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Application of Hamilton's and divisor methods to degressively proportional allocation functions

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Abstract

The most recognizable historically approved methods of proportional division of mandates in collegiate bodies refer to an ideal assumption where each vote is associated with identical number of representatives. Proportional methods of distribution such as Hamilton's and divisor methods of Jefferson, Adams or Webster cannot be directly applied to the allocation of seats between the Member States in the European Parliament because of the wide variation in their population. A desire to ensure appropriate representation have triggered legal acceptance of degressive proportionality rule contained in the Lisbon Treaty. The new principle, however, does not allow determining an unambiguous solution. The article presents the allocations which can be obtained reaching the classical methods of proportional division, taking into account degressively proportional allocation functions.

Keywords: degressive proportionality; allocation functions; proportional division; elections; apportionment problem.

1. Introduction

The principle of proportional representation as a general rule od allocation of seats in the electoral law is tied with the necessity to determine a method of converting real proportions to the integers. The number of developed solutions, however, shows that there is no ideal one. Each of the proposed methods of rounding the exact proportions has certain drawbacks, known in literature as a proneness to certain paradoxes. The situation is even more complex

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in the case of apportionment of mandates in the European Parliament. Lack of a clear indication defining the allocation degressiveness additionally complicates seeking the proper solution – there are many possible to be used.

The aim of the article is to show the applicability of the methods used for the proportional division to degressively proportional allocations. Basing on the known families of allocation functions the real apportionments are calculated and then, using classical methods, apportionments with integer values are determined. This approach allows to obtain weakly degressively proportional allocations in the sense of the Cambridge Compromise. Moreover, a method enabling to specify the measure of the equitable spread of degressive proportionality weight over all countries is indicated and on its basis an accordingly optimal allocation is selected.

2. Proportional methods of apportionment

The apportionment problem concerns determining a division of a given integer number of seats $H \geq 0$ proportionally among a set of *n* states according to their populations p_i, $i = 1,2,...,n$. The problem arises when the values of number of citizens of the state i divided by the total population $P = p_1 + p_2 + ... + p_n$ are not integer. One need to find then a vector $a = (a_1, a_2, ..., a_n)$ of nonnegative integers such that $\sum_{i=1}^{n} a_i = H$ (Balinski & Young, 1980).

The most well-known classical methods giving a solution of determining sought vector **a** are methods of Hamilton, Jefferson, Adams, Webster, Dean and Hill. First of them, also known as the method of largest reminder, was proposed by the U.S. Secretary of the Treasury Alexander Hamilton in 1792. The procedure of finding the apportionment is as follows. For a given vector of populations of the states $p = (p_1, p_2, ..., p_n)$ compute a vector of quotas $q = (q_1, q_2, ..., q_n)$, where $q_i = p_i H / P$. Next, order the fractional reminders $d_i = q_i - [q_i]$, where [*t*] denotes rounding downwards, in descending sequence $d_{i_1} \geq d_{i_2} \geq ... \geq d_{i_n}$. Assign each state *i* [q_i] seats and the remaining $m = H - \sum_{i=1}^{n} [q_i]$ ones to states $d_{i_1}, d_{i_2}, ..., d_{i_m}$ (Balinski & Young, 1977).

Jefferson's method, proposed by Thomas Jefferson, Secretary of State, was an alternative for Hamilton's solution, who was his main opponent in the U.S. government. According to this procedure one should find a divisor *d* such that $\sum_{i=1}^{n} [p_i / d] = H$. For any vector *p* and total number of seats *H* there always exists such a divisor. Furthermore, there usually is an interval of divisors returning the same allocation (Young, 1994).

Adams' and Webster's methods are simple modifications of Jefferson's method. Author of the first one proposed that values p_i / d should be rounded upwards – one, therefore, needs to find a divisor *d* such that $\sum_{i=1}^{n} [p_i / d] = H$ (where [*t*] denotes the smallest integer equal ore more than *t*). In Webster's solution fractions are rounded to the nearest integer, that is $[p_i / d + 1/2]$. As in method of Jefferson there always exists a divisor giving sought allocation (Young, 1994).

