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Linearity of the Sharpe-Lintner version of the Capital Asset Pricing Model

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Abstract

In the paper the Capital Asset Pricing Model (CAPM) in the original form considered and developed by William Sharpe and John Lintner is entertained and investigated for its empirical validity. The CAPM is one the underlying building blocks of Modern Portfolio Theory and as such is constructed on a number of strong theoretical assumptions concerning the behaviour of financial markets and of investors. In consequence, this model establishes a linear relationship of risky assets returns excess of the riskless rate to market portfolio returns excess of the riskless rate. Its conclusions are weighty and its functional relationship can be deemed as restrictive. On many a ground, the CAPM is thus challenged from the perspective of both a theorist and a practitioner. This empirical study revisits empirical validity of the linear functional form of the CAPM with respect to recent data.

1. Introduction

Traditional microeconomic consumer theory does not accommodate the assumption that consumers make their decisions under risk and, consequently, it can go freely into explaining the behaviour of consumers and describing the establishing of equilibria at non-financial markets. The fact that it does not account for risk associated with some consumer decisions makes it grossly insufficient to explain the behaviour of investors at financial markets whose consumer decisions are made in the presence of a good amount of risk. This was felt to be a drawback (Sharpe, 1964) and led researches to construct a specialized microeconomic theory explaining investment behaviour in

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conditions of risk. In the bosom of the Markowitzian modern portfolio theory, a theory of risky asset pricing emerged in the 1960's and materialized into various variants of the Capital Asset Pricing Model (conveniently abbreviated as "CAPM"). This theory, with weighty implications, was built under the abstraction of a number of idealistic assumptions and has been subjected to massive criticism and has been legitimately questioned on many grounds. Yet, after some initial mistrust with practitioners and a tentative approval by academic circles, the CAPM has come to be a popular tool of investment management and has been applied throughout the entire investment and corporate management, not only in the area of investing but also in corporate financial analysis, corporate performance measurement, company evaluation or in project selection.

Having warranted the existence of the market portfolio and of a riskless asset, the CAPM asserts that the expected return on risky assets in excess of the riskless rate are linearly linked with the expected return on the market portfolio in excess of the riskless rate, and that there exists a unique coefficient which determines this linear relationship. This coefficient is (owing to its usual notation) addressed as "beta" and – in the exposition of the CAPM theory – is the perfect measure of risk and comprises all the information that can be learned about riskiness of the given risky asset. Clearly, the assumptions and conclusions of the CAPM permit a good opportunity to dismissive judgements and critical appraisal, but attention here in the paper is devoted solely to the linear function form that is vital to the model provided the theory holds.

Under this circumscription of scope, the paper revisits validity of the original version of the CAPM, the version of Sharpe (1964) and Lintner (1965), in regard to the linear functional form that is necessitated between excess returns on risky assets and excess market returns, above the riskless rate. To this end, a random draw of 10 stocks represented in the S&P 500 Index is made so as to ensure that each stock is represented as per one of the ten GICS sectors that compose the underlying index. Although admitting the fundamental critique of Roll's (Roll, 1977), the S&P 500 Index is taken here as a proxy of the market portfolio, and the riskless rate is proxied by the nominal one-year interest rate on U. S. government securities. For the period of 10 years spanning from 2003 to 2012 divided into two subsequent non-overlapping 5-year subperiods, using monthly returns, it is inspected as to whether the relationship claimed by the model is linear and that the beta coefficient does not change over time. In most cases, no evidence contrary to the CAPM is found on the scale of linearity and its constancy over the two subperiods.

The method pursued in the paper is new as the analysis does not rest on the distributional assumption that is usual for a study of this sort. Instead, the scatter analysis of data exploration is employed and combined with the bootstrap method of statistics in the way that allows avoiding this possibly erroneous assumptions. Another trait of the approach is that it uses no rigorous statistical testing in inspecting empirical validity of the model, whereas it is well understood that traditional statistical testing only induces a number of associated difficulties that are inherent in it and here it would result in the multiple comparison problem.

Save the introductory and concluding section, the body of the paper consists of two sections. The ensuing section sets out the theory of the model and exposits the fundamental necessary to this empirical exercise, and is continued by the methodological section. Graphical results are postponed to appendices and discussion is embodied in the conclusion.

2. Theory of the CAPM

It must be said that there are several versions of the model and numerous modifications stemming from attempts to best reflect reality of financial markets. In principle, the CAPM may appear either in its original version as developed by Sharpe (1964) and Lintner (1965), or in the version entertained by Black (1972). The distinction consists in the authors' attitude towards the existence of lending and borrowing facilities at the riskless rate of interest. Whilst Sharpe and Lintner admit that an investor is able to borrow or lend at a riskless rate of interest, the Black version operates in the milieu where a riskless asset is absent and not available. The exposition here is only restricted to the Sharpe-Lintner version of the CAPM. Note that the development of this version is sometimes also attributed to Mossin (1966) and Treynor (LeRoy & Werner, 2001).

