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Suite of statistical models forecasting Latvian GDP

Andrejs Bessonovs[ab]*

[a]*University of Latvia, Faculty of Economics and Management, Aspazijas Boulevard 5, Riga, LV-1050, Latvia*
[b]*Bank of Latvia, Kr.Valdemara str. 2a, Riga, LV-1050, Latvia*

Abstract

We develop a suite of statistical models to forecast Latvian GDP. We employ various univariate and multivariate econometric techniques to obtain short-term GDP projections and to assess the performance of the models. We also comprise the information contained in components of GDP and obtain short-term GDP projections from disaggregated perspective. We run out-of-sample forecasting procedures to evaluate GDP projections and to assess forecasting accuracy of all individual statistical models. We conclude that factor and bridge models are among the best individually performing models in the suite. Forecasting accuracy obtained using disaggregated models of factor and bridge models is noteworthy and might be considered as a good alternative to aggregated ones. Furthermore, weighted combination of the forecasts of the statistical models allows obtaining robust and accurate forecasts which leads to a reduction of forecasted errors.

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1. Introduction

Timely information on economic developments is highly important for economic policy analysis and decision making. It's essential for economic policy makers and the business community to recognize the economic environment they operate in, to be able adequately assess the operative information and to make appropriate and effective decisions about their future behaviour.

* Corresponding author. Tel.: +371 28265333.
E-mail address: Andrejs.Bessonovs@bank.lv
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In this paper we develop a suite of statistical models in order to track Latvian gross domestic product (GDP) for nowcasting and forecasting purposes. We employ various econometric techniques to process the most recent operative statistical information in a suitable manner to obtain short-term projections. We run forecast evaluation exercise in order to assess the performance of individual statistical models out-of-sample and compare them against standard benchmark model. Moreover, we study weighted combinations of the individual forecasts.

Commencing with the simplest univariate models, we proceed to more advanced models as bridge and factor models. We expand our analysis by modelling GDP from disaggregated point of view. This is because disaggregated components appropriately captures structural changes of GDP and the explanatory variables that are tailor-made to predict particular component of GDP might raise an estimation "noise" in predicting GDP directly if a contribution of the component to total GDP is small. We develop disaggregated sub-models in order to forecast individual components of GDP both from expenditure and output side and further aggregate them to obtain GDP forecasts.

We compile quarterly GDP real-time database (RTD). RTD database contains monthly vintages (releases) of GDP, i.e. the vintages that were available each month starting in January 2004. We exploit RTD database in order to take into account GDP data revisions for forecasting purposes. We collect also large dataset of monthly series, which serves as dataset of predictors estimating the statistical models.

We conclude that factor models and bridge models outperform other models in the individual competition and are regarded to deliver more accurate forecasts. Moreover we find that combination of forecasts is a powerful tool to hedge the "risks" against poor performance of some individual models and argue that combination of forecasts is the optimal solution to choose the forecast.

We proceed as follows. Section 2 discusses the data we exploit in this paper. Section 3 reviews the statistical models and discusses model specifications. Section 4 reports the empirical results of the statistical models.

2. Data

2.1. Monthly data

We consider large dataset for Latvian economy. The data are collected on the main aspects of economy comprising business and consumer surveys of European Commission, industrial production, retail sales, consumer price indices, producer price indices, foreign trade, labour market, monetary statistics, exchange rates and interest rates, balance of payments and fiscal statistics (see Table 1). There are 187 variables in the database and all the time series are with monthly frequency[1].

Table 1. Description of the database of monthly variables

Category	Number of variables	Category	Number of variables
Confidence indicators	48	Interest rates	4
Industry	22	Exchange rates	4
Retail trade	16	Monetary statistics	9
CPI	13	Fiscal statistics	9
PPI	11	Balance of payments	7
Foreign trade	40	Labour market	2
		Others	2
		TOTAL	187

Time span of monthly variables is from January 1996 till January 2013. Most of the monthly series are subject to seasonal adjustment. Therefore all time series are seasonally adjusted by X-12-ARIMA method with specifications set by default, except interest rates and exchange rates, and those times series that already are published by statistical offices in seasonally adjusted form. We transform data to make it stationary, i.e. most data are log differenced, but those data with negative values are one time differenced. In addition, input data for factor model is normalized prior factor estimation in order to neutralize differences in scale of variables.

[1] Complete description of monthly variables is available on the request.

Evidently, all the monthly variables are supplied by statistical offices and respective officials with some delay or within individual schedule of publication as current month passes by. Therefore at any moment of time we inevitably observe missing observations and unbalanced panel of data, having so called ragged edge of data (see Table 2).