The other two divisor methods of Dean and Hill differ only in the way of rounding as well. Hill's procedure orders to assign state $i[p_i / d]$ seats if the value p_i / d is less than the geometric mean of the two nearest integers and $[p_i / d]$ otherwise. In Deans' solution rounding is based on comparison to the harmonic mean (Young, 1994).

3. Degressively proportional allocation of seats

Some cases, however, preclude the proportional representation of citizens. Such situation occurs in the European Parliament due to large diversity of populations of the Member States.

Therefore seats in the European Parliament are allocated in accordance with the "Degressive Proportionality Principle". It was introduced in art. 1 point 15 of the Lisbon Treaty: "*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats*." (Lisbon Treaty, 2007) and firstly interpreted in 2007 in the Report of the Committee on Constitutional Affairs and the European Parliament Resolution (Lamassoure & Severin, 2007):

"*The principle of degressive proportionality provided for in Article [9a] of the Treaty on European Union shall be applied as follows:*

- *the minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States;*
- *the larger the population of a country, the greater its entitlement to a large number of seats;*
- *the larger the population of a country, the more inhabitants are represented by each of its Members of the European Parliament."*

Therefore, on the basis of these two legal acts the formal definition of degressive proportionality is as follows.

Definition 1. Sequence $s = (s_1, s_2, ..., s_n)$, $s_i \geq 0$, is degressively proportional with respect to $p = (p_1, p_2, ..., p_n)$, $p_i \geq 0$ and $p_1 \leq p_2 \leq ... \leq p_n$, if it satisfies following conditions:

$$s_1 \leq s_2 \leq ... \leq s_n , \tag{1}$$

$$\frac{p_1}{s_1} \leq \frac{p_2}{s_2} \leq ... \leq \frac{p_n}{s_n} . \tag{2}$$

Definition 2. Sequence $s = (s_1, s_2, ..., s_n)$, $s_i \geq 0$, is degressively proportional with boundary conditions with respect $p = (p_1, p_2, ..., p_n)$, $p_i \geq 0$ and $p_1 \leq p_2 \leq ... \leq p_n$, if it is degressively proportional and for a triple (m, M, H) such that $m \leq M$ and $m + M \leq H$ satisfies:

$$s_1 = m , \; s_n = M \text{ and } \sum_{i=1}^{n} s_i = H . \tag{3}$$

In given definition M means the greatest suggested number, m – the smallest and H – the total number of seats in the European Parliament. Sequences s and p describe the number of seats and population of particular member states. At present we have $m = 6$, $M = 96$ and $H = 751$.

Too late ratification of the Lisbon Treaty which entered into force in December 2009 – that is after the beginning of seventh European Parliament term – caused the adoption of allocation of seats that wasn't degressively proportional. In 2013 Committee on Constitutional Affairs presented new Report on the composition of the European Parliament with a view to the 2014 elections in which interpretation of degressive proportionality has been changed (Gualtieri & Trzaskowski, 2013):

"In the application of the principle of degressive proportionality provided for in the first subparagraph of Article 14 (2) TEU, the following principles shall apply: 14(2) TEU, the following principles shall apply:

- *the allocation of seats in the European Parliament shall fully utilise the minimum and maximum numbers set by the Treaty in order to reflect as closely as possible the sizes of the respective populations of Member States;*
- *the ratio between the population and the number of seats of each Member State* ***before rounding to whole numbers*** *shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats;*"

In practice the new interpretation, known as the Cambridge Compromise, weakens the rule enabling to determine an allocation that doesn't satisfy condition (2). Instead it must satisfy

$$\frac{p_1}{A(p_1)} \le \frac{p_2}{A(p_2)} \le \ldots \le \frac{p_n}{A(p_n)}, \tag{4}$$

where $A(x)$ is a defined in paragraph 4 allocation function with boundary conditions.

