

Augmentation of GNSS Autonomous Positioning Using a Ground Based ZigBee Phase Shift Distance Measurement

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Abstract. The article presents the impact of GNSS positioning system augmentation using a ZigBee device. Ground based augmentation is based on the distance measurement between two ZigBee devices – one embedded into the rover and one acting as an anchor. The paper describes the device that was used in the experiment along with experiment results. Experiment results shows the significant impact of an additional distance measurement on the positioning results, especially in the case when there is a small number of satellites in view.

Keywords: GNSS, ZigBee, positioning system, GBAS.

Conference topic: Technologies of Geodesy and Cadastre.

Introduction

Global Navigation Satellite Systems (GNSS) are commonly used for navigation and positioning in many industrial, civilian and military applications. These systems are available worldwide in any place where the sky is not obstructed. In cases, the obstructions of the sky are making its use difficult or impossible. Low number of visible satellites along with poor geometry of the positioning solution are causing low accuracy and reliability of the solution.

To enhance the accuracy and availability of GNSS in certain applications, a local ground-based augmentation systems (GBAS) are used. Usually these systems provide an improvement in accuracy by correcting the phase or pseudorange measurements. This way the impact of common errors (ephemeris and satellite clock errors, ionospheric and tropospheric delays) is minimized Braff and Shively (2005). The main disadvantage of this method is the open sky requirement. If there is not enough satellites in view or the satellites geometry is poor, the resulting position accuracy may not be satisfactory. This problem exists especially in heavy urbanized places where buildings and other construction objects obstruct the sky.

To mitigate this problem a different approach to ground based augmentation can be introduced. Instead of transmitting pseudorange or phase observation corrections, an additional range observation from ground based transmitter can be used. It provides a geometry improvement which should result in better accuracy and availability of a navigation system. This idea was investigated by various researchers in publications related to pseudolites Cellmer and Rapinski (2010), Cobb (1997), Rapinski *et al.* (2012a, 2012b), Rapinski (2011), Rapinski *et al.* (2012c), Rzepecka *et al.* (2006), Stone and Powell (1998). The modern communication networks allow to measure distance using much cheaper, mobile and easily available devices than pseudolites. The distance can be obtained using various techniques: radio signal strength indicator (RSSI), time of flight (TOF), time of arrival (TOA) or differential time of arrival (TDOA) Chen *et al.* (2012), Farid *et al.* (2013). These techniques are applicable to most of the modern network communication systems like WiFi, Bluetooth or ZigBee providing ranging capabilities next to the communication. Notwithstanding, the accuracy of the methods presented above is low comparing to GNSS observations. Ranging based on ZigBee signal phase shift measurements is an emerging technology allowing to obtain relatively good accuracy while maintaining low costs and very low energy consumption Rapinski (2015), Rapinski and Cellmer (2015), Rapinski and Smieja (2015), Janicka and Rapinski (2015).

The main goal of the research presented in this paper is to investigate the influence of additional ZigBee phase shift based range observations on GNSS single point positioning. The paper presents theoretical assumptions and experiment results.

Ranging using ZigBee phase shift measurements

ZigBee is a specification of communication protocol based in the IEEE 802.15.4 specification. Its low rate of 250kbps usually designates it to the use in data transmission from an industrial sensor or other input device.

The phase shift measurement ranging technique was first introduced to ZigBee device in ATRF233 from Atmel. The analysis presented in Rapinski and Smieja (2015) shows that the accuracy of raw phase shift range observations are low and burdened with many outliers. Therefore the algorithm of smoothing the observations presented in Rapinski and Smieja (2015) is used in this paper. Tests performed with this device and algorithm shows the mean relative

accuracy of ranging on the 2% level Rapinski and Smieja (2015). To perform ranging two identical devices are required. One of them acts as a rover (a device which coordinates are to be estimated) while another one acts as an anchor (a fixed device with known coordinates). ZigBee communication is performed on the 2.4GHz ISM band but the ranging procedure is performed on the intermediate frequency according to the following procedure: devices initialization, range request sent by rover, ranging initialization, time synchronization, ranging start, ranging measurement, request measurement result, transfer data and calculate distance.

All of the steps above are conducted according to IEEE 802.15, except for point 6 which is Atmel proprietary. The ranging procedure and achieved accuracy are described in details in Rapinski (2015), Rapinski and Cellmer (2015), Rapinski and Smieja (2015), Janicka and Rapinski (2015).

Equipment considerations

In order to provide the augmentation capability, a standard GNSS receiver is not sufficient. An additional ZigBee ATRF233 transceiver is necessary. It should be connected to the user interface device or be embedded in it. An scheme of the device requirements is depicted in Figure 1.

