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On a decision rule for mixed strategy searching under uncertainty on the basis of the coefficient of optimism

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Abstract

The paper presents a new decision rule which may be applied for mixed strategy searching under uncertainty, which means that the decision maker (DM) is able to prepare a payoffs' matrix with possible alternatives and scenarios, but he does not know the probability of the occurrence of particular states of nature. The advantage of the new method, called the β-decision rule for mixed strategies, is the fact that it takes into account the DM's coefficient of optimism on the basis of which a set of the most probable events is suggested and an optimization model is formulated and solved.

Keywords: uncertainty; coefficient of optimism; mixed strategy; optimization model; decision rule; scenarios.

1. Introduction

In the paper we will try to figure out an optimization model allowing the decision maker to find, under uncertainty, a mixed strategy considering his or her level of optimism.

We will assume that the *uncertainty* occurs when one is not able to anticipate the future effectively. One may just forecast various phenomena and events, but it is difficult to estimate the exact value of particular parameters (company profit, demand for a product, product prices etc.). Then, the decision maker (DM) has to choose the appropriate alternative (decision, action, strategy, project) on the basis of some scenarios (states of nature, events) predicted by experts, him- or herself, but the DM does not know the probability of these scenarios. If the likelihoods

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were known one would make the decisions under *risk*, DMUR – decision making under risk, not under uncertainty, DMUU – decision making under uncertainty (Groenewald & Pretorius, 2011; Render, Stair, & Hanna, 2006; Chronopoulos, Reyck, & Siddiqui, 2011; Officer & Anderson, 1968; Sikora, 2008; Trzaskalik, 2008).

(Knight, 1921) first introduced the idea to apply risk and uncertainty in economics, but these two categories were formally integrated in economic theory by (Neumann & Morgenstern, 1944).

Let us also add that we will even focus on cases for which the probability distribution aforementioned exists, but the crucial fact is that the decision maker does not know it or is not willing to use those available data. (Williams, Smith, & Young, 1997) and (Courtney, Kirkland, & Viquerie, 1997) call such a type of uncertainty – the *subjective uncertainty* (in contrast to *certainty* where the future results are known, *objective uncertainty* where parameters are presented by means of random variables with a given probability distribution, or *ignorance* which implies the lack of information concerning scenarios and possible values of parameters considered).

On the other hand, (Teczke, 1996) names it *strategic uncertainty* (in contradiction to *probabilistic uncertainty*, within which the probability distribution is known, and *stochastic uncertainty* where the likelihood can be calculated on the basis of a sample).

Hence, the result of the choice made by the decision maker under uncertainty depends on two factors: which decision (or which combination of decisions) will be selected and which scenario (state of nature) will occur in the future.

The DMUU may be presented with the aid of a profits' matrix or payoffs' matrix (Table 1) where m is the number of mutually exclusive scenarios (let us denote them by S_1, S_2, …, S_m), n signifies the number of decisions (D_1, D_2, …, D_n) and a_{ij} is the profit connected with the scenario S_i and the alternative D_j. We will assume that outcomes in the matrix are monetary payoffs.

Table 1. Payoffs' matrix / decision table (general case)

Scenarios and decisions	D_1	D_j	D_n
S_1	a_{11}	a_{1j}	a_{1n}
S_i	a_{i1}	a_{ij}	a_{in}
S_m	a_{m1}	a_{mj}	a_{mn}

There exist many decision rules applied within the decision making under uncertainty (e.g. Wald, 1950; Hurwicz, 1951; Hurwicz, 1952; Savage, 1961; Basili, 2006; Basili & Zappia, 2010; Basili, Chateauneuf, & Fontini, 2008; Gilboa, 2009; Ghirardato, Maccheroni, & Marinacci, 2004; Ellsberg, 2001; Etner, Jeleva, & Tallon, 2012; Gaspars-Wieloch, 2013a; Gaspars-Wieloch, 2013b; Marinacci, 2002; Nakamura, 1986). Some of them are dedicated to optimal pure strategy's searching, other are designed for optimal mixed strategy's searching.

When we talk about *pure strategies*, the decision maker chooses and completely executes only one alternative. On the other side, a *mixed strategy* (mixed acts, mixed actions) implies that the decision maker selects and performs a weighted combination of several accessible alternatives, see e.g. bonds portfolio construction, cultivation of different plants (Officer & Anderson, 1968). This paper will concern the second case.