The CAPM was derived in order to describe establishing an economic equilibrium between rational agents (investors) who make decisions founded on the expectations of their future wealth under the belief that standard deviation is a best guidance in assessing the risk that they take. A set of nine assumptions is herewith necessary to

derive the model. They are the following (Harrington, 1987; Fabozzi, Focardi, & Kolm, 2006; Sharpe, Alexander, & Bailey, 1995):

- The objective of investors is to maximize the utility of their terminal wealth.
- Investors make choices by comparing return and risk. Whilst return is measured by the mean returns expected of a portfolio of assets, risk is measured by the standard deviation of these portfolio returns.
- Investors are rational and risk-averse, i.e. they seek to construct mean-variance efficient portfolios.
- Investors have homogeneous expectations of risk and return.
- Investors have identical time horizons for investment.
- Information is freely and simultaneously available to investors and immediately reflected in asset prices.
- There exists a riskless asset, and investors can borrow and lend at the riskless rate of interest without any restriction.
- There are no taxes, transaction costs, restrictions on selling short or other forms of market imperfections.
- All assets are marketable and divisible.

Note that the first sextet of them are connected to the efficient-market hypothesis and the Markowitzian mean-variance maxim of investors at financial markets. It is self-evident that these assumptions are freely available to considerable discussion. Their defense and empirical failure is addressed in detail by Harrington (1987).

Some rules of behaviour result for investors from these assumptions. One feature is that investors construct their optimal portfolios as a linear combination of risky assets and the riskless asset. Although each investor selects the portfolio that best reflects his utility (in consequence of which portfolios do differ across investors); each investor chooses the same combination of risky assets and difference between investors arises only in the way in which they mix this combination of risky assets and the riskless asset. This means that each investor spreads his funds amongst risky assets in the same relative proportions, adding to these risky assets a portion of the riskless asset so as to optimize his tradeoff between return and risk. This property of the CAPM is called as the separation theorem.

Another essential property of the CAPM is that in equilibrium each risky asset must have a non-zero proportion in the composition of portfolios selected by investors. Furthermore, the proportions of risky assets answer, in the state of equilibrium, to the proportions of the market portfolio, which is defined to be a portfolio of all risky assets that are globally available in the market and that participate on it in the weight proportional to their market value. Although it is a purely theoretical concept, it transpires that the market portfolio is central in the CAPM as investors at equilibrium create portfolios that are a linear combination of the market portfolio and the riskless asset.

Finally, the equilibrium relationship between return and risk of risky assets is described by the so-called Security Market Line (contracted conventionally to "SML"), which depicts the position of equilibrium portfolios chosen by investors. Some notation is necessary for the following exposition. Let R_i denote the return of risky asset i, R_m stand for the return on the market portfolio and R_f be the return on the riskless asset (i. e. the riskless rate of interest). Naturally, both risky asset returns and market portfolio returns are interpreted as random variables, and the existence and finiteness of their first two expectations is assumed.

Confronting the assumptions of the CAPM, the riskless rate is stipulated non-stochastic and constant over time. This is not the case in practice and will have to be taken into account later. Nonetheless, the treatment here is adjusted here so as to allow for possible stochasticity and non-constancy of the riskless rate, which is not at odds with usual exposition under the riskless rate being a constant. If the riskless rate R_f is set to a constant in the following, the entire presentation collapses to the traditional textbook outline. Another two variables are introduced: $\gamma_i := R_i - R_f$ represents the return on risky asset i in excess of the riskless rate and, similarly $\gamma_m := R_m - R_f$ signifies the excess return on the market portfolio (often referred to as market premium or risk premium). The fundamental consequence of the CAPM is that the following relationship between excess returns on any risky asset i and excess market returns (measured with respect to an arbitrary investment horizon) holds

$$E(\gamma_i) = \beta_i E(\gamma_m) \tag{1}$$

where the beta coefficient of risky asset i, i.e. β_i, is governed by the formula

$$\beta_i = \text{cov}(\gamma_i, \gamma_m) / \text{var}(\gamma_m) \tag{2}$$

A few implications of the CAPM and of (1) and (2) can be stated and elaborated, but the focus of the paper here is reduced only to the empirical question of linearity in (1) and whether the beta coefficient obeys (2). Before proceeding to fulfilling these goals and formulating the methodology, some comments are pertinent on the associated issues.