Table 2. Timeliness of the selected monthly variables in the dataset on 4th February 2013

date \ variable	ESI index	Industrial production	Retail sales	Nominal Exports	HICP	M3 money supply
nov.2012	✓	✓	✓	✓	✓	✓
dec.2012	✓	✓	✓	na	✓	✓
jan.2013	✓	na	na	na	na	na
feb.2013	na	na	na	na	na	na

* With (✓) mark is denoted available data. (na) means that data were not yet published on 4th February 2013.

Studies show that it's crucial to exploit the most recent statistical information to provide more accurate forecasts (Banbura & Runstler, 2011; Banbura & Modugno, 2012). Therefore we employ *Expectation-Maximisation* algorithm to fill out missing observations in the database, obtain balanced panel of data and to take into account all the timely information (for details see Stock & Watson, 2002a).

Timeliness of monthly variables in Table 2 suggests the amount of information is published on 4th February 2013, which we exploit to forecast GDP in February 2013. Knowing a systematic regularity of statistical information published by officials one could assume that the similar pattern of ragged edge appears on any month earlier. For example, to construct dataset on 4th January 2013 we preserve the same ragged edge of dataset as on 4th February 2013 only assuming one observation less for each variable. Rolling backwards the dataset available on 4th February 2013, we simulate the patterns of data and obtain pseudo real-time monthly vintages of monthly variables. Thus we ensure that only timely available statistical information in the past is exploited in out-of-sample forecasting evaluation.

2.2. Real-time GDP database

Numerous studies show that real-time data are relevant either in monetary policy analysis or in forecasting. Diebold & Rudebusch (1991) and Orphanides (2001) stress the importance of real-time data constructing leading indicators and analysing monetary policy. Croushore & Stark (2001), Stark & Croushore (2002), Croushore (2011) among others argue that data revisions are a major source of uncertainty which remarkably affects the forecasts.

Therefore we compile real-time GDP database in order to take into account GDP data revisions over time, that is, on each out-of-sample iteration step we use respective GDP vintage, i.e., the first GDP data release which was available at that time. Real-time data vintages for GDP are collected from January 2004 till May 2013. Thus we have 111 vintages of quarterly GDP available on the monthly basis. It allows evaluate GDP forecasts out-of-sample starting from 2004Q1 till 2012Q4, in total 36 quarters.

Similarly, we collect vintages of GDP components on expenditure and output basis in order to make estimate projections from disaggregate perspective. GDP expenditure components contain private consumption (C), government consumption (G), gross capital formation (I), exports (X) and imports (M). GDP output components contain items of NACE 2.0 classification. However, there is a methodological structural break in September 2011 due to change from classification NACE 1.1 to NACE 2.0, which precludes estimating forecasts in the chosen out-of-sample period. Notwithstanding, we overcome this issue by modelling aggregated industries which are very close to each other between both classifications (Table 3).

Aggregated economic sections in Table 3 enable us to forecast GDP on output basis in real-time and to obtain comparable forecasting errors continuously between both classifications.

Having GDP vintages available we proceed out-of-sample forecasting as following. Assume that in September 2012 GDP data is published and the last actual observation of GDP is 2012Q2 (see Table 4).

Table 3. Correspondence of NACE 1.1 and NACE 2.0 in terms of economic sections (in parentheses are given the shares of GDP of respective economic sections in 2010, percents)

Economic sections	Classification in terms of NACE 1.1 *		Classification in terms of NACE 2.0 *	
Primary sector	A+B	(3.9)	A	(3.9)
Industry	C+D+E	(14.9)	B+C+D+E	(16.3)
Construction	F	(4.6)	F	(6.0)
Wholesale un retail trade, hotels and restaurants, transportation, storage and communication	G+H+I	(31.8)	G+H+I	(33.2)
Public services	L+M+N	(11.6)	O+P+Q	(10.6)
Commercial services	J+K+O	(25.6)	J+K+L+M+N+R+S+T+U	(22.7)
Net taxes	D21-D31	(7.5)	D21-D31	(7.4)

* Letters denoting economic section's description differ in NACE 1.1 and NACE 2.0. For more details see Eurostat (2008).