Existing decision rules for mixed strategies rather relate to the game theory, i.e. game between players (Czerwiński, 1969; Gilboa, 2009; Luce & Raiffa, 1957), than the game against nature (which constitutes a neutral opponent). Nevertheless one may find in the literature several approaches for the last situation.

Depending on the DM's target, it is possible to formulate and solve, among other things, the max-min problem (Wald's problem) or the Bayes' problem which allow to establish an optimal mixed strategy (see e.g. Puppe & Schlag, 2009; Sikora, 2008). The first rule is designed for radical pessimists intending to perform the selected solution only once. The second rule assumes that the DM is going to execute the recommended plan many times.

In this paper a new decision rule is presented. Its final aim consists in finding a suitable mixed strategy for a DM who is interested in a single execution of the chosen strategy and who is not a radical pessimist, but is rather moderate or even optimist and knows his level of optimism.

The remainder of the paper is organized as follows. In Section 2 the Wald's and Bayes' rules for mixed strategies are briefly described. Section 3 presents the new decision rule. Section 4 contains an illustration of the suggested approach. Conclusions are gathered in Section 5.

2. The Wald's and Bayes' decision rules for mixed strategies

The Wald's rule assumes that since the DM intends to perform the mixed strategy only once, it is reasonable to behave cautiously and to maximize the minimal benefit. Therefore, the optimization model corresponding to such an approach has the following structure (Officer & Anderson, 1968):

$$y \to \max \quad (1)$$

$$\sum_{j}^{n} a_{ij} x_j \geq y \qquad i = 1,\dots,m \quad (2)$$

$$\sum_{j}^{n} x_j = 1 \quad (3)$$

$$x_j \geq 0 \qquad j = 1,\dots,n \quad (4)$$

where y is the minimum guaranteed revenue, n denotes the number of decisions, a_{ij} is the payoff connected with the state S_i and the decision D_j, x_j signifies the share of the alternative D_j in the mixed strategy and m is still the number of scenarios.

The solution of such a model ensures that even if the least attractive scenario takes place, the income of the decision maker will not be lower than $y*$ which denotes the maximized minimum guaranteed revenue.

The Bayes' rule, as it was mentioned above, assumes that the DM will perform the selected plan many times. That is why, in such circumstances it seems to be logical to maximize the average income. Hence, the optimization model emerges in the following form:

$$\sum_{j=1}^{n} \overline{a_j} x_j \to \max \quad (5)$$

$$\sum_{j}^{n} x_j = 1 \quad (6)$$

$$x_j \geq 0 \qquad j = 1,\dots,n \quad (7)$$

$$\overline{a_j} = \frac{1}{m} \sum_{i=1}^{m} a_{ij} \qquad j = 1,\dots,n \quad (8)$$

where n denotes the number of decisions, $\overline{a_j}$ is the average payoff for the alternative D_j, x_j signifies the share of the alternative D_j in the mixed strategy, a_{ij} is the payoff connected with the state S_i and the decision D_j and m is the number of scenarios.

As one may observe, the maximized average income ($a*$) is at least as high as the maximized minimum guaranteed revenue ($y*$).

Obviously, from the DM's point of view, the most desirable situation would occur if such a state of nature S_i took place and such an alternative D_j was chosen, for which the payoff a_{ij} fulfills the Equation (9), compare with the max-max rule for pure strategies (Trzaskalik, 2008) and see Equation (10).

$$a_{ij} = \max_{\substack{i=1,\dots,m \\ j=1,\dots,n}} \{a_{ij}\} = M* \quad (9)$$

$$M^* \geq a^* \geq y^* \tag{10}$$

The decision maker may introduce some additional constraints to the models (e.g. the share of the strategy D_1 in the mixed strategy may not exceed the share of the strategy D_2). In these circumstances the maximized minimum guaranteed revenue and the maximized average income are usually lower than y^* and a^* computed by means of Equations (1)–(4) and (5)–(8) respectively. Such supplementary conditions also unable to obtain the level M^*.

3. The β-decision rule for mixed strategies

Now, let us check how we can formulate the optimization model when the decision maker intends to perform the mixed strategy only once but reckons on a higher income than y^* (the maximized minimum guaranteed income) and is able to estimate his or her coefficient of optimism (β) which satisfies, of course, the condition (11):

$$\beta \in [0,1] \tag{11}$$

One possible approach may consist in applying the concept of the Hurwicz's rule (Hurwicz, 1951; Hurwicz, 1952) which was originally proposed for pure strategy searching.

