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## Selection of orthogonal investment portfolio using Evolino RNN trading model

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### Abstract

Investing in financial market require the reliable predicting of expecting returns, assessment of risk and reliability. Principle of portfolio orthogonality was using to reduce the risk of the investment. An artificial intelligence system may reveal new opportunities for using this principle. Prediction of recurrent neural networks ensemble is stochastically informative distribution, which is helpful for portfolio selection. Shape and parameters of distribution influence decision making in currency market. Assessment of portfolio riskiness, finding most orthogonal elements of portfolio, influence better results for trading in real market.

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### 1. Introduction

The decision to choose an investment portfolio is always taken on uncertainty. It reflects a certain expectation that this or any other financial market instrument in the future will behave according to certain rules. The expectations it is mostly based on the financial instruments behavior on the past. It assumes that future events can be predicted based on the past.

The most common financial market forecasts generated using different averages. Latane (1959) first described the use of the geometric mean of the investment strategy, in order to optimize the investment strategy of choice under uncertainty. Authors (Weide, Peterson, & Maier, 1977) examine the investment strategies of the return when investment is increasing if the geometric average of the expected return on investment is maximized. Authors tested

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conditions when return on investment is a discrete distribution. (Roll, 1983) drew attention to the importance of forecasting accuracy of expected return on the arithmetic average and compared it with other priority-setting methods. (Jean, 1984) compared the arithmetic and harmonic averages of the use of the expected return setting. (Cooper, 1996) declares the preference to the arithmetic average, offering forecasting method using the arithmetic and geometric averages hybrid. (Indro & Lee, 1997) shows that weighted arithmetic and geometric averages of the hybrid method can achieve better prediction accuracy for the long-term than the arithmetic or geometric averages alone.

Geometric averages for forecasting financial markets are very popular, but (Missiakoulis, Vasiliou, & Eriotis, 2007) proves that the geometric mean of portfolio assessment is not appropriate. (Missiakoulis, Vasiliou, & Eriotis, 2010) proposed a stock market forecasting tools approach based on the arithmetic average for the long-term investment. (Santiago & Estrada, 2011) assessed the potential, the observed and simulated dynamics of the geometric mean. (Zhang, 2012) theoretically justified that the geometric mean is an important stock market returns the defining quality. In practice, the using of stochastic models of self-financing assumption supported the theoretical results. (Markowitz, 2012) examined the accuracy of six forecasting methods using arithmetic and geometric averages, two databases. (Missiakoulis, Vasiliou, & Eriotis, 2012) compared the all three averages, made an empirical analysis of the predictive accuracy and prefers harmonic average, arguing that in the long term, the harmonic mean best reflects the investors' expectations.

Predictions obtained through artificial intelligence systems, was also used. (Kimoto, Asakawa, Yoda, & Takeoka, 1990) suggested a number of training algorithms and prediction methods for TOPIX (Exchange Tokyo Stock Price Index) forecasting system. Purposed algorithms can predict the best time to buy and sell the stock.

Profits are more important than the statistical results for investors (Ang & Quek, 2006) proposed stock trading model RSPOP (Rough Set Based Pseudo-Outlet Product), which predicts the stock price difference. Neuro Fuzzy system-based model uses a moving average trading rules. The model tested in the real market showed, that the using of the model provided more accurate forecasts than the free trade model. (Raudys & Zliobaite, 2006) researched the financial markets from a different angle – as a rapidly changing chaotic process. The stock price does not change in isolation, (Choudhry & Garg, 2008) has noticed that such stock prices are significantly correlated with other stock prices, so some stock prices can be predicted by the other stock prices. (Rutkauskas & Ramanauskas, 2009) proposed artificial stock market model based on the interaction of heterogeneous agents, whose behavior is formed by self learning algorithms in combination with certain evolutionary selection procedures.