Table 4. Timeliness of the forecasts

Current date:	Sep. 2012	Oct. 2012	Nov. 2012	Dec. 2012	Jan. 2013	Feb. 2013
GDP data up to:	2012Q2	2012Q2	2012Q2	2012Q3	2012Q3	2012Q3
	1st month	2nd month	3rd month	1st month	2nd month	3rd month
+ 1 quarter ahead:	2012Q3	2012Q3	2012Q3	2012Q4	2012Q4	2012Q4
+2 quarters ahead:	2012Q4	2012Q4	2012Q4	2013Q1	2013Q1	2013Q1

In September 2012 we forecast out-of-sample one and two periods ahead, respectively 2012Q3 and 2012Q4 and denote September as the 1st month when the forecast is made. Consequently, we may forecast 2012Q3 and 2012Q4 in October and November (respectively 2nd and 3rd month) up to December 2012 when next release of GDP is available. Rolling recursively backwards and estimating out-of-sample forecasts from 2004Q1 till 2012Q4 we are able to evaluate one and two quarters ahead for three consecutive months. Note that in every consecutive month we have more monthly information than a month before, which may potentially enhance forecasting accuracy.

3. The Suite of statistical models

In this section we review the statistical models used in forecasting Latvian GDP. We gather the most common econometric techniques used in short-term forecasting procedures around the world at leading research institutes. We obtain forecasts of Latvian GDP exploiting autoregression model, bridge model, factor model, VAR and BVAR models.

We expand the suite of models and develop disaggregated versions of bridge and factor models. We use both approaches – expenditure and output basis. We develop disaggregated sub-models in order to forecast individual components of GDP. The purpose is threefold: first, disaggregated data properly captures structural changes; second, we incorporate in our analysis more statistical information to infer about forecasting performance and third, we study whether disaggregated models are helpful in forecasting procedure in terms of forecasting accuracy.

3.1. Univariate models

3.1.1. Random walk (RW)

The very simple model is random walk model. It assumes no change in variable of interest. The model is given as following:

$$y_t = y_{t-1} + \varepsilon_t \tag{1}$$

where y_t is annual growth rate of real GDP. The *h*-step ahead forecast of the RW model is the following:

$$\hat{y}_{t+h|t} = y_t \tag{2}$$

where $\hat{y}_{t+h|t}$ is the *h*-step forecast of annual growth rate of real GDP with given information up to time t .

Typically random walk is referred as a benchmark model in comparison with other more sophisticated econometric models in a way that random walk could bring the easiest and the simplest guess we are able to obtain without pretending to use too much information.

3.1.2. Autoregression (AR)

Autoregression models are ones of the simplest econometric models. It is easy to construct and apply the autoregression model for economic forecasting. The main idea is to find the best and most appropriate time series model, which observations are modelled as a function of past observations. General form is the following:

$$y_t = c + \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t \tag{3}$$

where y_t is quarterly growth rate of real GDP, ϕ_i and c are coefficients to be estimated and p is the order of *AR* terms, $\varepsilon_t \sim i.i.d.N(0, \sigma^2)$.

We iterate recursively equation (3) forward and obtain the forecast as following:

$$\hat{y}_{t+h|t} = \hat{c} + \sum_{i=1}^{p} \hat{\varphi}_i y_{t-i+h} \tag{4}$$

where $\hat{y}_{t+h|t}$ is the h-step forecast of quarterly growth rate of real GDP with given information up to time t.

We select automatically lag structure of autoregression model according to Schwarz information criteria (SIC) each out-of-sample period.

3.2. Multivariate models

3.2.1. Bridge models (BM)

To assess the latest developments in economic activity, economic agents and forecasters take great emphasis on economic conjuncture indicators that are available much faster than the official GDP release and mostly at monthly frequency. These indicators typically are volume of industrial production, real retail trade turnover, business and consumer surveys, financial indicators etc. Therefore, the monthly figures can be used in forecasting model by means of bridging them to quarterly GDP growth estimates.

Bridge models are successfully applied to developed countries forecasting economic activity. Runstler & Sedillot (2003), Baffigi, Golinelli, & Parigi (2004) and Benkovskis (2008) conclude that bridge equations significantly improve the quality of the forecasts in comparison with conventional ARIMA model forecasts.

The bridge model considers the following form:

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{k} \sum_{m=0}^{s} \delta_{j,m} x_{j,t-m} + \varepsilon_t \tag{5}$$

where y_t is the quarterly growth rate of real GDP, p is number of lags of GDP growth rate, $x_{j,t}$ are monthly indicators; φ_i and $\delta_{j,m}$ are coefficients, k is the number of monthly indicators, s is the number of lags of monthly indicators, $\varepsilon_t \sim i.i.d.N(0, \sigma^2)$.