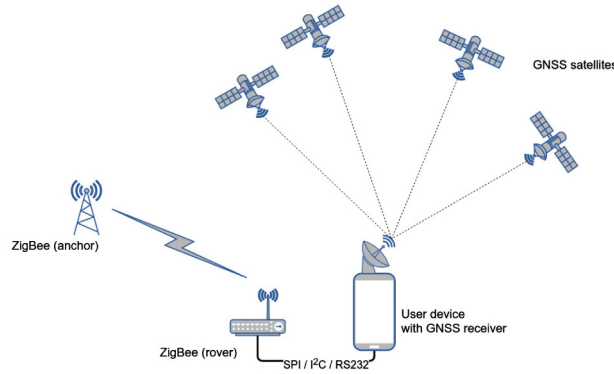


Fig. 1. Configuration of devices required for augmentation

Observation model

Despite the fact that observations from GNSS and ZigBee are range measurements, they cannot be treated exactly the same in the observation model. Differences in distance and different atmospheric effects are causing the observations to be treated separately.

GNSS pseudorange observation model

In this paper the pseudorange observations were modelled according to the following equation:

$$P = \rho - c\Delta\tau_{rec} + c\Delta\tau_{sat} + \delta_{iono} + \delta_{trop} + \xi, \quad (1)$$

where: c = speed of light in vacuum; τ_{rec} = receiver clock offset; τ_{sat} = satellite clock offset; δ_{iono} = ionospheric correction; δ_{trop} = tropospheric correction; ξ = not modeled noise;

and:

$$\rho = \sqrt{(x_{rec} - x_{sat})^2 + (y_{rec} - y_{sat})^2 + (z_{rec} - z_{sat})^2}, \quad (2)$$

where: x_{rec} , y_{rec} , z_{rec} = user coordinates; x_{sat} , y_{sat} , z_{sat} = satellite coordinates.

The δ_{trop} parameter is calculated on the basis of the Saastamoinen model and mean atmospheric parameters Katsougiannopoulos *et al.* (2006). Ionospheric delay and satellite clock correction are calculated on the basis of the broadcast clock and Klobuchar model parameters Stepniak *et al.* (2014) (transmitted in the ephemeris). In each equation there are four parameters to be estimated: X, Y and Z coordinates of the receiver and receiver clock correction τ_{rec} .

ZigBee observation model

ZigBee transmitter is relatively close to the receiver (approximate maximum range is about 300 m). Its signal travels up to few hundred meters in troposphere only. The influence of the troposphere on the measured distance is largely independent from the signal frequency, so a model used for pseudolite can be used Dai *et al.* (2000):

$$\delta_{trop} = \left(77.6 \frac{P}{T} + 3.73 \cdot 10^5 \frac{e\omega}{T^2} \omega \right) 10^{-6} d, \quad (3)$$

where P – atmospheric pressure in hectopascals, $e\omega$ – partial water vapour pressure in hectopascals, T – temperature in Kelvin, d – distance.

For standard atmospheric parameters ($T = 20$ °C, $P = 1013$ hPa, $e\omega = 25$ hPa) and 300 m distance the tropospheric correction is equal to 11 cm. For the distance of 19 m which is used in the experiment described below, this influence is equal to 7 mm which is negligible. Since there are no clock errors and the signal does not travel through ionosphere the ZigBee observation model is more simple then GNSS pseudorange model. It can be described using the following equation:

$$P_{ZigBee} = \rho + \delta_{trop} + \xi_{ZigBee}, \quad (4)$$

where ξ_{ZigBee} is not modeled observation noise and ρ can be denoted as (index anchor stands for a stationary reference transceiver):

$$\rho = \sqrt{(x_{rec} - x_{anchor})^2 + (y_{rec} - y_{anchor})^2 + (z_{rec} - z_{anchor})^2}. \quad (5)$$

Weighting of observations

Range observations from GNSS satellites and from ZigBee transceivers have various accuracy so an appropriate weighting scheme is required.

According to Parkinson (1996) the accuracy of a pseudorange observation is equal $\frac{1}{100}$ of the code chip (about 3 m for the C/A code). In addition the GNSS satellite signal degrades along with decreasing elevation Tay and Marais (2013) which should be taken into account. In the tests presented below the weighting function of GNSS pseudoranges is given by (6).

$$\omega = \frac{1}{0.01 + e^{-\alpha}}, \quad (6)$$

where α stands for a satellite elevation angle.

For a ZigBee observation, the signal travels relatively short distance always in troposphere, therefore the elevation related weighing is not appropriate. On the basis of Rapinski (2015), Rapinski and Cellmer (2015), Rapinski and Smieja (2015), Janicka and Rapinski (2015) the weighting function was arbitrary chosen to be:

$$\omega = \frac{1}{0.02 \cdot d}. \quad (7)$$

Resulting weighting functions for both GNSS and ZigBee range observations are depicted in Figure 2.