The all-important ingredient of these restrictions on the relationship of excess returns on risky assets and excess market returns is the implication that the beta coefficient β_i is a complete measure of the risk of risky asset i an no other measure of risk appears in (1) (Fama & MacBeth, 1973); in this sense, β_i must exhaust the entire variation of asset excess returns (Campbell, Lo, & MacKinlay, 1996). The beta coefficient is estimated on a regular basis for publicly traded stocks and some other financial assets and published with descriptives on the economic and financial conditions of individual assets; hence its popularity despite the fact that the true beta coefficient is a perfectly theoretical quantity and must inevitably differ more or less from its estimated value.

Permitting constancy of the riskless rate R_f, (1) transforms into an alternative representation,

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f \right] \tag{3}$$

which is the formula of the SML. The SML is shown in Fig. 1. As indicated in Fig. 1 and as readily results from (3), there is only one portfolio whose beta coefficient equals 1, and this portfolio is the market portfolio. Other portfolios have a different beta coefficient and this positions them in a distinctive relation to the market portfolio.
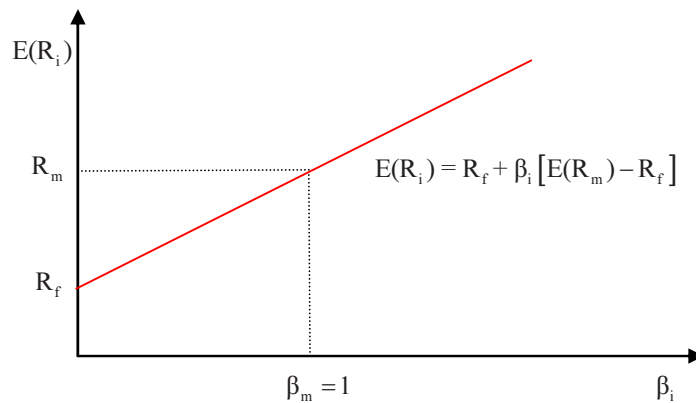

Fig. 1. Security Market Line

It should be made clear that there are numerous problems inherent in the CAPM. Grave criticism of the CAPM was brought forward by Roll (1977), which has come to be known as the Roll Critique. Roll (1977) questioned a possibility of testing the CAPM and made thereat one crucial point. In his view, the market portfolio is not observable because it comprises not only publicly traded financial assets but consists also of non-traded financial assets (such as human capital) or financial assets not represented in indices (such as real estates). In light of this assertion, the CAPM formula can be neither estimable or verifiable (via statistical testing), and the "tests" of the CAPM were invalid. He conceded that to test the CAPM a mean-efficient benchmark is sufficient to replace the market portfolio but discarded of finding a valid substitute and even knowing that this benchmark is really mean-efficient in the sense of Markowitz.

This theoretically based rejection of the CAPM is complemented with findings of empirical studies whose authors did not evaluate the situation as desperate as Roll had done and attempted to test it. Empirical studies concentrated on possible misspecification and inadequacy of the CAPM and pointed out that there is a great deal of anomalous

evidence against the CAPM suggesting that some other factors should be included in theoretical considerations (such as the size of the firm, P/E ratio and other financial indicators). An overview of these studies and their results is given in Harrington (1987) or Brown & Walter (2013).

In spite of the fact that the CAPM is built on expected quantities and theoretical relationships between random variates, its practical implementation is linked with historical data from which all necessary quantities are computed. This approach is perhaps only practicable though it demands the underlying assumption that the past is a good and trustworthy predictor of the future development. No way should this be deemed as contrary to the expectational interpretation of the CAPM, which is entertained by Harrington (1987) or the *ex ante* interpretation of the model, which is accentuated by Brown & Walter (2013).

As declared afore, this paper is a study into validity of the CAPM and centres on the basic properties of the CAPM as specified by (1) and (2): linearity in (1) and specialty of the beta coefficient in (2). The methodological procedure is clarified in detail in the next section.

3. Methodological setup

In specializing (1) and (2) into an observational form, assume that T historical excess returns $\gamma_{i1}, \gamma_{i2},..., \gamma_{iT}$ on risky asset i and T historical excess market returns $\gamma_{m1}, \gamma_{m2},..., \gamma_{mT}$ are available. Having these measurements on γ_i and γ_m, it is possible to write that $E(\gamma_i) = \gamma_{it} + \zeta_{it}$ and $E(\gamma_m) = \gamma_{mt} + \zeta_{mt}$ for any $t \in \{1,...,T\}$, where ζ_{it} and ζ_{mt} are random variates with zero expectations. By the assumption that γ_i and γ_m have existent and finite second expectations, this property is shared with ζ_{it} and ζ_{mt}. Then (1) becomes

$$\gamma_{it} + \zeta_{it} = \beta_i \left(\gamma_{mt} + \zeta_{mt} \right), \tag{4}$$

which after the introduction of $\xi_{it} := \beta_i \zeta_{mt} - \zeta_{it}$ goes into

$$\gamma_{it} = \beta_i \gamma_{mt} + \xi_{it} . \tag{5}$$

Of the additive disturbance component ξ_{it} no additional assumptions are formed as to its distribution. It is but required that for each risky asset i the sequence $\{\xi_{it}\}_{t \in \{1,...,T\}}$ represents a white noise process.