The model could then emerge in the following form:

$$\sum_{j=1}^{n} h_j x_j \to \max \tag{12}$$

$$\sum_{j}^{n} x_j = 1 \tag{13}$$

$$x_j \geq 0 \qquad j = 1,\ldots,n \tag{14}$$

$$h_j = \beta \cdot \max_i \{a_{ij}\} + (1-\beta) \cdot \min_i \{a_{ij}\} \qquad j = 1,\ldots,n \tag{15}$$

where h_j is the Hurwicz's index of the decision D_j.

However, it is worth emphasizing that the Hurwicz's rule takes into consideration only extreme payoffs connected with a given alternative, not the frequency of occurrence of the intermediate ones (see Gaspars-Wieloch, 2012; Gaspars-Wieloch, 2013a; Gaspars-Wieloch, 2014), which may lead to quite illogical recommendations.

Additionally, the Hurwicz's rule uses the information about the DM's risk aversion to predict the maximum and the minimum payoffs related to particular decisions and not to forecast the true scenario. Such reasoning means that usually the best and the worst state of nature vary depending on the decision considered (Gaspars-Wieloch, 2013b) – the scenario S_1 may be the best for the alternative D_1 and may be at the same time the worst for the alternative D_2.

Therefore, the approach presented below will be significantly different in order to avoid drawbacks of the original Hurwicz's rule.

We will assume that when β tends to 1, then the DM is an optimist and expects a very high income, close to the payoff characterized by the formula (9), i.e. M^*. On the other hand, when β tends to 0, the decision maker is less demanding and wants to gain at least y^*.

Additionally, the parameter β will have an impact on the set of the most likely scenarios. One may make the following assumption – higher the value of this parameter is, less probable are, in the DM's opinion, states of nature offering low payoffs. Therefore, such scenarios will not be considered in the optimization model.

The optimization model corresponding to the new rule, called the β-decision rule for mixed strategies, may be formulated as follows:

$$\sum_{i \in K} \max\{g_i, 0\} \to \min \tag{16}$$

$$\sum_{j}^{n} a_{ij} x_j = r_\beta - g_i \qquad i \in K \tag{17}$$

$$\sum_{j}^{n} x_j = 1 \tag{18}$$

$$x_j \geq 0 \qquad j = 1,...,n \tag{19}$$

$$r_\beta = \beta(M^* - y^*) + y^* \tag{20}$$

$$g_{i^*} \leq 0 \tag{21}$$

where K denotes the set of the most probable events, r_β is the expected level of revenue dependent on β, g_i is the deviation from r_β of the income achieved by the DM if the scenario S_i occurs, n is the number of decisions, a_{ij} is the payoff connected with the state S_i and the decision D_j, x_j is the share of the alternative D_j in the mixed strategy.

The constraint (20) shows the relationship between the coefficient of optimism and the expected revenue. Both sides of the condition (17) present the true revenue obtained if the shares for a particular mixed strategy equal x_1, x_2, …, x_n and the scenario S_i takes place. The aim (Equation 16) is to minimize, within the set K, the sum of all deviations between the true incomes and the expected ones. Notice that only positive deviations are treated as unwanted since then the expected revenue exceeds the true income. If the decision maker claims that a concrete event (let us say S_{i^*}) will take place, he or she may additionally introduce the constraint (21) which enables to set such values of the variables x_1, x_2, …, x_n that guarantee that for this particular state of nature the true income will be not lower than the expected one. Nevertheless, it is worth emphasizing that the event selected by the DM must obligatorily fulfill the first condition given in Equation (22). Otherwise, the problem is contradictory and does not have any feasible solution.

One of the main question is how to select the most probable scenarios on the basis of the coefficient of optimism declared by the decision maker. Theoretically, there are many possible approaches (see e.g. Gaspars-Wieloch, 2013b), but it is difficult to state which one indicates the most suitable set of events. Let us say that this set should contain all scenarios that satisfy the conditions (22)–(25):

$$S_i \in K \Leftrightarrow \left(\underset{a_{ij}\,(j=1,...,n)}{\exists} a_{ij} \geq r_\beta \right) \vee \left(d_i \geq d_\beta \right) \tag{22}$$

$$d_\beta = \beta(d_{\max} - d_{\min}) + d_{\min} \tag{23}$$

$$d_i = \sum_{j=1}^{n} d_{ij} \tag{24}$$

$$d_{ij} = m - \max\{p(a_{ij})\} \qquad i = 1,...,m;\ j = 1,...,n \tag{25}$$

where d_{ij} denotes the number of payoffs related to the alternative D_j which are worse than the payoff a_{ij}. The symbol m still signifies the number of scenarios and $p(a_{ij})$ is the position of the payoff a_{ij} in the non-increasing sequence of all profits connected with the decision D_j (when a_{ij} has the same value than other payoffs concerning a given alternative, then it is recommended to choose the farthest position of this payoff in the sequence – see Equation 25).