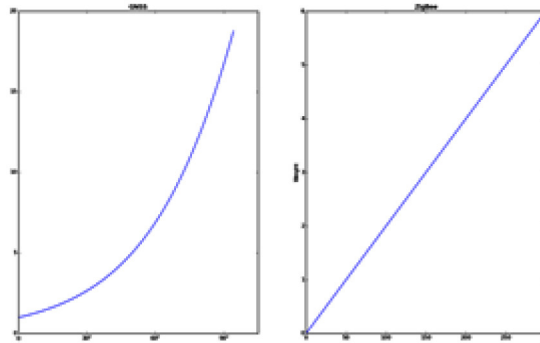


Fig. 2. Weighting of GNSS and ZigBee observations

Objective function

For the range-based positioning the objective function is described by Equation (8).

$$\sum_{i=1}^n \omega (\rho - P)^2 = \min, \quad (5)$$

where ρ is a range calculated on the basis of initial coordinates of rover and fixed coordinates of anchor and P is an GNSS pseudorange or ZigBee ranging observation. In these equations there are four unknown parameters mentioned in previous section. For the ZigBee observation equations the d_{trec} parameter is always assumed to be equal zero. Various approaches to minimize the objective function can be applied in order to obtain estimated parameters. In this paper the numerical Nelder-Mead method is used Olsson and Nelson (1975), Rapinski and Cellmer (2015), Skala-Szymańska *et al.* (2014). It is a numeric, downhill simplex method that does not require the observation equations to be expanded into Taylor series. This is especially important since the difference in receiver – satellite and receiver – ZigBee anchor distances are very significant. The non linear pseudorange equations can be easily linearised using first terms of Taylor series expansion. First terms are sufficient since the radius of the sphere (which is the graphical representation of pseudorange equation) is very large (20 000 km). In case of much shorter distance to ZigBee transceiver first terms of Taylor series expansion are not approximating the function well enough. The problems of transforming this kind of observation equations into linear form was described in details in Cellmer and Rapinski (2010), Rapinski and Cellmer (2015).

The experiment and the results

To confirm the possibility of ground based augmentation of GNSS positioning system using ZigBee phase shift ranging, the experiment was conducted. The equipment used for the experiment consisted of uBlox LEA 6T GNSS receiver connected to PC computer using RS232 to USB converter and ATRF233 evaluation kit connected to the same PC. The evaluation kit consists of AT86RF233 ZigBee communication module, microcontroller performing calculations, RF switch and two antennas used for the antenna diversity and multipath mitigation. The GNSS receiver was transmitting raw C/A code observations to the PC and all further calculations were done in postprocessing. Each GNSS epoch was triggering a ZigBee measurement request. The devices used in the experiment are depicted in Figure 3.

In this paper a static application is considered, hence the difference in time between GNSS epoch and ZigBee observation time is neglected. From authors experience the time offset is not larger then one third of a second, so the method described in this paper is applicable also for slowly moving objects (like pedestrians). ZigBee transmitter was placed about 19 m away from the receiver in a window at fourth floor of the building (about 9 m from the ground). Epoch 2016-05-20 18:00:00 was selected to perform the tests. The skyplot with the satellites (and ZigBee) distribution is depicted in Figure 4. At this epoch there were 13 satellites in view with elevation varying from 3.6 to 75.6 degree. For this epoch, with all satellites in view, at the point of observation PDOP value is 1.06. This indicates a very good geometric distribution of the satellites.

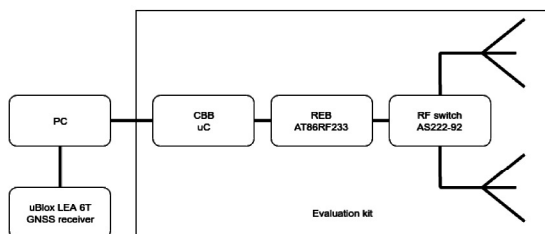


Fig. 3. Devices used in the experiment

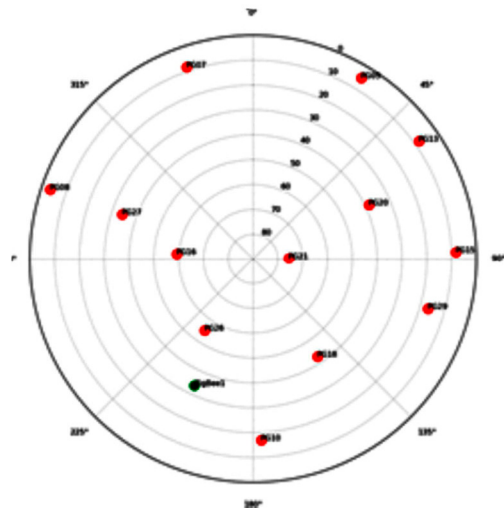


Fig. 4. Configuration of satellites during the experiment

To test the influence of additional ZigBee observation on positioning accuracy, selected combinations of possible satellite distribution were used to calculate rover position. This simulates various obstacles shading the signal from the satellites in all of the possible combinations. Four cases were analyzed – with three, four, seven and eleven satellites in view. Results with and without the additional, ground based ZigBee transmitter were compared for these cases. For each case, all possible combinations of satellites in view were used. Hence for each case, a set of displacements from the true position to the resulting position for all possible combinations of satellites is presented. True position was calculated previously on the basis of the GNSS static session and is considered errorless in the analyses. The displacements were calculated in the local zone of UTM in order to present its horizontal component. The linear distortion of