4. Allocation functions

Allocation functions used to determine the degressive proportional division are associated with classical methods of proportional distribution, where elements of the sequence of quotas q are values of the linear function $f(x) = ax$. In the case of degressive proportionality it is usually impossible to use it – linear function does not meet the boundary conditions m and M, for example. Therefore, we define degressively proportional allocation function as follows.

Definition 3. Function $A:[p_1, p_n] \to [m, M]$ is for a given $p = (p_1, p_2, \ldots, p_n)$ an allocation function with boundary conditions (m, M, H) if:

$q = (q_1, q_2, \ldots, q_n)$, where $q_i = A(p_i)$ is degressively proportional with respect to p. (5)

$$A(p_1) = m, \; A(p_n) = M. \tag{6}$$

We call the allocation function with boundary conditions exact if additionally $\sum_{i=1}^{n} A(p_i) = H$ holds.

There are indubitably many functions satisfying conditions of definition 3. Five very natural families of allocation functions always returning degressively proportional allocation are described in (Słomczyński & Życzkowski, 2011). Depending on sequence p and values of m, M, H they are characterized by various properties. With a view to adapt them to the European Parliament, we may consider the following families:

$$A_I(x) = \min\left(m + \frac{x - p_1}{\delta}, M\right), \tag{7}$$

where $\frac{p_1}{m} \le \delta \le \frac{p_n - p_1}{M - m}$;

$$A_{II}(x) = \min\left(\frac{mx}{p_1}, M + \frac{x - p_n}{\delta}\right), \tag{8}$$

where $\delta \ge \frac{p_n - p_1}{M - m}$;

$$A_{III}(x) = \left(\frac{x - p_1}{p_n - p_1}\frac{M}{p_n} + \frac{p_n - x}{p_n - p_1}\frac{m}{p_1}\right)x - \delta(x - p_1)(p_n - x), \tag{9}$$

where $0 \ge \delta - \frac{m / p_1 - M / p_n}{p_n - p_1} \ge \frac{\min\left(M - m, mp_n / p_1 - M\right)}{\left(p_n - p_1\right)^2}$;

$$A_{IV}(x) = M\frac{x^{\delta} - p_1^{\delta}}{p_n^{\delta} - p_1^{\delta}} + m\frac{p_n^{\delta} - x^{\delta}}{p_n^{\delta} - p_1^{\delta}}, \tag{10}$$

where $0 \le \delta \le 1$ and $\left(M / m - 1\right)\delta \le \left(p_n / p_1\right)^{\delta} - 1$;

$$A_V(x) = \frac{M\left(x / M - \delta\right)(x - p_1) + m\left(x / m - \delta\right)(p_n - x)}{\left(p_n / M - \delta\right)(x - p_1) + \left(p_1 / m - \delta\right)(p_n - x)}, \tag{11}$$

where $\delta \le p_1 / M$.

In each family parameter δ is set by the constraint that the total number of seats is fixed.

5. Composition of the EP with the use of classical methods

Hamilton's and divisor methods give a solution of finding an allocation of indivisible goods in the most proportionate way by treating the vector of quotas as an "ideal" division. The allocation functions may be considered in a similar manner in the context of degressively proportional allocations. A sequence $(A(p_1), A(p_2),..., A(p_n))$ may be used as a degressively proportional pattern on the basis of which an allocation using classical methods is determined. Achieved allocation fulfills the "weakened" definition of degressive proportionality, i.e. it satisfies conditions (1), (2'), (3).

Table 1 presents degressively proportional bases determined for each class of the specified allocation functions. Calculations were based on data from 2012. Such choice was dictated by the use of them in 2013 Report.