This rewriting is suggestive that the linear relationship as displayed in (1) in the case of expectations must also hold for the observational form presented in (5). The beta coefficient β_i could be estimated in analogy to (2) by the sample observational estimator

$$b_i = est.cov(\gamma_{it}, \gamma_{mt}) / est.var(\gamma_{mt}) , \tag{6}$$

where the prefix "est." indicates the respective sample moment estimator (for covariance and variance). Note that (6) is the traditional ordinary least squares (contracted hereinafter as "OLS") estimator of the slope coefficient in regression (5) extended of intercept. Posing the problem as in (5) and (6) led many researchers to the explicit modification of (5) into a regression model with intercept and estimating its parameters by the OLS. This is understood here and later utilized. Sometimes distributional assumptions are imposed on the disturbance term ξ_{it}, which is in contrast to the approach here; and it should be highlighted that the distribution is left free.

In empirical research in this field, it is necessary to take some substitutes in place of the market portfolio and of the riskless rate as these are theoretical concepts only. The market portfolio is not observable and is usually substituted by a market index, which represents the market of risky assets under consideration. Of course, it may be sometimes a matter of subjective preference as to what index choose specifically, but the fact remains that investors use them as benchmarks and they thus still deliver useful information. Commonly, for the U. S. stock market it is the S&P 500 Index that serves as the market proxy. As far as the riskless asset is concerned, it must needs be admitted that no asset is itself free of risk and the recent development of financial markets and of the global financial system over the last decade has confirmed and proven this statement. The riskless rate is usually proxied by a return rate of

government fixed-income securities. Specifically, for the U. S. economy usually the 90-day Treasury bill rate is made use of and for the economies of European Union member states also the EURIBOR rate is sometimes considered. Of course, in both cases it is still an approximation of desired quantities. Another point is neither proxy of the riskless rate is constant; as a matter fact, in empirical studies, interest rates and derived variables are always considered stochastic.

Similar substitutes are necessary for this study as well. The market proxy is the S&P 500 Index and the riskless rate is proxied by the nominal one-year interest rate on U. S. government securities (Treasury constant maturities, Table H.15 of the Federal Reserve Bank). Ten risky assets, stocks, included in the S&P 500 Index were selected to participate in the study in a random design. From each of the 10 different GICS sectors represented in the S&P 500 Index one stock was picked at random so as to enable a variety of stocks stratified across various industries. The list of the stocks selected is presented in Table 1.

Table 1. The stocks participating in the empirical exercise

GICS sector	Consumer Discretionary	Consumer Staples	Energy	Financial	Health Care
Stock (& ticker)	Lennar Corp. (LEN)	ConAgra Foods Inc. (CAG)	Denbury Resources Inc. (DNR)	Capital One Financial Corp. (COF)	Quest Diagnostics Inc. (DGX)
GICS sector	Industrials	Information Technology	Materials	Telecommunications Services	Utilities
Stock (& ticker)	Avery Dennison Corp. (AVY)	Harris Corp. (HRS)	Cliffs Natural Resources Inc. (CLF)	Crown Castle Int. Corp. (CCI)	Exelon Corp. (EXC)

The time span of 10 years from 2003 to 2012 was used and the data used were related to a monthly frequency. From monthly S&P 500 Index returns and returns on the 10 selected stocks, monthly averages of the nominal one-year government securities interest rate (expressed as p.m.) were deducted so that excess returns of the S&P 500 Index and on the 10 stocks were constructed. The database counted $10 \times 12 = 120$ monthly observations of excess returns. The entire 10-year period was divided into two subsequent non-overlapping subperiods: from 2003 to 2007 (including 60 observations) and from 2008 to 2012 (comprising 60 observations).

For each of the 10 selected stocks and the entire 10-year period and both of the 5-year subperiods, CAPM beta coefficients were estimated by means of formula (6). The estimates are reported in Table 2. Also a graphical visualization is provided and is moved to Appendix A, to retain the clarity of presentation. In Appendix A, for each stock, a scatterplot of excess returns against excess S&P 500 Index returns is produced. Individual observations are marked and coloured according to their classification to the proper subperiod. For each subperiod, a regression line is displayed whose slope is the estimated CAPM beta coefficient (this line was estimated by the OLS). Alongside each axis, a boxplot of excess returns is shown and provides an insight into the distribution of excess returns. In eight scatterplots (for LEN, DNR, COF, DGX, HRS, CLF, CCI and EXC), some unusual observations are clearly suspected and they are identified and labelled. In the opinion of the authors, graphical presentation in Appendix A is more informative and demonstrative than numerical values reported in Table 2.