the UTM can be neglected in such a small area. For vertical component an ellipsoidal height was used. Considering the size of the result sets (286 for three, 715 for four, 1716 for seven and 78 for eleven satellites in view) the results are summarized in tables (Table 1, 2, 3, 4) containing mean value, standard deviation, minimum value, first, second, third quartile and maximum value for calculated displacements in x, y and z coordinate (components of linear displacement), length of displacement (Δ) and RMS of the solutions. Resulting horizontal displacements are depicted in Figures 5, 8, 9, 12, 13, 16, 17. These figures show the displacements in the horizontal plane along with histograms of displacements in X and Y directions. Vertical displacements are depicted as histograms in Figures 6, 10, 14, 18. To make the plots readable, the axes are limited to the range -30 to 30 meters. ZigBee position is marked in the figures using an arrow pointing from the true rover coordinates to ZigBee location. To describe the geometry of the solution DOP factors are used. Figures 7, 11, 15 and 19 depicts the distribution of PDOP values for examined cases (only values lower than 30 are shown).

In the case with only three satellites in view, it is not possible to calculate GNSS receiver position without using additional observation (such as ZigBee). The number of unknown parameters ($x, y, z, \Delta t$) is greater than the number of possible equations, so the set of equations is singular. In this scenario only the case with additional ZigBee transceiver is analyzed.

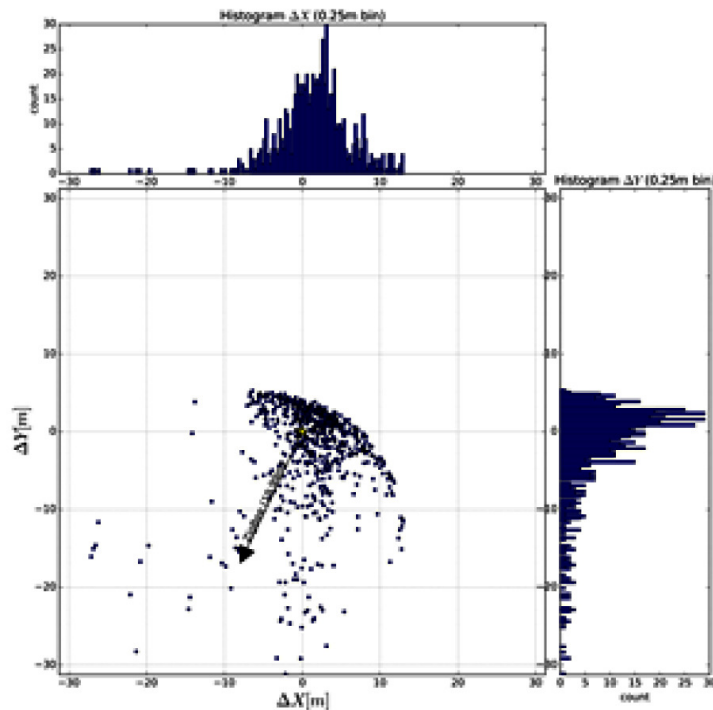


Fig. 5. Horizontal displacements with 3 satellites and one ZigBee in view

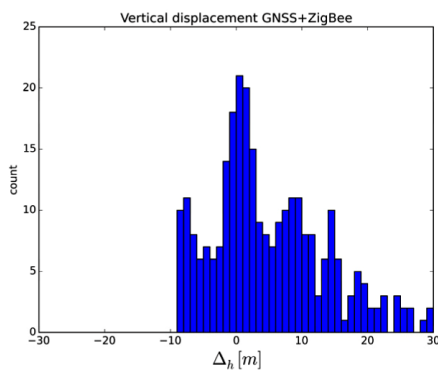


Fig. 6. Histogram of vertical displacements with 3 satellites and one ZigBee in view

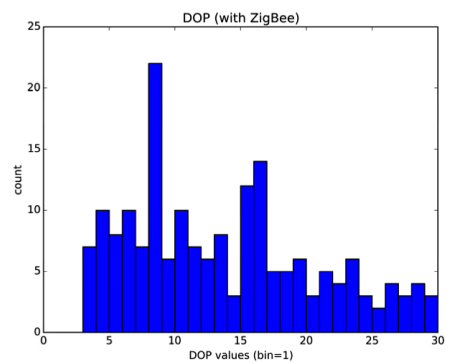


Fig. 7. Histogram of PDOP for three satellites and one ZigBee in view

Table 1. Comparison of results for three satellites and one ZigBee and three satellites in view