Table 1. Degressively proportional bases determined by allocation functions

Country	Population	$A_I(p)$	$A_{II}(p)$	$A_{III}(p)$	$A_{IV}(p)$	$A_V(p)$
Malta	416,110	6	6	6	6	6
Luxemburg	524,853	6.132021	7.567994	6.14401	6.18152	6.145657
Cyprus	862,011	6.541352	8.763056	6.59011	6.723476	6.596742
Estonia	1,339,622	7.121203	9.277559	7.220998	7.45653	7.234386
Latvia	2,041,763	7.973647	10.03394	8.146242	8.489082	8.168923
Slovenia	2,055,496	7.99032	10.04873	8.164312	8.508879	8.187167
Lithuania	3,007,758	9.146427	11.07455	9.414837	9.85392	9.449092
Ireland	4,582,769	11.05859	12.77121	11.47244	11.99104	11.52268
Finland	5,401,267	12.0523	13.65294	12.53646	13.07104	12.59364
Slovakia	5,404,322	12.05601	13.65623	12.54042	13.07504	12.59763
Denmark	5,580,516	12.26992	13.84603	12.76899	13.3053	12.82757
Bulgaria	7,327,224	14.39054	15.72766	15.02589	15.55241	15.09594
Austria	8,443,018	15.74518	16.92964	16.45899	16.95918	16.53438
Sweden	9,482,855	17.00761	18.04979	17.78849	18.25355	17.86754
Hungary	9,957,731	17.58414	18.56135	18.39371	18.83988	18.47403
Czech Rep.	10,505,445	18.2491	19.15137	19.09026	19.51271	19.17172
Portugal	10,541,840	18.29328	19.19058	19.13649	19.55729	19.21802
Belgium	11,041,266	18.89962	19.72858	19.77012	20.16755	19.85239
Greece	11,290,067	19.20168	19.9966	20.08528	20.47054	20.16782

Country	Population	$A_I(p)$	$A_{II}(p)$	$A_{III}(p)$	$A_{IV}(p)$	$A_V(p)$
Netherlands	16,730,348	25.80653	25.8571	26.89312	26.95219	26.96695
Romania	21,355,849	31.42218	30.83989	32.55595	32.29508	32.60397
Poland	38,538,447	52.28296	49.34971	52.58262	51.29248	52.46152
Spain	46,196,276	61.58005	57.59904	60.99559	59.46046	60.79963
Italy	60,820,764	79.33512	73.35316	76.18442	74.70396	75.92431
U. Kingdom	62,989,550	81.96816	75.68947	78.33879	76.93125	78.08287
France	65,397,912	84.89207	78.28386	80.70146	79.39565	80.45543
Germany	81,843,743	96	96	96	96	96
Total	–	751	751	751	751	751
Δ	–	823679.8	928296.3	1.60E-13	0.9005	–3621087

It is easy to see that the first class favors more populated countries, second - the smaller ones. Other families return similar sequences. Therefore, willing to eliminate the effect of deviation, it is desirable to indicate a measure enabling to compare the results. For that purpose one may use an index of internal degression describing the biggest deviation from proportionality, proposed by J. Łyko (Łyko, 2013). The measure is based on a number $U_I = \max_i(|u_i|)$, where $u_i = \frac{z_i - u^*}{z_i + u^*}$, $z_i = \frac{p_{i+1} - p_i}{A(p_{i+1}) - A(p_i)}$, $u^* = \frac{p_n - p_1}{A(p_n) - A(p_1)}$, for $i = 1, 2, ..., n-1$. Table 2 shows values of the coefficients u_i and U_I for each class of allocation functions.