d_i is the total number of "dominance cases" related to the state S_i, d_{max} and d_{min} are the highest and the lowest number of "dominance cases" respectively. A scenario S_i may belong to K if and only if it contains at least one payoff not lower than r_β or its number of "dominance cases" is sufficiently close to d_{max} (Equation 22). The scenario with d_{max} and with at least one payoff equal to M^* might be treated as the best state of nature, but in many decision problems such an event does not exist.

Hence, the β-decision rule for mixed strategies includes the following steps:

- Step 1: Given a set of potential decisions and a payoffs' matrix for the decision problem considered, choose an appropriate value of the parameter β according to your level of optimism.
- Step 2: Solve the max-min problem in order to calculate the maximized minimum guaranteed income (y^*), Equations (1)–(4).
- Step 3: Find the maximum revenue (M^*), Equation (9).
- Step 4: Find the set of the most probable scenarios (K) with the aid of the Equations (22)–(25).
- Step 5: Solve the β-decision problem using Equations (16)–(21) or (16)–(20) and additional constraints if necessary. The optimal solution represents the mixed strategy reflecting the level of optimism declared.

4. Illustration

Now let us analyze a simple decision problem. Assume that the decision maker wants to invest his capital in at least one out of 4 alternatives (D_1, D_2, D_3, D_4) giving an annual income dependent on the state of nature. Experts claim that there are 4 possible scenarios (S_1, S_2, S_3, S_4). The payoffs' matrix is presented in Table 2.

Each payoff a_{ij} represents the annual income (in mln Euro) provided that the decision maker allocates 100% of his capital in the investment D_j and that the scenario S_i happens. Assume also that the income connected with the alternative D_j is directly proportional to the quantity of capital allocated in this decision. The decision maker intends to find an appropriate mixed strategy taking into account his level of optimism.

Table 2. Payoffs' matrix (case study)

Scenarios and decisions	D_1	D_2	D_3	D_4
S_1	5	7	6	7
S_2	3	5	2	10
S_3	8	3	1	3
S_4	9	9	11	2

In order to solve this problem let us apply the β-decision rule for mixed strategies.

- Step 1: Assume that β=0.6.
- Step 2: The solution of the max-min problem for the case study is as follows: x_1=0.5714, x_2=0.0286, x_3=0, x_4=0.4, y^*=5.857. So, in order to gain 5.86 mln Euro in one year (this is the maximized minimum guaranteed revenue), the DM ought to assign about 57% of his capital to the investment D_1, almost 3% to the investment D_2 and 40% to the investment D_4.
- Step 3: The maximum revenue equals M^*=11.
- Step 4: All scenarios are Pareto-optimal, but according to the results presented below the state S_3 does not belong to the set K. The expected income equals $r_\beta = 0.6(11-5.857)+5.857 = 8.9429$. Thus the scenarios S_2 and S_4 are the only ones that contain at least one payoff not lower than 8.9429. On the other hand $d_\beta = 0.6(9-3)+3 = 6.6$. This means that the events S_1 and S_4 are the only ones which satisfy the second part of the condition (22), see Table 3. The scenarios S_1, S_2 and S_4 fulfill at least one requirement. Therefore they form the set K.

Table 3. "Dominance cases" (case study)

Scenarios and decisions	D_1	D_2	D_3	D_4	d_i (d_β=6.6)
S_1	1	2	2	2	7 (ok.)
S_2	0	1	1	3	5
S_3	2	0	0	1	3
S_4	3	3	3	0	9 (ok.)