	ZigBee					No ZigBee				
	dx [m]	dy[m]	dz[m]	Δ [m]	RMS	dx[m]	dy[m]	dz[m]	Δ [m]	RMS
count	286.00	286.00	286.00	286.00	—	—	—	—	—	—
mean	-0.09	-4.40	5.48	9.15	—	—	—	—	—	—
std	7.74	8.04	9.43	7.74	—	—	—	—	—	—
Min	-27.13	-31.06	-8.82	0.27	—	—	—	—	—	—
25%	-3.25	-8.62	-0.84	3.59	—	—	—	—	—	—
50%	1.20	1.96	-3.00	6.58	—	—	—	—	—	—
75%	4.74	1.37	10.49	12.36	—	—	—	—	—	—
max	12.81	5.15	30.91	35.46	—	—	—	—	—	—

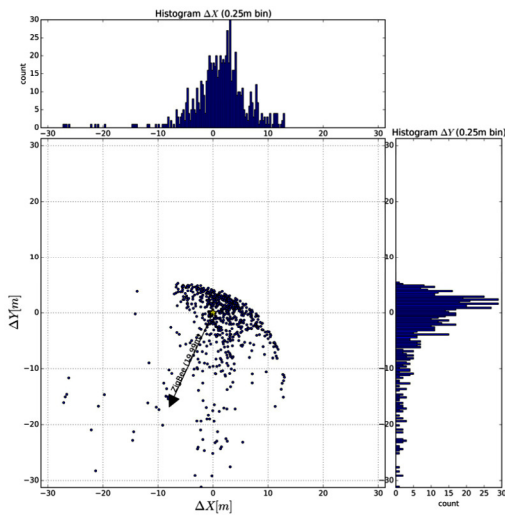


Fig. 8. Horizontal displacements with 4 satellites and one ZigBee in view

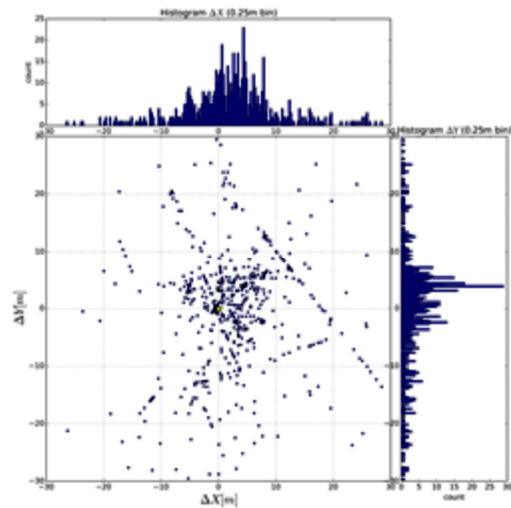


Fig. 9. Horizontal displacements with 4 satellites in view

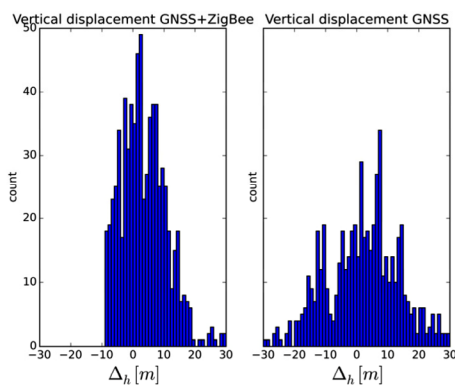


Fig. 10. Histogram of vertical displacements with 4 satellites and one ZigBee in view

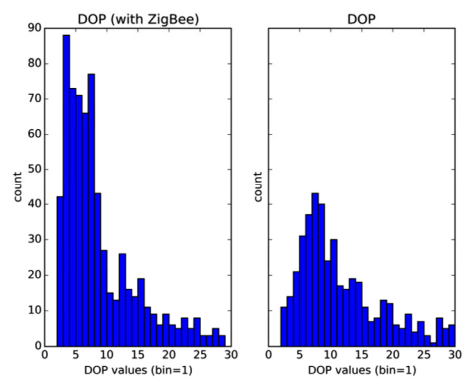


Fig. 11. Histogram of PDOP for four satellites and one ZigBee and four satellites in view

Table 2. Comparison of results for four satellites and one ZigBee and four satellites in view