Table 2. Internal degression of allocation functions

Country	Population	$A_I(p)$	$A_{II}(p)$	$A_{III}(p)$	$A_{IV}(p)$	$A_V(p)$
Malta	416,110	–	–	–	–	–
Luxemburg	524,853	0.04690	0.85761	0.09015	0.20327	0.09579
Cyprus	862,011	0.04690	0.52459	0.08971	0.18511	0.09522
Estonia	1,339,622	0.04691	0.01284	0.08888	0.16270	0.09416
Latvia	2,041,763	0.04690	0.01284	0.08769	0.14182	0.09264
Slovenia	2,055,496	0.04691	0.01297	0.08696	0.13204	0.09171
Lithuania	3,007,758	0.04690	0.01284	0.08598	0.12201	0.09048
Ireland	4,582,769	0.04690	0.01285	0.08340	0.10219	0.08724
Finland	5,401,267	0.04691	0.01284	0.08094	0.08834	0.08417
Slovakia	5,404,322	0.04704	0.01299	0.07952	0.08451	0.08327
Denmark	5,580,516	0.04690	0.01285	0.07991	0.08357	0.08288
Bulgaria	7,327,224	0.04691	0.01284	0.07792	0.07576	0.08044
Austria	8,443,018	0.04690	0.01284	0.07495	0.06572	0.07680
Sweden	9,482,855	0.04691	0.01285	0.07269	0.05936	0.07406
Hungary	9,957,731	0.04690	0.01284	0.07110	0.05531	0.07214
Czech Rep.	10,505,445	0.04690	0.01285	0.07003	0.05277	0.07084
Portugal	10,541,840	0.04684	0.01280	0.06944	0.05133	0.07019
Belgium	11,041,266	0.04691	0.01285	0.06885	0.05012	0.06943
Greece	11,290,067	0.04690	0.01284	0.06806	0.04844	0.06849
Netherlands	16,730,348	0.04690	0.01284	0.06199	0.03751	0.06136
Romania	21,355,849	0.04690	0.01284	0.05108	0.02204	0.04880
Poland	38,538,447	0.04690	0.01284	0.02653	0.00016	0.02229

Country	Population	$A_I(p)$	$A_{II}(p)$	$A_{III}(p)$	$A_{IV}(p)$	$A_V(p)$
Spain	46,196,276	0.04690	0.01285	0.00302	0.01780	0.00749
Italy	60,820,764	0.04690	0.01284	0.03111	0.02931	0.03322
U. Kingdom	62,989,550	0.04690	0.01284	0.05333	0.03672	0.05236
France	65,397,912	0.04691	0.01284	0.05955	0.03853	0.05747
Germany	81,843,743	0.24139	0.01284	0.08599	0.04522	0.07807
Max	–	0.24139	0.85761	0.09015	0.20327	0.09579
Mean	–	0.05439	0.06503	0.06918	0.07660	0.07081
U_I /mean	–	4.438346	13.18852	**1.303109**	2.653558	1.352822

Function $A_{III}(p)$ is therefore most reliable in the sense of proportionality. Taking then into account the entire structure of the allocation, it determines the most preferable pattern division. Allocations based on function $A_{III}(p)$ shows Table 3. Columns 3–8 contain allocations s_{Ha}, s_{We}, s_{Je}, s_{Ad}, s_{Hi} and s_{De} which were obtained in the generalized methods of Hamilton, Webster, Jefferson, Adams, Hill and Dean. It is worth noting that the allocation obtained using the Hamilton method matches the result obtained by the Webster, Hill and Dean method. Apportionments for the remaining families of functions are presented in the Appendix.

Table 3. Proposal of allocation of seats in the European Parliament based on the function $A_{III}(x)$

Country	Population	s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}
Malta	416,110	6	6	6	6	6	6
Luxemburg	524,853	6	6	6	7	6	6
Cyprus	862,011	7	7	6	7	7	7
Estonia	1,339,622	7	7	7	8	7	7
Latvia	2,041,763	8	8	8	9	8	8
Slovenia	2,055,496	8	8	8	9	8	8
Lithuania	3,007,758	9	9	9	10	9	9
Ireland	4,582,769	11	11	11	12	11	11
Finland	5,401,267	13	13	12	13	13	13
Slovakia	5,404,322	13	13	12	13	13	13
Denmark	5,580,516	13	13	13	13	13	13
Bulgaria	7,327,224	15	15	15	15	15	15
Austria	8,443,018	16	16	16	16	16	16
Sweden	9,482,855	18	18	18	18	18	18
Hungary	9,957,731	18	18	18	18	18	18
Czech Rep.	10,505,445	19	19	19	19	19	19
Portugal	10,541,840	19	19	19	19	19	19
Belgium	11,041,266	20	20	20	20	20	20
Greece	11,290,067	20	20	20	20	20	20
Netherlands	16,730,348	27	27	27	26	27	27
Romania	21,355,849	33	33	33	32	33	33
Poland	38,538,447	53	53	53	52	53	53
Spain	46,196,276	61	61	62	60	61	61
Italy	60,820,764	76	76	77	75	76	76