The forecast is made rolling forward equation (5) and using available and timely information of monthly indicators as the following:

$$\hat{y}_{t+h|t} = \sum_{i=1}^{p} \hat{\varphi}_i y_{t-i+h} + \sum_{j=1}^{k} \sum_{m=0}^{s} \hat{\delta}_{j,m} x_{j,t-m+h} \tag{6}$$

where $\hat{y}_{t+h|t}$ is the *h*-step forecast of the quarterly growth rate of real GDP with given information up to time t.

Bridge model's feature precludes us to use many explanatory indicators. Relatively short times series and loss of degrees of freedom typically constrains analysis to $k \leq 6$. Therefore we are encouraged to use the most important information in order to effectively forecast using a bridge model.

We identify 6 indicators which tell us much about the economy and are timely available for the forecasting procedure. Then aggregated GDP is modelled as following:

GDP Aggregated

- GDP = f (IP, retail, M3, EX, IM, ESI)

where explanatory variables are real industrial production (IP), real retail sales (retail), money supply M3 (M3), nominal exports (EX) and imports (IM), economic sentiment indicator (ESI) from business and consumer surveys.

We exploit also additional information for disaggregated models and identify relevant variables. We follow economic reasoning in variables' selection for every component of GDP. In some cases we use proxies which in our opinion might be reasonable explanatory variables. Therefore bridge models for GDP components are modelled as the following:

GDP Expenditure side

- Private consumption = f (retail, IM, cons.esi)
- Government consumption = f (budget.exp)
- Gross capital formation = f (ESI)
- Exports = f (EX, ex.serv)
- Imports = f (IM, im.serv)

GDP Output side

- Primary sector = f (cons.esi)
- Industry = f (IP, indu.esi)
- Construction = f (buil.esi)
- Trade, Transportation, Accommodation = f (retail, port, IM)
- Public services = f (M3)
- Commercial services = f (M3)
- Net taxes = f (cons.esi, reta.esi)

where GDP components are the functions of real retail sales (*retail*), real industrial production (*IP*), nominal imports (*IM*), nominal budget expenditures (*budget.exp*), nominal exports (*EX*), export of services (*ex.serv*), imports of services (*im.serv*), money supply M3 (*M3*), total economic sentiment indicator (*tot.esi*), industry confidence indicator (*indu.esi*), consumption confidence indicator (*cons.esi*), construction confidence indicator (*buil.esi*), ports turnover (*port*), retail confidence indicator (*reta.esi*).

3.2.2. Factor models (FM)

Last two decades factor models proved to be a very effective tool in short-term forecasting and economic analysis. Studies claim that a small number of factors could explain the large part of variation among many macroeconomic variables. In this case, if the forecasters can accurately assess the unobserved factors, the prediction exercise becomes much easier, because instead of n variables, we could use just few r factors $(r \ll N)$.

Effectiveness of factor models varies across countries and methods but still most researchers stress a usefulness of factor models. Ajevskis & Davidsons (2008), Boivin & Ng (2006), Brisson, Campbell, & Galbraith (2003), Camacho & Sancho (2003), Shintani (2003), Siliverstovs & Kholodilin (2009), Stock & Watson (2002a) report significant improvements in the forecast accuracy using principal components.

In our study we exploit approximate dynamic factor model in the spirit of Stock and Watson (2002a) diffusion indices. Let's assume $\boldsymbol{F}_t = (F_{1t}, F_{2t}, \ldots, F_{rt})$ is a vector of unobservable static factors which have pervasive effect throughout the economy and explain dependent variable as following:

$$y_t = \alpha + \sum_{i=1}^{r} \beta_i F_{it} + \sum_{j=1}^{p} \gamma_j y_{t-j} + \varepsilon_t \tag{7}$$

where y_t is the quarterly growth rate of real GDP, $\boldsymbol{F_t}$ is $r \times 1$ vector of factor estimates, y_{t-j} is y_t j th lag variable, α and β_i are estimated coefficients, p is an order of autoregression, $\varepsilon_t \sim i.i.d.N(0, \sigma^2)$. Then the data admits the following factor structure:

$$\boldsymbol{X_t} = \boldsymbol{\Lambda F_t} + \boldsymbol{u_t} \tag{8}$$

where $\boldsymbol{X_t} = (X_{1t}, \ldots, X_{Nt})'$ is the vector of N variables at time $t = 1, \ldots, T$, $\boldsymbol{F_t}$ is $r \times 1$ vector of factors, $\boldsymbol{\Lambda}$ is $N \times r$ a vector of factor loadings, $\boldsymbol{u_t}$ is idiosyncratic error, which allowed to be serially correlated and weakly cross-sectionally correlated. Static factors in equation (8) are estimated by principal components.