	ZigBee					No ZigBee				
	dx[m]	dy[m]	dz[m]	Δ [m]	RMS	dx[m]	dy[m]	dz[m]	Δ [m]	RMS
count	715.00	715.00	715.00	715.00	715.00	715.00	715.00	715.00	715.00	715.00
mean	1.34	-2.23	3.51	6.47	1.05	18.29	-4.71	61.25	54.43	-
std	5.09	6.43	7.15	5.66	0.69	460.83	180.64	647.00	492.32	-
Min	-27.12	-31.16	-8.95	0.28	0.00	-1561.79	-3048.10	-134.96	0.27	-
25%	-0.83	-3.96	-1.84	3.09	0.52	-2.63	-7.76	-9.91	5.66	-
50%	1.68	-0.20	2.68	4.56	0.97	2.24	1.11	2.98	10.00	-
75%	3.93	2.05	8.03	7.81	1.50	6.37	5.51	13.49	21.65	-
max	12.95	5.28	29.83	35.41	3.58	11597.23	2317.12	12668.26	11991.11	-

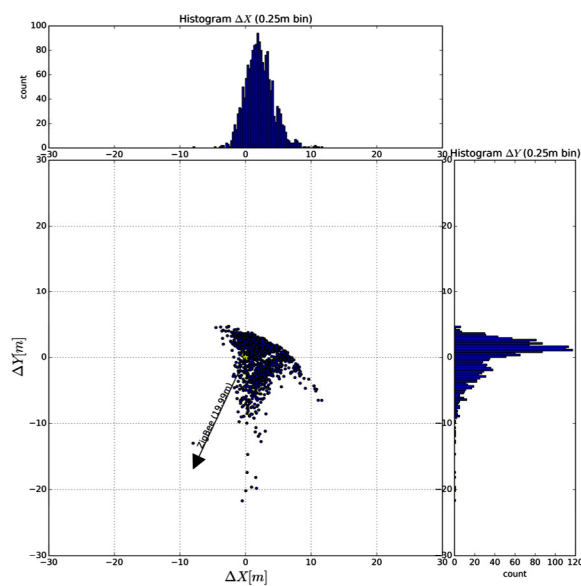


Fig. 12. Horizontal displacements with 7 satellites and one ZigBee in view

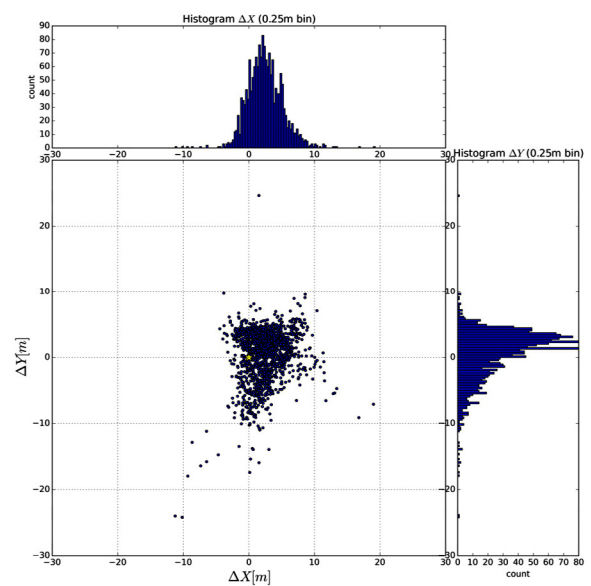


Fig. 13. Horizontal displacements with 7 satellites in view

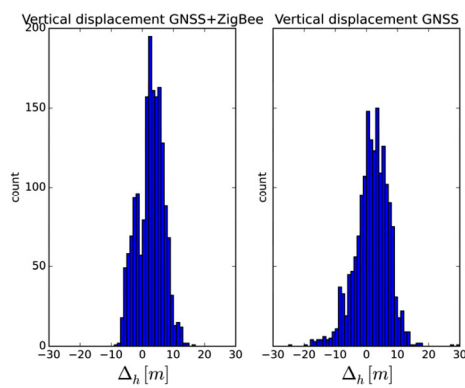


Fig. 14. Histogram of vertical displacements with 7 satellites and one ZigBee in view

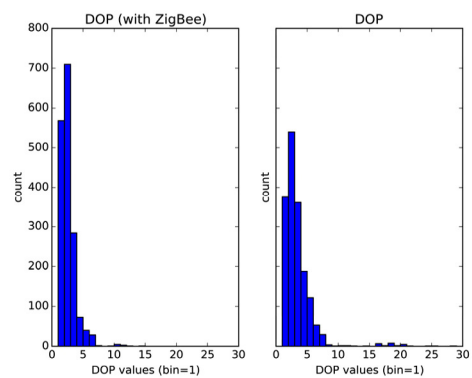


Fig. 15. Histogram of PDOP for seven satellites and one ZigBee and seven satellites in view

Table 3. Comparison of results for seven satellites and one ZigBee and seven satellites in view