Country	Population	s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}
U. Kingdom	62,989,550	78	78	79	78	78	78
France	65,397,912	81	81	81	80	81	81
Germany	81,843,743	96	96	96	96	96	96
Total	–	751	751	751	751	751	751

6. Conclusion

The analysis shows that in each of the considered five families of functions there is an unambiguously determined exact allocation function. Real degressively proportional allocations $(A(p_1), A(p_2), ..., A(p_n))$ generated by them substantially differ from each other. For functions $A_I(x)$ and $A_{II}(x)$, a bias may be clearly noticed. First of them favors more populated countries, second – the smaller ones. According to applied criterion of internal degression measure, the best solution is the adoption of the allocation function $A_{III}(x)$. It is difficult, therefore, to agree with the statement contained in the Report on the composition of the European Parliament with a view to the 2014 elections that proposition of the Cambridge Compromise using function $A_I(x)$ is "the most 'proportional' mechanism respecting degressive proportionality" (Gualtieri & Trzaskowski, 2013).

Appendix. Proposal of allocation of seats in the European Parliament based on the allocation function generated using classical methods (families I, II, IV, V)

Table 4. Proposal of allocation of seats in the European Parliament based on the allocation function generated using classical methods (families I, II)

Country	Population	$A_I(p)$						$A_{II}(p)$					
		s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}	s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}
Malta	416,110	6	6	6	6	6	6	6	6	6	6	6	6
Luxemburg	524,853	6	6	6	7	6	6	7	8	7	8	8	8
Cyprus	862,011	7	7	6	7	7	7	9	9	9	8	9	9
Estonia	1,339,622	7	7	7	8	7	7	9	9	10	9	9	9
Latvia	2,041,763	8	8	8	8	8	8	10	10	10	10	10	10
Slovenia	2,055,496	8	8	8	8	8	8	10	10	10	10	10	10
Lithuania	3,007,758	9	9	9	10	9	9	11	11	11	11	11	11
Ireland	4,582,769	11	11	11	11	11	11	13	13	13	13	13	13
Finland	5,401,267	12	12	12	12	12	12	14	13	14	13	13	13
Slovakia	5,404,322	12	12	12	12	12	12	14	14	14	13	14	14
Denmark	5,580,516	12	12	12	13	12	12	14	14	14	14	14	14
Bulgaria	7,327,224	14	14	14	15	14	14	16	16	16	16	16	16
Austria	8,443,018	16	16	15	16	16	16	17	17	17	17	17	17
Sweden	9,482,855	17	17	17	17	17	17	18	18	18	18	18	18
Hungary	9,957,731	18	18	17	18	18	18	18	18	19	18	18	18
Czech Rep.	10,505,445	18	18	18	18	18	18	19	19	19	19	19	19
Portugal	10,541,840	18	18	18	18	18	18	19	19	19	19	19	19
Belgium	11,041,266	19	19	19	19	19	19	20	20	20	20	20	20
Greece	11,290,067	19	19	19	19	19	19	20	20	20	20	20	20
Netherlands	16,730,348	26	26	26	26	26	26	26	26	26	26	26	26

Country	Population	$A_I(p)$						$A_{II}(p)$					
		s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}	s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}
Romania	21,355,849	32	31	31	31	31	31	31	31	31	31	31	31
Poland	38,538,447	52	52	53	52	52	52	49	49	49	49	49	49
Spain	46,196,276	62	62	62	61	62	62	58	58	57	58	58	58
Italy	60,820,764	79	80	80	78	80	80	73	73	73	74	73	73
U. Kingdom	62,989,550	82	82	83	81	82	82	76	76	75	76	76	76
France	65,397,912	85	85	86	84	85	85	78	78	78	79	78	78
Germany	81,843,743	96	96	96	96	96	96	96	96	96	96	96	96
Total	-	751	751	751	751	751	751	751	751	751	751	751	751