We obtain forecasts *h*-step ahead using direct multistep method (see Stock & Watson, 2002a) as following:

$$\hat{y}_{t+h|t} = \hat{\alpha} + \sum_{i=1}^{r} \beta_{ih} F_{it} + \sum_{j=1}^{p} \hat{\gamma}_j y_{t-j+h} \tag{9}$$

where $\hat{y}_{t+h|t}$ is the *h*-step forecast of quarterly growth of real GDP, F_{it} are estimated factors.

Forecasts of components of GDP are estimated as following. We estimate common factors in (8) using entire database and run regressions of GDP components on factor estimates in (7) and obtain forecasts in (9).

In our empirical application we proceed with one lag of dependent variable to keep moderate dynamics. We run formal *Bai-Ng* test to identify number of static factors (Bai & Ng, 2002). Number of factors is automatically estimated and chosen for each out-of-sample period.

3.2.3. Vector autoregressions (VAR)

By virtue of Sims (1980) empirical contribution to the economic analysis, vector autoregression (VAR) became very popular in the economic system analysis and forecasting. Nowadays small-scale VAR models are often used in monetary policy analysis and forecasting various economic variables, among others Sims (1992), Marcellino, Stock, & Watson (2003), Jacobson, Jansson, Vredin, & Warne (2001), Favero & Marcellino (2005), and Runstler *et al.* (2009).

Suppose $\boldsymbol{y_t}$ is $n \times 1$ vector of variables at time t. Then $\boldsymbol{y_t}$ dynamics can be described by the p-th order of the Gaussian autoregression model:

$$\boldsymbol{y_t} = \boldsymbol{c} + \boldsymbol{\Phi}_1 \mathbf{y_{t-1}} + \boldsymbol{\Phi}_2 \mathbf{y_{t-2}} + \ldots + \boldsymbol{\Phi}_\mathbf{p} \mathbf{y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t} \tag{10}$$

$$E\left(\boldsymbol{\varepsilon}_\mathbf{t} \boldsymbol{\varepsilon}_\mathbf{s}'\right) = \boldsymbol{\Omega}, \text{ ja } t = s$$

$$E\left(\boldsymbol{\varepsilon}_\mathbf{t} \boldsymbol{\varepsilon}_\mathbf{s}'\right) = 0, \text{ ja } t \neq s$$

$$E\left(\boldsymbol{\varepsilon}_\mathbf{t}\right) = 0$$

where $\boldsymbol{y_t}$ is a vector of variables of interest, $\boldsymbol{\Phi}_\mathbf{i}$ are matrices of coefficients, $i = 1, \ldots, p$, $\varepsilon_\mathrm{t} \sim N(0, \boldsymbol{\Omega})$.

The VAR form easily allows to obtain forecasts by iterating equation (10) *h* steps ahead:

$$\hat{\boldsymbol{y}}_{\boldsymbol{t+h|t}} = \hat{\boldsymbol{c}} + \hat{\boldsymbol{\Phi}}_1 \mathbf{y_{t-1+h}} + \hat{\boldsymbol{\Phi}}_2 \mathbf{y_{t-2+h}} + \ldots + \hat{\boldsymbol{\Phi}}_\mathbf{p} \mathbf{y_{t-p+h}} \tag{11}$$

where $\hat{\boldsymbol{y}}_{\boldsymbol{t+h|t}}$ is *h*-step ahead forecast of the vector of variables.

The standard VAR model typically includes three variables: real GDP, consumer price index and interest rates. Taking into account the feature of the Latvian economy, we augment VAR also with money supply M3, forming so called monetary VAR. Three-month Euribor serves as interest rate in our VAR. The lag order of VAR, p, is selected by SIC. However, we restrict lag order, $p \leq p_{max} = 4$.

3.2.4. Bayesian vector autoregressions (BVAR)

Bayesian vector autoregression models (BVAR) are known as models to provide better and more accurate results than VAR models. Bayesian estimator helps to avoid overparametrisation problem and thus allows exploiting a greater number of variables in a model. It seems very attractive to apply Bayesian techniques to VAR modelling in the case of Latvia due to relatively short time series of macroeconomic variables.