	ZigBee					No ZigBee				
	dx[m]	dy[m]	dz[m]	Δ [m]	RMS	dx[m]	dy[m]	dz[m]	Δ [m]	RMS
count	1716.00	1716.00	1716.00	1716.00	1716.00	1716.00	1716.00	1716.00	1716.00	1716.00
mean	2.15	0.00	2.63	3.75	1.89	2.47	0.53	1.59	4.68	1.86
std	2.14	2.99	4.14	2.01	0.55	2.70	3.84	6.02	2.56	0.62
Min	-7.98	-21.70	-8.55	0.19	0.50	-11.25	-24.17	-55.74	0.08	0.05
25%	0.70	-1.51	-0.39	2.61	1.49	0.73	-1.37	-1.30	3.16	1.45
50%	2.00	0.95	2.85	3.44	1.85	2.27	1.43	2.06	4.28	1.78
75%	3.42	1.95	5.62	4.43	2.33	4.08	3.15	5.59	5.70	2.30
max	11.60	4.70	16.22	21.70	3.07	19.00	24.66	29.29	26.49	3.26

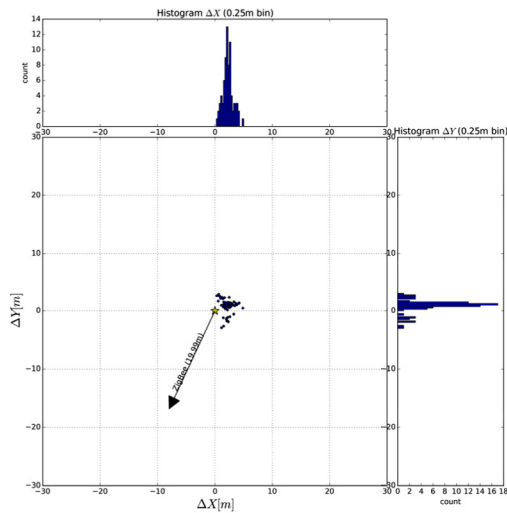


Fig.16. Horizontal displacements with 11 satellites and one ZigBee in view

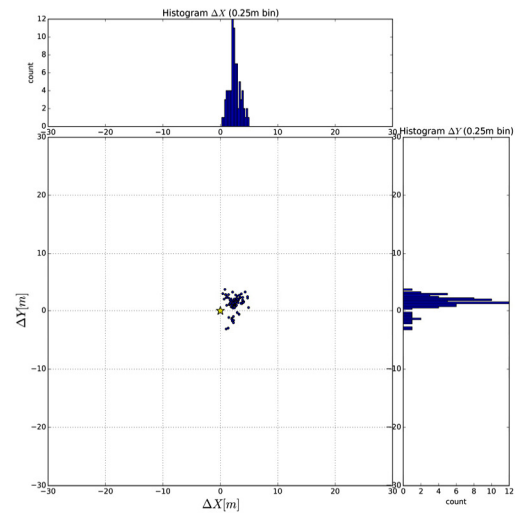


Fig.17. Horizontal displacements with 11 satellites in view

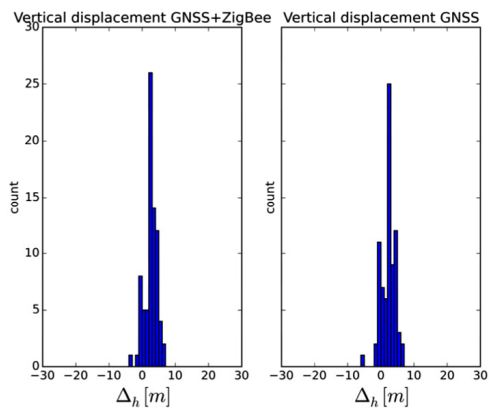


Fig. 18. Histogram of vertical displacements with 11 satellites and one ZigBee in view

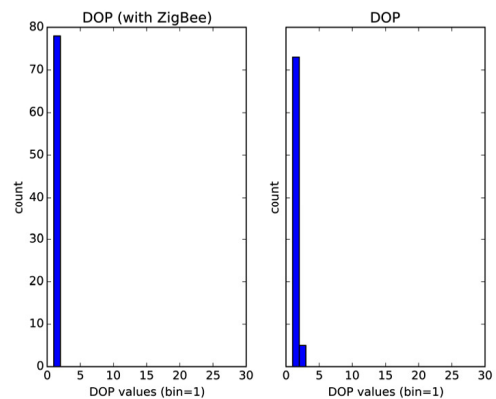


Fig. 19. Histogram of PDOP for eleven satellites and one ZigBee and eleven satellites in view

Table 4. Comparison of results for eleven satellites and one ZigBee and eleven satellites in view