Table 5. Proposal of allocation of seats in the European Parliament based on the allocation function generated using classical methods (families IV, V)

Country	Population	$A_{IV}(p)$						$A_V(p)$					
		s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}	s_{Ha}	s_{We}	s_{Je}	s_{Ad}	s_{Hi}	s_{De}
Malta	416,110	6	6	6	6	6	6	6	6	6	6	6	6
Luxemburg	524,853	6	6	6	7	6	6	6	6	6	7	6	6
Cyprus	862,011	7	7	6	7	7	7	7	7	6	7	7	7
Estonia	1,339,622	7	7	7	8	7	7	7	7	7	8	7	7
Latvia	2,041,763	9	8	8	9	9	9	8	8	8	9	8	8
Slovenia	2,055,496	9	9	8	9	9	9	8	8	8	9	8	8
Lithuania	3,007,758	10	10	10	10	10	10	9	9	9	10	9	9
Ireland	4,582,769	12	12	12	12	12	12	12	12	11	12	12	12
Finland	5,401,267	13	13	13	13	13	13	13	13	12	13	13	13
Slovakia	5,404,322	13	13	13	13	13	13	13	13	12	13	13	13
Denmark	5,580,516	13	13	13	13	13	13	13	13	13	13	13	13
Bulgaria	7,327,224	16	16	16	16	16	16	15	15	15	15	15	15
Austria	8,443,018	17	17	17	17	17	17	17	17	17	16	17	17
Sweden	9,482,855	18	18	18	18	18	18	18	18	18	18	18	18
Hungary	9,957,731	19	19	19	19	19	19	18	18	19	18	18	18
Czech Rep.	10,505,445	20	20	20	19	20	20	19	19	19	19	19	19
Portugal	10,541,840	20	20	20	19	20	20	19	19	19	19	19	19
Belgium	11,041,266	20	20	20	20	20	20	20	20	20	20	20	20
Greece	11,290,067	20	21	21	20	20	20	20	20	20	20	20	20
Netherlands	16,730,348	27	27	27	27	27	27	27	27	27	26	27	27
Romania	21,355,849	32	32	33	32	32	32	33	33	33	32	33	33
Poland	38,538,447	51	51	51	51	51	51	52	52	53	52	52	52
Spain	46,196,276	59	59	60	59	59	59	61	61	62	60	61	61
Italy	60,820,764	75	75	75	75	75	75	76	76	76	75	76	76
U. Kingdom	62,989,550	77	77	77	77	77	77	78	78	78	78	78	78
France	65,397,912	79	79	79	79	79	79	80	80	81	80	80	80
Germany	81,843,743	96	96	96	96	96	96	96	96	96	96	96	96
Total	–	751	751	751	751	751	751	751	751	751	751	751	751

Table 6. Minimal and maximal number of seats obtained using allocation functions (families I–V)

Country	Population	Min	Max
Malta	416,110	6	6
Luxemburg	524,853	6	7
Cyprus	862,011	6	9
Estonia	1,339,622	7	10
Latvia	2,041,763	8	10
Slovenia	2,055,496	8	10
Lithuania	3,007,758	9	11
Ireland	4,582,769	11	13
Finland	5,401,267	12	14
Slovakia	5,404,322	12	14
Denmark	5,580,516	12	14
Bulgaria	7,327,224	14	16
Austria	8,443,018	15	17
Sweden	9,482,855	17	18
Hungary	9,957,731	17	19
Czech Rep.	10,505,445	18	20
Portugal	10,541,840	18	20
Belgium	11,041,266	19	20
Greece	11,290,067	19	21
Netherlands	16,730,348	26	27
Romania	21,355,849	31	33
Poland	38,538,447	49	53
Spain	46,196,276	57	62
Italy	60,820,764	73	80
U. Kingdom	62,989,550	75	83
France	65,397,912	78	86
Germany	81,843,743	96	96

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