Table 2. The estimated CAPM betas for individual stocks

(Sub)period	LEN	CAG	DNR	COF	DGX
2003 – 2012	1.802	0.706	1.433	1.769	0.583
2003 – 2007	1.816	0.731	0.645	1.959	0.336
2008 – 2012	1.849	0.714	1.569	1.743	0.640
(Sub)period	AVY	HRS	CLF	CCI	EXC
2003 – 2012	1.287	1.120	2.642	1.319	0.521
2003 – 2007	0.660	1.083	2.862	1.494	0.370
2008 – 2012	1.415	1.117	2.574	1.263	0.530

Furthermore, scatterplots can be used to assess whether the relationship between excess S&P 500 Index returns and excess returns on individual stocks is linear. The CAPM implies not only that the relationship between excess market returns and excess returns on risky asset (here excess S&P 500 Index returns and excess returns on stocks) must be linear but also that is linear in partial periods. Therefore, the linearity consequence of the CAPM requires

not only that individual monthly observations of excess returns are scattered alongside a line but also that this line does not change its slope in partial periods. Therefore, on individual plots parallel lines must be seen, and in Table 2 estimates of the CAPM beta coefficients for individual stocks must match. An absence of linearity between returns or a presence of a significant deviation from parallelism can be interpreted as evidence contrary to the CAPM. In determining what is a "significant" deviation and what is a "minor" deviation, a procedure was devised based on bootstrap mimicking the data generation process.

Recall that (5) in conjunction with (6) is frequently written as a linear regression model with intercept (where intercept is formally included and estimated but later inspected as to whether it is zero indeed). Thinking of (5) and (6) in these terms, one is possible to use the bootstrap procedure so as to construct the underlying distribution of the estimator b_i in (6). To this purpose, the error resampling was utilized, in which it is assumed that the regression specification is correct but is free of any distributional assumption concerning errors (Davison & Hinkley, 1997). In (5), the error term is represented by ξ_{it} and satisfies, by the previous constructions, the assumptions of the classical linear regression model but no distributional assumption is formed. The bootstrap procedure was used to compare the distributions of the beta coefficient estimators in the two 5-year subperiods. A total of 10 000 bootstrap draws were made for each stock and for each subperiod.

Many researchers would go into inventing statistical tests or into using some conventional ones in responding to this task. Nonetheless, this would be at risk of mistaking some particular distribution for ξ_{it} and associated with troubles of making multiple comparisons. The authors opine that procedures oriented on verification of the CAPM should be kept distributional free and are possibly reducible to exploratory data analysis as here in the paper. It is a common practise to assume that ξ_{it} is a Gaussian variate (Campbell, Lo, & MacKinley, 1996), but there is an abundance of accumulated experience that this is not appropriate (Campbell, Lo, & MacKinley, 1996). This is avoided here and the comparison of regression slope coefficients is effected via the comparison of bootstrap distributions of the underlying estimator. Kernel density estimates of the bootstrap distribution of the CAPM beta coefficient (using the Gaussian kernel and the Sheather-Jones method to select bandwith) for the two 5-year subperiods are reported in Appendix B. For each stock, two kernel density estimates are plotted in one graph and the indication of the subperiod is done by a different colouring. Horizontal lines identify point estimates of the CAPM beta coefficients that were already shown in Table 2. The more distant are estimated CAPM coefficients of the two 5-year subperiods, having in mind also the shape of the two distributions and their intersection, the higher is belief that they are not truly equal.

Before concluding the results and discussing them, a necessary remark is in order related to the use of software: In computations and preparing graphical presentations, the software R 3.0.1 (R Core Team, 2013) was employed and several of its libraries, PerformanceAnalytics (Carl, Peterson, Boudt, & Zivot, 2013), FRBdata (Takayanagi, 2011), car (Fox & Weisberg, 2013). All the codes were prepared by the authors.