(Guresen, Kayakutlu, & Daim, 2011) compared the number of different well-known neural network's ability to predict stock index. (Bhattacharyya, Kar, & Majumder, 2011) entrusted the portfolio selection problem for hybrid of fuzzy diffusion and genetic algorithms. Optimistic, pessimistic and combined portfolios demonstrated the effective trading in stock market. (Chakravarty & Dash, 2012) compared six different diffusion and fuzzy neural network systems, its abilities to predict the stock price indices, and evaluated the forecasting accuracy. (AbuHamad, Mohd, & Salim, 2013) model the indicators of technical and fundamental analysis by artificial neural networks, successfully predicts the three exchange rates with the US dollar. (Giebel & Rainer, 2013) increased the efficiency of neural network learning and reduced the amount of data required for learning to last 10 days, predicted chaotic stock prices changes and currency fluctuations with sufficient precision.

Although, the choice of predictions methods is very important, but for the selection of the investment portfolio under uncertainty it is not enough. The modern portfolio theory pioneer (Markowitz, 1952) used a profit-risk dimensions and his works was efficiently complemented by (Black, 1974) and many other works. (Anderson & Francke, 1980), (Jorion, 1986) researches the problem of portfolio diversification, experimentally analyzed how many financial tools must be in portfolio for the safe investment.

One of the risk mitigation techniques is constructing orthogonal portfolio. (Roll, 1980) first formulated the term of portfolio orthogonality. Some authors (Jobson & Korkie, 1982), (Asgharian, 2011) proposed asset allocation models with latent factors based on the portfolio orthogonality term.

Rutkauskas (2000) offered an adequate investment portfolio, which combines the profitability, riskiness and reliability. Portfolio, which has the lowest risk for a given profitability with a certain probability or the highest return for a given level of risk associated with a certain probability (Rutkauskas, 2006).

Investment decisions are always made under uncertainty; it is possible only in a heuristic approach to the maximum result. Potential distributions best reflect the investment opportunities. The aim of paper is to select

investment portfolio using parameters of potential distribution obtained by the model based on ensemble of recurrent neural networks.

**2. Principals of investment portfolio selection**

A portfolio consisting of p assets:  $P_t$  – the value of the portfolio at time t; and  $w_t$  – the weight of the portfolio value invested in asset i;  $P_{it} = w_i p_t$  – the value of asset i. Planning profit of portfolio:

$$ER_p = \sum_{i=1}^n W_i ER_i \tag{1}$$

where  $W_i$  is present of many, investing in asset i,  $ER_i$  – planing profit from asset i,  $n$  – number of assets in portfolio.

$$COV_{ij} = r_{ij} \sigma_i \sigma_j \tag{2}$$

where  $COV_{ij}$  is covariation between assets i and j,  $r_{ij}$  – correlation between assets i and j;  $\sigma$  – standard deviation. Portfolio risk:

$$\sigma^2 = \sum_{i=1}^n W_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1}^n W_i W_j r_{ij} \sigma_i \sigma_j \tag{3}$$

Each searching of portfolio assets consist from such steps (Fig. 1):

- *Historical data monitoring and the accumulation of experience.* Changing of financial market prices is chaotic process. It has a memory – what was in the past – effects future. The amount of data and the form of storage is determined by the chosen method of forecasting.
- *Future forecasting.* It is known big amount of different methods of forecasting, based in different means: arithmetic, geometric, harmonic and its hybrids. Artificial intelligence systems, like neural networks, genetic algorithms, Fuzzy systems, expert systems, are also used for forecasting financial markets.
- *Asset allocation and assessment of reliability.* Selection of portfolio elements from different financial and asset markets and divide the invested funds for all elements with best result – is main aim of each investor. The biggest gain is selecting for a given level of risk or the lowest risk is selecting for a given level of profit.