	ZigBee					No ZigBee				
	dx[m]	dy[m]	dz[m]	Δ [m]	RMS	dx[m]	dy[m]	dz[m]	Δ [m]	RMS
count	78.00	78.00	78.00	78.00	78.00	78.00	78.00	78.00	78.00	78.00
mean	2.29	0.81	2.64	2.75	0.29	2.50	1.27	2.26	3.16	2.36
std	0.89	1.15	1.88	0.67	0.29	1.0	1.38	2.13	0.86	0.32
Min	0.32	-2.89	-3.28	1.47	1.51	0.43	-3.12	5.44	1.46	1.47
25%	1.80	0.64	1.96	2.34	2.16	1.99	0.95	0.68	2.53	2.14
50%	2.23	1.04	2.84	2.64	2.33	2.38	1.47	2.58	2.98	2.42
75%	2.71	1.32	3.59	3.06	2.53	3.07	2.14	3.26	3.79	2.63
max	4.86	2.84	6.67	4.88	2.59	4.90	3.65	6.69	5.39	2.69

Discussion

Summary of consecutive scenarios:

– Scenario with three satellites in view.

In this scenario, all possible combinations of three satellites (from thirteen satellites in view) were analysed. In this case positioning with no ZigBee is ambiguous due to not sufficient number of satellites in view (therefore there is no RMS in the Table 1 since the number of observations is equal to the number of unknown parameters). When GNSS was augmented with one additional ZigBee transceiver 286 solutions were obtained. The mean linear displacement is equal 9.15 m with 7.74 m standard deviation. The distribution of the results show strong influence of ZigBee observation. In the layout of consecutive results a pattern radial to a ZigBee transceiver location can be observed (causing the histograms not to be symmetrical).

– Scenario with four satellites in view.

In the scenario with four satellites in view both GNSS augmented with ZigBee and GNSS only positioning is possible (with no RMS values for GNSS only). The mean linear displacement with ZigBee is equal 6.47 m with 5.66 m standard deviation. Without ZigBee the mean linear displacement is equal 54.43 m with 492.32 m standard deviation (which is 8.41 times bigger displacement with 86.98 times bigger standard deviation then with ZigBee). In the GNSS only scenario, there is a lot of outliers and the displacement histograms are very wide. After adding ZigBee observation, the histograms are much more narrow and no significant outliers are visible. These results indicates that when the number of satellites in view is small, adding observation from ZigBee transceiver significantly improves the accuracy of positioning. This is also confirmed by the PDOP values depicted in Figures 7 and 11 (as histograms).

– Scenario with seven satellites in view.

In scenario with seven satellites in view, the mean linear displacement with ZigBee is equal 3.75 m with 2.01 m standard deviation. Without ZigBee the mean linear displacement is equal 4.68 m with 2.56 m standard deviation (which is 1.25 times bigger displacement with 1.27 times bigger standard deviation then with ZigBee). In this case the results are more similar to each other, however additional observation is slightly improving the accuracy. Also smaller number of outliers exists when the ZigBee transceiver is included in the solution.

– Scenario with eleven satellites in view.

In scenarios with eleven satellites in view, the mean linear displacement with ZigBee is equal 2.75 m with 0.67 m standard deviation. Without ZigBee the mean linear displacement is equal 3.16 m with 0.86 m standard deviation (which is 1.15 times bigger displacement with 1.28 times bigger standard deviation then with ZigBee).

Summary of results

In each case adding a short, ground based distance measurement (from ZigBee transceiver) to the point position estimation process improved the positioning accuracy. Histograms of displacements in both horizontal and vertical planes are narrower if ZigBee observation is included. In the case with no ZigBee the histograms are more symmetrical. Skewness of displacement histograms, when ZigBee is in use, is resulting from the selected method of objective function minimization. This is reflecting how the Nelder-Mead method (which is a downhill method and initial coordinates for the adjustment are equal to the ZigBee transceiver coordinates) is iterating towards the correct solution. If maximum number of Nelder-Mead simplex iterations or function evaluation is exceeded, the iterations ends closer to the initial coordinates. Ground based augmentation can improve both positioning accuracy and position reliability. The influence of additional ZigBee observation is growing with reducing number of satellites in view. 4.3. Further work This paper does not exhaust the topic of ground based augmentation of GNSS positioning using a ZigBee transceivers. Further

research on the topic of this paper will be conducted by authors. Variable number of ZigBee transceivers in various layouts, different method of objective function minimization and different possible applications will be analysed. Moreover the application of this method with receiver in motion will require additional research due to the time offset between GNSS and ZigBee observations. Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DOP: Dilution Of Precision
GBAS: Ground Based Augmentation System
GNSS: Global Navigation Satellite System
ISM: Industrial, Scientific and Medical
PDOP: Position Dilution Of Precision
RMS: Root Mean Square
TDOA: Time Differential Of Arrival
TOA: Time Of Arrival
TOF: Time Of Flight
UTM: Universal Transverse Mercator

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