4. Conclusion and discussion

The CAPM itself does not impose on the strength of the linear relationship in (5). Non-linear relationships between excess S&P 500 Index returns and excess returns of individual stocks are not detectable. In some cases (such as with CAG, DNR or CLF), if present at all, the linear relationship is very weak, and in other cases (such as with COF, DGX and EXC), the presence of linear relationship is apparent. This holds not only for the entire 10-year period but also for the two 5-year subperiods. As seen in Table 2 or in Appendix A, for most stocks, there is almost a perfect match in the slope of regression lines as they seem parallel or nearly parallel, the alarming disparity is manifested with 3 stocks DNR, DGX and AVY. Regression line slopes in the two subperiods for these 3 stocks clearly differ. Another insight can be gained by assessing the distributions of the estimator of the CAPM beta coefficients and comparing them between the 5-year subperiods in Appendix B. If they beta coefficients in the two subperiods are to be identical, density curves must overlap at point estimates or one must be encompassed by another. A good agreement between point estimates of the beta coefficient in terms of the bootstrap distributions is ascertained with 4 stocks (LEN, CAG, HRS and EXC), quite a match is found with 3 stocks (COF, CLF and CCI) and an acceptable overlap of bootstrap densities is the situation of 2 stocks (DNR and DGX). The only visual statistical difference in the centre of bootstrap distributions and their overlap is found with AVY, in which case

point estimates of the CAPM beta coefficients are located distant from each other and are deep in the tail of the bootstrap distribution that is constructed the other subperiod.

The analysis is suggestive that there is no material evidence contradictive of the linearity property of the CAPM with respect to the selection of 10 assets made on the proviso that one accepts the selected proxies for the market portfolio and for the riskless rate. Contra-evidence arises when the entire evaluation period of ten years from 2003 to 2012 is divided into two subperiods. For one stock participating in the underlying S&P 500 Index, AVY, changes in the betas are found which is in contrast to the linearity property of the CAPM. An interesting point is that the CAPM will continue to prosper and enjoy great popularity for its intuitive and understandable construction resulting from a set of assumptions peculiar to the environment of developed capital markets.