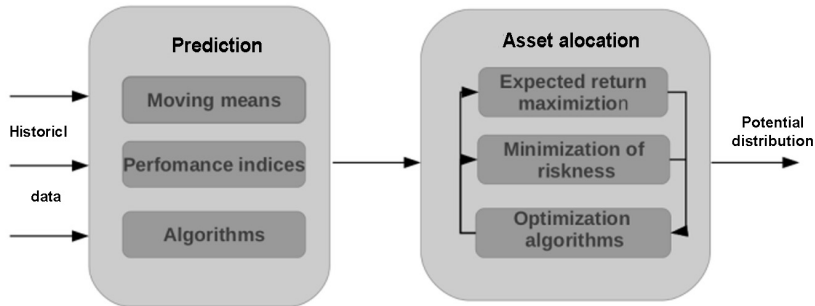


Fig. 1. The constructing the efficient portfolio

The basic rule of portfolio optimization is to maximize return, minimize risk, maximize reliability (Markowitz, 1952; Rutkauskas, 2000):

$$\begin{cases} \max ER_p \\ \min \sigma^2 \\ \max P_p \end{cases} \tag{4}$$

$$\left\{ \begin{array}{l} ER_p = \sum_{i=1}^n W_i ER_i \\ \sigma^2 = \sum_{i=1}^n W_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=1}^n W_i W_j r_{ij} \sigma_i \sigma_j \\ P_p = \prod_i \frac{f_i}{N} \end{array} \right. \quad (5)$$

where  $f_i$  is frequency of the distribution mode,  $N$  is number of predictions and  $\sum W_i = 1$ .

Roll (1983) introduced the term of portfolio orthogonality

$$\sum_{i=1}^n r_{ij} \sigma_i \sigma_j = 0 \quad (6)$$

We enter some  $\varepsilon$  which is the scale of proximity to portfolio orthogonality:

$$\sum_{i=1}^n r_{ij} \sigma_i \sigma_j = \varepsilon \quad (7)$$

The close to zero  $\varepsilon$  means better orthogonality. It is two ways to get less  $\varepsilon$ : to select asset with different signs of  $r_{ij}$ , or to look for asset with less  $\sigma_i$  and  $\sigma_j$ . This principle of orthogonality should be checked experimentally in real time in Forex market.

### 3. Selection of investment portfolio and testing it in real market

Authors (Wierstra, Gomez, & Schmidhuber, 2005), (Schmidhuber, Wierstra, & Gomez, 2005) proposed a new class of learning algorithms for supervised recurrent neural networks – RNN Evolino. Evolution of recurrent systems with Optimal LINEar Output. Evolino-based LSTM recurrent networks learn to solve several previously unlearnable tasks. "Evolino-based LSTM was able to learn up to 5 sins, certain context-sensitive grammars, and the Mackey-Glass time series, which is not a very good RNN benchmark though, since even feed forward nets can learn it well" (Schmidhuber, Wierstra, Gagliolo, & Gomez, 2007).

It is known, that ensembles can predict more accurate, then single RNN. Authors constructed Evolino RNN-based prediction model (Fig. 2), ensemble from 176 RNN. Influence of number of RNN in ensemble was investigated by authors in earlier work (Maknickiene & Maknickas, 2013).

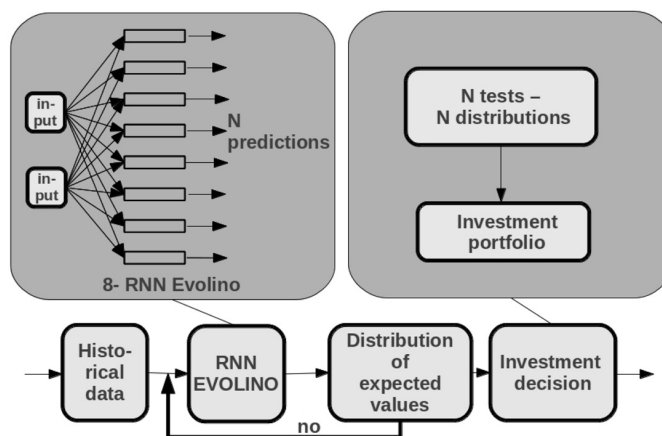


Fig. 2. Forecasting model, using 176 Evolino RNN ensemble

Predictions of exchange rates are getting by the following steps:

- *Data step.* Getting historical financial markets data from Meta Trader – Alpari, we choose for prediction EUR/USD (Euro and American Dollar), EUR/JPY (Euro and Japanese Yen), USD/JPY (American Dollar and Japanese Yen), USD/CHF (American dollar and Swiss Franc), GBP/USD (United Kingdom and American dollar), GBP/AUD (), NZD/CAD () exchange rates and their historical data for the first input, and for the second input, two years historical data for XAUUSD (gold price in USA dollars), XAGUSD (Silver price of USA dollars), QM (Oil price in USA dollars), and QG (Gas price in USD dollars). At the end of this step we have a basis of historical data.
- *Input step.* The python script calculates the ranges of orthogonality of the last 80--140 points of the exchange rate historical data chosen for prediction, and an adequate interval from the two years historical data of XAUUSD, XAGUSD, QM, and QG. A value closer to zero indicates higher orthogonality of the input base pairs. Intervals with the best orthogonality were used for the inputs to the Evolino recurrent neural network. Influence of data orthogonality was research in earlier work (Maknickas & Maknickiene, 2012).
- *Prediction step.* For calculation of big amount of ensembles software and hardware acceleration were employed. Every predicting neural network from ensemble could be calculated separately. So, calculations could be done in parallel. MPI wrapper mpi4py (Dalcin, 2012) were used for this purpose. Cycle of each predicting neural network was divided into equal intervals and every interval were calculated on separate processor node. There are not needs for communication between mpi threads, so obtained equal to one efficiency of parallelism, where efficiency is described as follow (Fox, Johnson, Lyzenga, Otto, Salmon, & Walker 1988), (Kumar, Gmma, & Anshul, 1994). When  $n > 120$ , the forecasting assumes the shape of the distribution. At the end of this step, we have a distribution with all parameters of it – mean, median, mode, skewness, kurtosis and end et.

Distributions are not normal, they are multimodal. Examples of different shapes of distributions are shown in Fig. 3. The tight distribution (Fig. 3 a) predict clear direction of currency price changing,  $ER_p = p_t - Mo$ , where  $p_t$  last known value of predicting exchange rate,  $Mo$  – mode of distribution, like most probable value of exchange rate in moment  $t+1$ . Scattered distribution (Fig. 3 b) shows high risk, and its standard deviation  $\sigma$  is high. Multimodal distribution (Fig. 3 c) warns about several forces operating in the financial market. Skewed distribution (Fig. 3 d) shows different probability of markets changing directions. Investors decision in all cases depends from  $p_t$  position in respect of  $Mo$ .

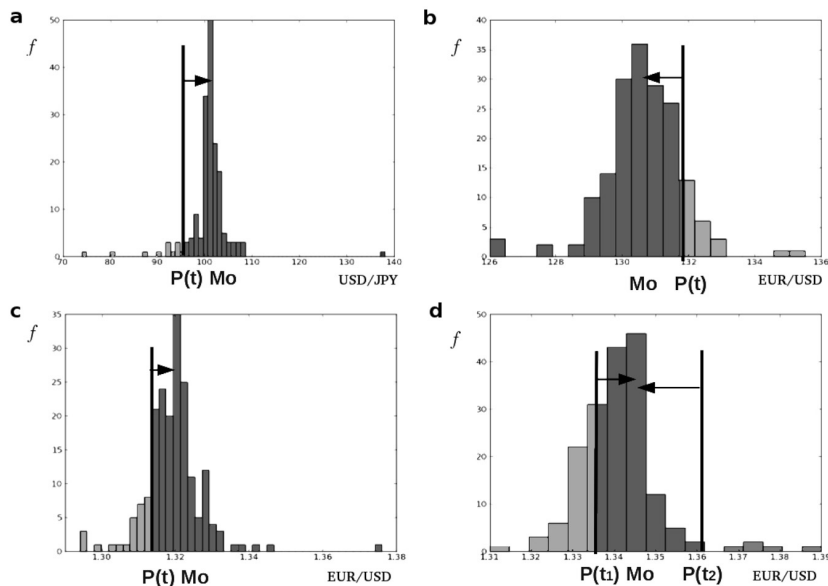


Fig. 3. Examples of distributions: a) tight, decision "buy"; b) scattered, decision "sell", but big riskiness; c) multimodal, decision "buy"; d) skewed to left, if  $p_{t1}$  decision "buy" with big riskiness, if  $p_{t2}$  – "sell"

For selecting exchange rates for investment was used term of portfolio orthogonality. Portfolio orthogonality (Eq. 7) depends on correlation coefficient  $r_{ij}$ . Calculated values are presented in Table 1.