Doan, Litterman, & Sims (1984) and Litterman (1986) works give great impetus to BVAR model development and implementation to macroeconomic forecasting. Recent literature on BVAR models, Banbura, Giannone, & Reichlin (2010), Bloor & Matheson (2011), Koop (2010) show how Bayesian techniques allow us to exploit large number of variables in VAR models. Let write BVAR model as follows:

$$\boldsymbol{y_t} = \boldsymbol{c} + \mathbf{B}_1\mathbf{y}_{\mathbf{t-1}} + \mathbf{B}_2\mathbf{y}_{\mathbf{t-2}} + \ldots + \mathbf{B_p}\mathbf{y}_{\mathbf{t-p}} + \boldsymbol{\upsilon_t} \tag{12}$$

$$E\left(\boldsymbol{\upsilon_t}\boldsymbol{\upsilon_s}'\right) = \boldsymbol{\Sigma}, t = s$$

$$E\left(\boldsymbol{\upsilon_t}\boldsymbol{\upsilon_s}'\right) = 0, t \neq s$$

$$E\left(\boldsymbol{\upsilon_t}\right) = 0$$

where $\boldsymbol{y_t}$ is $n \times 1$ vector of variables at time $t = 1,\ldots,T$; $\{\boldsymbol{c}, \boldsymbol{B}_1,\ldots, \boldsymbol{B_p}, \boldsymbol{\Sigma}\}$ are parameters of the model. Let put model's coefficients in one vector, $\boldsymbol{\theta} = \{\boldsymbol{c}, \boldsymbol{B}_1,\ldots, \boldsymbol{B_p}\}'$, then the prior information is given by $p(\boldsymbol{\theta}) \sim N(\boldsymbol{\theta}_0, \boldsymbol{\Sigma}_0)$, where $\boldsymbol{\theta}_0$ is a mean and $\boldsymbol{\Sigma}_0$ is diagonal variance matrix. Analogically, BVAR forecasts are obtained by iterating equation (12) *h* steps ahead:

$$\hat{\boldsymbol{y}}_{\boldsymbol{t+h|t}} = \hat{\boldsymbol{c}} + \hat{\mathbf{B}}_1\mathbf{y}_{\mathbf{t-1+h}} + \hat{\mathbf{B}}_2\mathbf{y}_{\mathbf{t-2+h}} + \ldots + \hat{\mathbf{B}}_\mathbf{p}\mathbf{y}_{\mathbf{t-p+h}} \tag{13}$$

where $\hat{\boldsymbol{y}}_{\boldsymbol{t+h|t}}$ is *h*-step ahead forecast of the vector of variables.

There are various schemes how priors could be identified in order to estimate a model. We employ the simplest Minnesota or Litterman prior (Litterman, 1986) which incorporates the belief that each element of y_t follows $AR(1)$ process or a random walk, but the prior variance assumed to be diagonal and controlled by hyperparameters. Error covariance matrix, Σ, is assumed to be known, however could be replaced by estimated error covariance matrix, $\hat{\Sigma}$. Hyperparameters depend on three parameters: λ_1 controls the variance of the prior on own lags, λ_2 controls the variance of the prior on lags of variables other than dependent, λ_3 controls relative tightness of the variance of lags. To identify BVAR model in our suite of models we set four lags. We impose "industry standard" values for prior beliefs, namely, $\lambda_1 = 0.2$, $\lambda_2 = 0.5$, $\lambda_3 = 1$ (see Canova, 2007; Litterman, 1986; Kapetanios, Labhard, & Price, 2008) to keep the model simple. Admittedly, the best priors there might be chosen by grid searching over parameters space and respectively evaluating forecasts.

3.3. Combination of forecasts

Early paper of Bates & Granger (1969) stresses that two separate sets of forecasts (provided by different models) of the same variable can yield lower mean squared error than either of the original forecasts. This conclusion about forecast combination is viewed and proved to be very effective how to robustify forecasting performance over the individual models. Forecast combination has received significant attention by academics and practitioners. Hendry & Clements (2002), Aiolfi & Timmermann (2006) and Aiolfi, Capistran, & Timmermann (2011), stress that individual models are differently affected by structural breaks, thus forecast combinations may be justified. Clemen (1989) argue that idea of combining forecasts implicitly assumes that one could not identify underlying process. There are possibilities to misspecify the underlying model, parameter estimates and generated forecasts, therefore individual models could be subject to misspecification bias.

There are a lot of papers that study weighting schemes of forecasts. Bates & Granger (1969), Granger & Ramanathan (1984), Diebold & Pauly (1990), Stock & Watson (2004), among others, exploit linear and time-varying methods to estimate forecast weights. As noted by Aiolfi *et al.* (2011) and others, that equal-weighted forecast is surprisingly difficult to beat. Stock & Watson (2004) point out combination methods with the lowest MSFEs are the simplest, either with equal weights or with weights that are very nearly equal and change little over time.