The empirical analysis was conducted under the set of implications. On the condition that the CAPM is a correct theoretical model, then the expected excess return on a risky asset is in a linear relation to the expected excess return on the market portfolio, which also implies that the excess return on this risky asset itself is linearly related to the excess return on the market portfolio itself. This relationship must hold for any period of time or any subperiod of it. This consideration sets out the method for this empirical study. As opposed to previous studies, no attempts are made at rigorous statistical testing since it introduces further complications connected with breaking some of the assumptions formed therewith. Instead, scatter exploratory data analysis is complemented with bootstrapping the linear regression model that can be discovered behind the formula of the CAPM and utilized in the paper for the purpose of verification of the CAPM. To the knowledge of the authors, empirical research has so far focused more on rigorous statistical testing and has not approached this problem from this perspective. The road taken in the paper is distribution-free and simplifies the assumptions of statistical testing considerably – removing the most frequent assumption of the Gaussian distribution. Of course, any approach to verification of the CAPM must rely on some proxies and so does the method embraced in the paper.
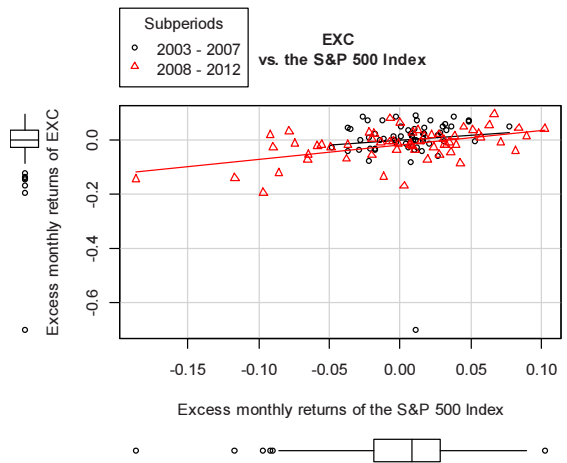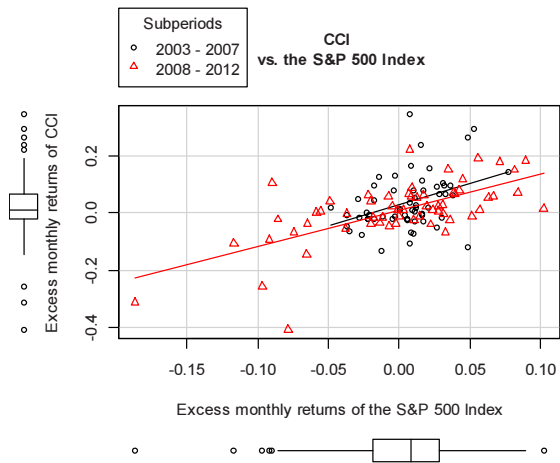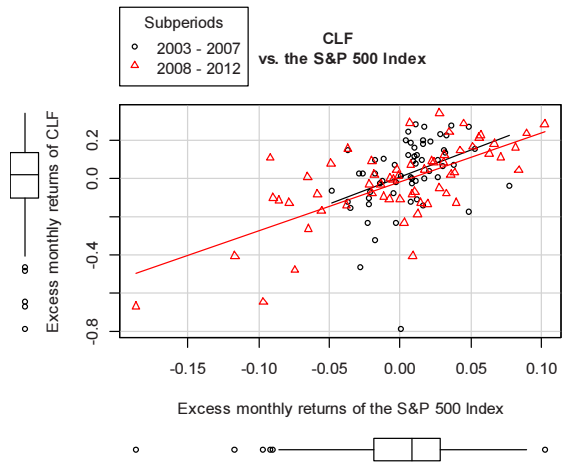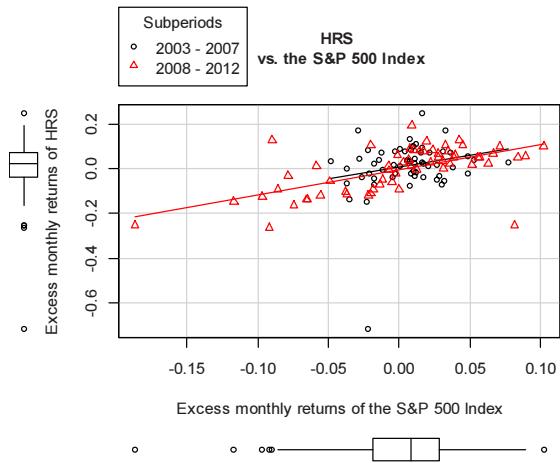
There are some remarks of interest for the future work in this area. It is indisputably needful to set some measurement criteria for comparison of the likeness/difference of estimated CAPM beta coefficients in terms of kernel density estimates of their bootstrap distribution (and avoid relying on the eyeball test), to consider all the assets represented in the S&P 500 Index as well as those that are not its components. Furthermore, longer time periods should be investigated and divided into several subperiods.
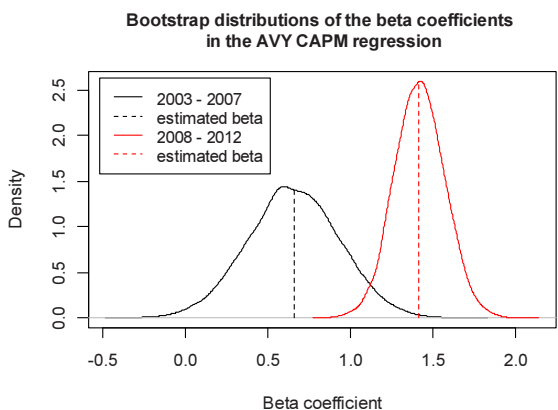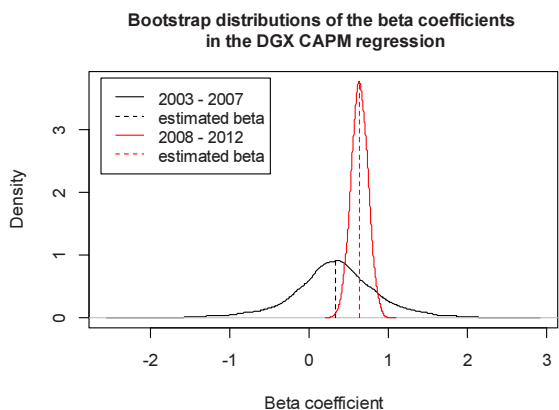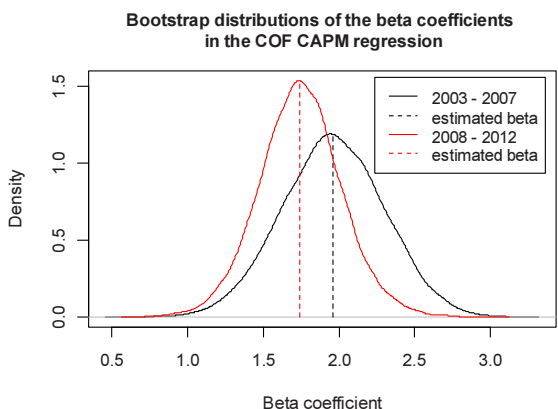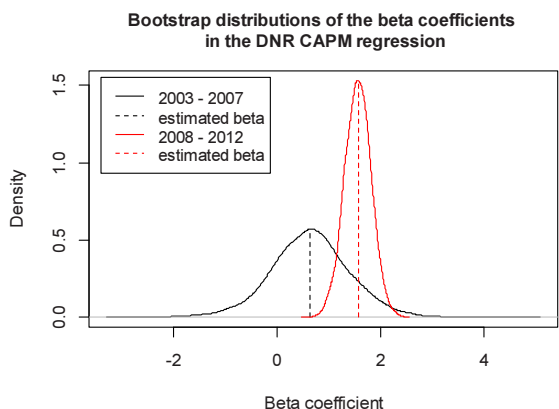
Acknowledgements

