper location of new forms of land use is an extremely important issue as regards space management. While searching for an optimum solution to the decision making problem as regards a change to the use of an area, a number of significant factors need to be taken into account. Each of the performed optimizations only concerns one variant – social, economic, or ecological one. While searching for a solution that would be the best in a particular time and place, it is necessary to combine and analyze all variants, and to choose the most beneficial variant which also takes all objectives into account. While formulating the polyoptimization task, it is necessary to find such functions and parameters so that sub-criteria had maximum values, with specific limitations fulfilled, and then to carry out an analysis of a set of compromise variants. Appropriate weights i.e. degrees of membership in the optimum use were determined for the objective functions using the linear weighing procedure. They range from 0.0 (hardly useful) to 1.0 (very useful).

Results of the survey indicate which functions, and to what extent, are most useful for social optimization. On the other hand, analysis of the real estate market shows which functions are most useful for economic income optimization. In the economic cost optimization we take advantage of the potential of space characteristics while minimizing the costs of transformation. The selection and determination of suitability and relevance for the designed areas during ecological optimization is determined, to a large extent, by the natural conditions, the nature of the area, and possibilities for the introduction of particular natural forms. When the overlaying method (Hejmanowska, Hnat 2009) is used, the optimum state of land development is obtained as the product of individual results of the optimization. The analysis performed on the basis of strict criteria determined during the process of social, economic, and ecological optimization enabled the determination of optimum states of area development (Table 4).

Table 4. Summary of the optimum forms of land use (Source: own elaboration)

	Optimization of land use – function				Optimum form of land use
No	Social	Economic – income	Economic – cost	Ecological	
Area 1	U	U	U	ZP	U
Area 2	P	P	MW	ZL	P
Area 3	U	U	U	ZP	U
Area 4	U	U	MW	ZP	U
Area 5	US	US	UC	US	US
Area 6	U	U	U	ZP	U
Area 7	U	U	MN	ZL	U
Area 8	U	U	MN	ZP	U
Area 9	MN	U	MN	ZP	MN
Area 10	P	P	P	ZL	P

The aim of polyoptimization is to find the best solution while taking account of several criteria at the same time, which typically involves finding compromise solutions (polyoptimum variants in a set of acceptable variants) While determining the polyoptimization criteria and defining limitations, appropriate degrees of membership in the objective function should be designated to particular forms of optimization, depending on the model. Summary of the degree of membership for particular forms of land use is presented in Table 5.

Table 5. Summary of the degree of membership for optimum forms of land use (Source: own elaboration)

<table>
<tr><th rowspan="2">Function</th><th colspan="4">Summary of the degree of membership for optimum forms of land use</th></tr>
<tr><th>Social</th><th>Economic – income</th><th>Economic – cost</th><th>Ecological</th></tr>
<tr><td>MN</td><td>0.64</td><td>0.40</td><td rowspan="7">The degree of membership for particular functions is determined by the existing characteristics of the space</td><td>0.00</td></tr>
<tr><td>MW</td><td>0.64</td><td>0.42</td><td>0.00</td></tr>
<tr><td>U</td><td>1.00</td><td>1.00</td><td>0.00</td></tr>
<tr><td>US</td><td>0.24</td><td>0.32</td><td>0.55</td></tr>
<tr><td>P</td><td>0.80</td><td>0.52</td><td>0.00</td></tr>
<tr><td>ZP</td><td>0.40</td><td>0.00</td><td>0.70</td></tr>
<tr><td>ZL</td><td>0.12</td><td>0.15</td><td>0.80</td></tr>
</table>

The criteria adopted for analysis concern the necessity of satisfying specific social, economic, ecological, and technical conditions by a particular function. The analysis of suitability, performed on the basis of criteria determined during the optimization process, as well as the analysis of a set of compromise variants enabled the determination of polyoptimum states of development of selected areas in the city of Grudziądz – Table 6.

Table 6. Summary of the optimum forms of land use (Source: own elaboration)

No	Optimization of land use				Polyoptimum (compromise) land use
	Social	Economic – income	Economic – cost	Ecological	
Area 1	U/1.00	U/1.00	U/1.00	ZP/0.70	U/ZP
Area 2	P/0.80	P/0.52	MW/1.00	ZL/0.80	P
Area 3	U/1.00	U/1.00	U/1.00	ZP/0.70	U/ZP
Area 4	U/1.00	U/1.00	MW/1.00	ZP/0.70	U/ZP
Area 5	US/0.24	US/0.32	UC/1.00	US/0.55	US
Area 6	U/1.00	U/1.00	U/1.00	ZP/0.70	U/ZP
Area 7	U/1.00	U/1.00	MN/1.00	ZL/0.80	U/MN
Area 8	U/1.00	U/1.00	MN/1.00	ZP/0.70	U/ZP
Area 9	MN/0.64	U/1.00	MN/1.00	ZP/0.70	MN/ZP
Area 10	P/0.80	P/0.52	P/1.00	ZL/0.80	P

The conducted analysis reveals that social expectations most often coincide with proposals of economic optimization. In all analyzed cases, the function with the superior sum of degrees of membership was adopted as the optimum function in a particular area. In most cases, where the conducted analysis failed to clearly indicate the optimum function, the selection was carried out by combining the least conflicting functions with each other.

Conclusions

The application of polyoptimization method allows one to find the best solution while taking several criteria into account at the same time. Polyoptimum analysis carried out while selecting the optimum land use showed the possibilities for its application as a tool supporting the process of making planning decisions. Social, economic, and ecological criteria adopted for the analysis provide a basis for sustainable development of a particular area, and should be considered in the process of planning the optimum area development.

Optimisation of the functions of the urban space of the city of Grudziądz was aimed at the verification of the most unmatched functions of the area, and a proposal for turning them into functions that are best matched to the existing social, economic and ecological demand. The resulting image of the space is neither extremely risky nor devoid of any risks. Grudziądz is a city with great potential which, however, is not taken advantage of in an optimum manner. Reconciliation of various groups' interests is an extremely complicated task. The application of polyoptimization procedure in spatial analyses will enable both the elimination of conflict areas from the city and determination of the hierarchy of the proposal of changes. The proposed changes will primarily increase the surface area of industrial areas by more than 5 ha, and of service areas by more than 15 ha. At the same time, the surface area of allotments and areas of uncontrolled greenery will decrease. The proposed compromise necessitated the combination of the least conflicting functions, e.g. services and single-family buildings with areas of controlled greenery, and services (e.g. those with no adverse effects) and single-family buildings. This will enable the combination of the proverbial business (useful functions of U and MN) with pleasure (ZP). The developed decision-making procedure with clearly defined activities necessary for the performance of the set task may support the space management system whose main instrument is the operations of land use transformations.

Acknowledgements

The study was prepared as a result of implementation of research project No. UMO-2014/13/B/HS4/00171 financed from the funds of the National Science Centre.

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