4. Empirical results

4.1. Estimation issues

Typically forecasting accuracy is measured by a loss function. There are various types of loss functions (e.g. see Gooijer & Hyndman, 2006). Conventional measure is root mean square forecasting error ($RMSFE$):

$$RMSFE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i - y_i^f\right)^2} \tag{14}$$

where y_i is actual realization, y_i^f is forecast value, N is number of out-of-sample forecasts.

Intuition of RMSFE is straightforward. It measures average deviation of forecasts from actual observations and defined in the same units as analysed indicator. Our study reports forecasting errors in terms of annual growth rates of the quarterly GDP. However, note that statistical models provide quarter-on-quarter growth rates. We convert quarterly growth rates to annual growth rates and compare with outturns. The reason for this is that quarterly growth rates of Latvian GDP (seasonally adjusted data) are largely revised from one release to another. Therefore forecasting accuracy measure would contain large portion of data measurement error, but not model error.

4.2. Evaluation of individual forecasts

We estimate statistical models recursively for every out-of-sample period. We start in 2004Q1 and proceed till 2012Q4 having estimated in total 36 quarters. Note that we have GDP monthly vintages, meaning we may estimate forecasts every month. We report forecasting accuracy results for numerous models in Table 5. We obtain 12 individual forecasts, including 6 forecasts of aggregated models and 6 forecasts of models from expenditure and output side. Results are given for the forecasts one and two quarters ahead, where 1st, 2nd and 3rd denote respective month since new GDP data release is available (explanation was given in Table 4). RMSFE in Table 5 is given in comparison with Random Walk (RW) model, i.e. relative RMSFE, thus the number greater (less) than one indicates that particular model is less (more) accurate than RW. Therefore RW model in the first line is equal to one. Loss functions expressed in relative terms provide comparability of the performance of models.

Table 5. Relative RMSFE results for the suite of statistical models

Model	1 quarter ahead			2 quarters ahead		
	1st	2nd	3rd	1st	2nd	3rd
RW	1.00	1.00	1.00	1.00	1.00	1.00
AR	0.97	0.97	0.97	1.01	1.07	1.07
BM	0.78	0.78	0.78	na	na	Na
FM	0.75	0.75	0.75	0.73	0.74	0.74
VAR	0.95	0.95	0.95	0.97	1.00	1.00
BVAR	0.93	0.93	0.93	0.98	1.03	1.04
AR_EXP	1.11	1.11	1.11	1.19	1.19	1.18
BM_EXP	0.97	0.97	0.97	na	na	na
FM_EXP	0.78	0.78	0.78	0.74	0.70	0.70
AR_OUT	1.31	1.31	1.31	1.26	1.31	1.31
BM_OUT	0.82	0.82	0.82	na	na	na
FM_OUT	0.76	0.76	0.76	0.82	0.83	0.82

Model acronyms stand for random walk (RW), autoregression (AR); bridge model (BM); factor model (FM); vector autoregression (VAR); Bayesian vector autoregression (BVAR); suffix (EXP) denotes disaggregated model on expenditure basis and suffix (OUT) denotes output basis of respective model. (na) denotes that forecast is not available.

Results in Table 5 show that most of the models outperform a simple benchmark model. Performance of AR model is very close to RW, although poor performance is obtained exploiting disaggregated versions of AR model. Factor model (FM) and bridge model (BM) are among the best performing models. The forecast accuracy gains of FM and BM comprise about 25% compared to simple RW model. Accuracy gains of VAR and BVAR on average are less than 5% and reflect modest improvement upon RW model. Forecasting performance obtained using disaggregated models of FM are remarkable (FM_EXP and FM_OUT). They outperform univariate models on average by 23% in both horizons. Disaggregated bridge model performs well from output side (BM_OUT), but improvement of disaggregated bridge model from expenditure side (BM_EXP) upon RW model is limited. Muted performance mainly stems from investments component of GDP, which is highly volatile and hardly predictable, thereby significantly raises the forecasting error. Overall, disaggregated forecasts seem very promising in a case when timely monthly information is taken into account.

4.3. Evaluation of combined forecasts

Standard approach of forecast combination techniques is the weighted average of the individual forecasts. One could obtain combined forecast applying particular weighting scheme, where the standard form is the following:

$$y_{t+h|t}^{c} = \sum_{i=1}^{n} w_{it} y_{t+h|t} \tag{15}$$

where $y_{t+h|t}^{c}$ is a combination of forecasts, $y_{t+h|t}$ is an individual forecast made at the time t for period h, w_{it} is the weight of model i at the time t.