Appendix A. Scatterplots of market excess returns
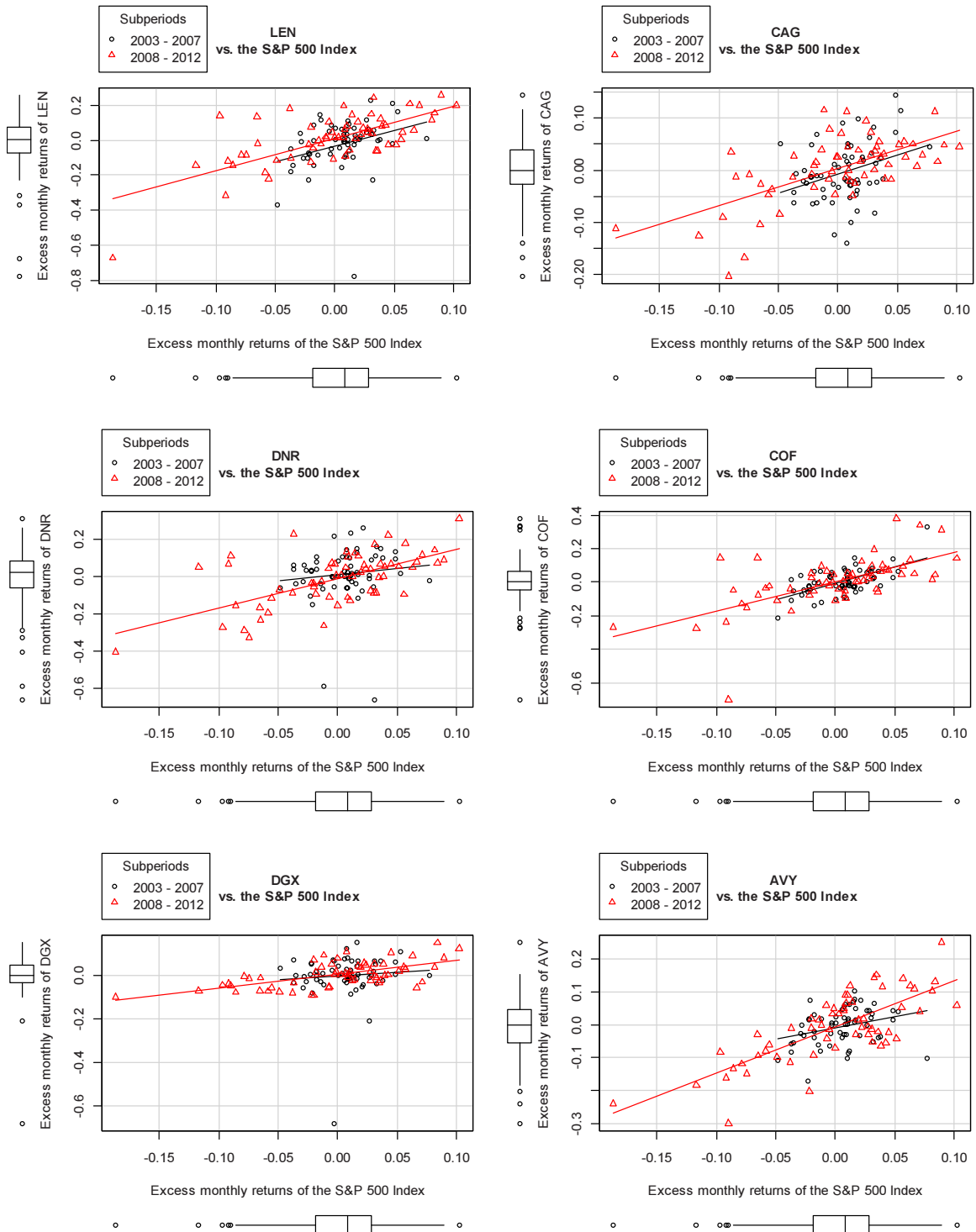
Appendix B. Kernel density estimates of the bootstrap distributions of the CAPM beta coefficient estimators

Bootstrap distributions of the beta coefficients in the LEN CAPM regression

Bootstrap distributions of the beta coefficients in the CAG CAPM regression

Bootstrap distributions of the beta coefficients in the DNR CAPM regression

Bootstrap distributions of the beta coefficients in the COF CAPM regression

Bootstrap distributions of the beta coefficients in the DGX CAPM regression

Bootstrap distributions of the beta coefficients in the AVY CAPM regression

Bootstrap distributions of the beta coefficients in the HRS CAPM regression

Bootstrap distributions of the beta coefficients in the CLF CAPM regression

Bootstrap distributions of the beta coefficients in the CCI CAPM regression

Bootstrap distributions of the beta coefficients in the EXC CAPM regression

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