- Step 5: Now one may formulate and solve the β-decision problem (assume that not a scenario is treated by the DM as completely sure):

$$\max\{g_1,0\}+\max\{g_2,0\}+\max\{g_4,0\}\rightarrow\min \tag{26}$$

$$5x_1+7x_2+6x_3+7x_4=8.9429-g_1 \tag{27}$$

$$3x_1+5x_2+2x_3+10x_4=8.9429-g_2 \tag{28}$$

$$9x_1+9x_2+11x_3+2x_4=8.9429-g_4 \tag{29}$$

$$x_1+x_2+x_3+x_4=1 \tag{30}$$

$$x_1,x_2,x_3,x_4\geq 0 \tag{31}$$

As one can see, we did not use the Equation (21) because the DM does not favour any state of nature. The solution is as follows: x_1=0, x_2=0.9918, x_3=0, x_4=0.0082, g_1=1.9429, g_2=3.9020, g_3=5.9429 (this variable was not minimized!), g_4=0. Both sides of the constraints (27)–(29) equal 7, 5.04, 8.94 respectively, which means that the most desirable events for our DM are S_1 and S_4 provided that he applies the mixed strategy recommended by the β-decision rule.

We also remark that with such an investment structure the income connected with the scenario S_3 amounts only to 3 mln Euro. It is a very low revenue, significantly lower than the minimum guaranteed revenue y^*, but notice that the decision maker, whose coefficient of optimism is equal to 0.6, just does not expect the occurrence of such an unattractive state of nature – let us recall that its index $d_3 = d_{min} = 3$. If the DM maintained that a particular scenario will happen, e.g. S_2, than the β-decision rule would suggest solving an extended optimization problem with the constraint (32),

$$g_2\leq 0 \tag{32}$$

which leads to the following solution: x_1=0, x_2=0.2114, x_3=0, x_4=0.7886, g_1=1.9429, g_2=0, g_3=5.9426 (this variable was not minimized!), g_4=5.4629 (this variable was minimized, but it concerns the best state of nature which is especially vital for decision makers whose coefficient of optimism is equal to 1 or almost 1). Both sides of the constraints (27)–(29) equal 7, 8.94, 3.48 respectively.

5. Conclusions

The paper presents a new decision rule which may be applied for mixed strategy's searching and which takes into account the decision maker's coefficient of optimism (β). This parameter is used to evaluate the income expected by the DM and to suggest the most probable scenarios, i.e. the events which correspond most to the DM's attitude. These secondary data enable to formulate and solve an optimization problem. The solutions obtained may facilitate the decision making process.

Notice that when the parameter β is equal to 0, then the set of the most probable states of nature (K) contains m elements (i.e. all scenarios) and the estimated expected income r_β is equal to the maximized minimum guaranteed revenue (y*), which means that in such circumstances the optimal mixed strategy generated by the β-decision rule is equivalent to the strategy recommended by the max-min model (i.e. the Wald's rule for mixed strategies). Perhaps it is quite astonishing to include S_4 in K when the DM is a pessimist (S_4 is a very attractive scenario), but let us recall that when the decision maker has such attitude towards risk, that implies that he or she wants to be prepared for the future in the best way possible, i.e. he or she intends to gain at least y* regardless the event which will finally take place.

The target of the new approach is to indicate a suitable mixed strategy provided that the decision maker intends to perform it only once. This assumption is quite significant, because one should be aware of the fact that after the realization of a mixed strategy, the decision maker may change his or her preferences, i.e. the level of optimism, and then a new optimization problem ought to be solved.

In the paper the set K is formed on the basis of two criteria (the expected income r_β and the number of "dominance cases" d_β). Of course, this is only a suggestion – one may choose other indices. Here, we will just explain why it is recommended to consider both r_β and d_β, not merely the first criterion. When the maximum payoff M* is much higher than the remaining payoffs in the matrix, then, even for low values of β, the index r_β becomes so high that only the scenario offering M* meets the criterion r_β. This means that regardless the level of optimism, just one state of nature is treated as the most probable.

Table 4. Payoffs' matrix with a high value M* (case study)

Scenarios and decisions	D_1	D_2	D_3	D_4	d_i
S_1	4	2	4	1	6
S_2	3	3	1	4	7
S_3	2	4	1	3	6
S_4	1	1	100	1	3

Let us analyze the example presented in Table 4. If we take into account only the expected income and the parameter β equals merely 0.02, the index r_β=0.02(100–2.975)+2.975=4.92. Hence, even if the DM was a radical pessimist (β=0.0106), the set K would contain only one element – the event S_4. Therefore, it is recommended to apply an additional criterion, e.g. the number of "dominance cases", which enables to keep also in this set all scenarios with an adequate index d_i. Thanks to the use of two criteria, for β<0.76 the set K={S_1, S_2, S_3, S_4} and for β≥0.76 the set K={S_2, S_4}.

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