In this paper we consider several forecast combination methods. We use standard equal weights, full sample RMSFE weights, full sample MSFE weights, recursive RMSFE weights and recursive MSFE weights. MSFE put more penalty on individual forecast errors compared to RMSFE due to quadratic form. However, full sample weights are tested against recursive ones, where recursive weights depend on historical performance.

Weights have the following general form:

$$w_{it} = m_{it}^{-1} / \sum_{j=1}^{n} m_{jt}^{-1}; \tag{16}$$

where m_{it} equals to:

$$m_{it} = 1, \text{ for all } i = 1,\ldots,n;\ t = T_0,\ldots,T \qquad \text{– equal weights} \tag{17}$$

$$m_{it} = \sqrt{\sum_{s=T_0}^{T} \left(y_{s+h} + y_{s+h|s}\right)^2} \qquad \text{– full sample RMSFE weights} \tag{18}$$

$$m_{it} = \sum_{s=T_0}^{T} \left(y_{s+h} + y_{s+h|s}\right)^2 \qquad \text{– full sample MSFE weights} \tag{19}$$

$$m_{it} = \sqrt{\sum_{s=T_0}^{t} \left(y_{s+h} + y_{s+h|s}\right)^2} \qquad \text{– recursive RMSFE weights} \tag{20}$$

$$m_{it} = \sum_{s=T_0}^{t} \left(y_{s+h} + y_{s+h|s}\right)^2 \qquad \text{– recursive MSFE weights} \tag{21}$$

Results of the combined forecasts are summarised in Table 6. Results are given in relative terms against RW model.

Results in Table 6 show that all forecast weighting schemes outperform RW model on average by 16% one quarter ahead and by 8% two quarters ahead. Discrimination by higher punishment of errors (MSFE weights) doesn't provide forecasting gains neither in full sample nor recursive scheme. Using full sample period forecasting gain is only marginally higher than recursive one. However, the best performance is attributed to equal weights.

Moreover, forecasts of equal weights outperform the most individual forecasts in Table 5 and at both horizons. Remarkably that all weighting methods perform better than univariate models in the suite of models contributing on average 8% and 15% higher accuracy respectively forecasting one quarter and two quarters ahead. In total, combination of forecasts immunes against the models' parameter instability and misspecification, therefore contributes to a better forecasting accuracy and leads to an optimal strategy which may be employed by forecaster.

Table 6. Relative RMSFE results for combination of forecasts

	+1 quarter ahead			+2 quarters ahead		
Type of weights	1st	2nd	3rd	1st	2nd	3rd
Equal	0.82	0.79	0.80	0.85	0.89	0.88
RMSFE full sample	0.85	0.82	0.83	0.89	0.92	0.92
MSFE full sample	0.87	0.85	0.86	0.92	0.96	0.96
RMSFE recursive	0.86	0.82	0.83	0.89	0.93	0.93
MSFE recursive	0.89	0.86	0.87	0.94	0.97	0.97

5. Conclusions

In this paper we develop the suite of statistical models forecasting Latvian GDP. We conduct forecast evaluation exercise in order to assess the performance of individual statistical models over out-of-sample period and compare them against standard benchmark model. Results show that factor (FM) and bridge (BM) models are among the best individually performing models. The forecast accuracy gains of FM and BM comprise about 25% compared to simple RW model forecasting GDP one quarter and two quarters ahead. Improvement of VAR and BVAR models upon benchmark are rather modest. Forecasting accuracy obtained using disaggregated models of FM and BM are remarkable, except BM model from expenditure side, which has limited accuracy gains due to hardly predictable investments component. Our study shows that modelling GDP from disaggregated perspective is a good alternative to aggregated ones. Moreover, we find that weighting and combining individual forecasts, one may persistently improve forecasting accuracy over the benchmark in both forecasting horizons. We find that equal weights are the best performing weighting scheme, which is very hard to beat.

Analysis in this paper virtually could be extended augmenting more statistical models, e.g., dynamic factor models, models with time-varying parameters and non-linear models as Markov-Switching (MS) or threshold models. A performance of latter ones is intriguing taking into account the magnitude of recent financial crisis affected Latvian economy. The paper would benefit making cross country comparison of model performances adding other Baltic states (Lithuania and Estonia), to our best knowledge, there is no such study made before.

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