Table 1. Correlation coefficients of exchange rates

Exchange rates	EUR/JPY	EUR/USD	USD/JPY	GBP/USD	GBP/AUD	NZD/CAD	USD/CHF
EUR/JPY	1	-0.188	0.786	-0.51	0.145	0.361	0.313
EUR/USD	-0.188	1	-0.756	0.808	0.244	-0.151	-0.964
USD/JPY	0.786	-0.756	1	-0.849	-0.059	0.337	0.815
GBP/USD	-0.51	0.808	-0.849	1	0.007	-0.346	-0.820
GBP/AUD	0.145	0.244	-0.059	0.007	1	-0.403	-0.307
NZD/CAD	0.361	-0.151	0.337	-0.346	-0.403	1	0.224
USD/CHF	0.313	-0.964	0.815	-0.82	-0.307	0.224	1

For investigating we are selecting two portfolios – "not orthogonal", with  $|\sum r_{ij}| = 1,418$  and "orthogonal" with  $|\sum r_{ij}| = 0,66$ .  $\sigma_i$  and  $\sigma_j$  is parameters of distribution and can't be calculated before selecting exchange rates. This parameters are important for dividing investment found for selected exchange rates.

Most simply diversification is  $1/n$  strategy, when investing funds are distributed equally for each exchange rate. The third portfolio was called "orthogonal-optimal". It was constructed using 5 equations. For calculating  $ER_p$ ,  $\sigma_p$ ,  $\sigma_j$  and  $P_p$  was used parameters of distribution and last known values of exchange rates.

Testing in Forex market, in real time shown in Fig. 3. The "orthogonal-optimal" portfolio was started later for technical reasons, but this did not prevent the best results. Problems by testing "not orthogonal" portfolio are conditioned by manipulations with Japanese yen.

Investing portfolio selected with best orthogonality, with  $\varepsilon$  close to zero gives better results in two portfolios – "orthogonal" and "orthogonal-optimal". Profit of "not orthogonal" portfolio with 2 times better orthogonality increase slowly, then orthogonal portfolios. The investment portfolio takes into account the orthogonality conditional allows for productive investment in the foreign exchange market.

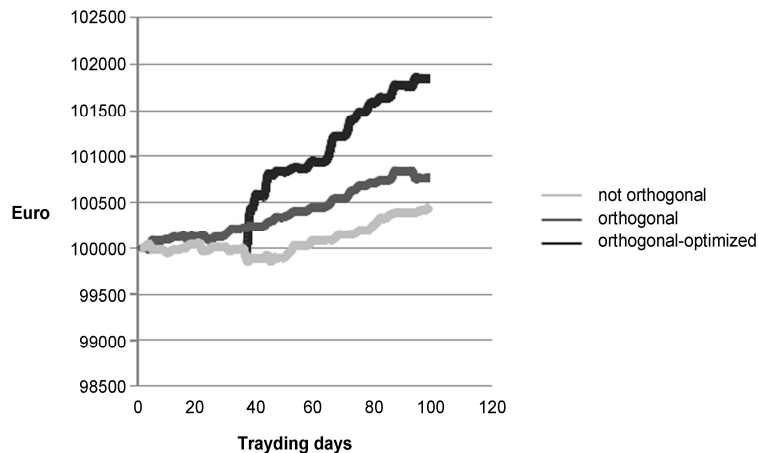


Fig. 4. Testing three different portfolios in Forex market in real time, from 2013-06-26 to 2013-10-07

## Conclusions

Main portfolio selection principals of modern portfolio theory- maximising expected return and minimising riskiness – are successfully using together with artificial intelligence.

Potential distributions best reflects the investment opportunities in conditions of uncertainty. Model, using ensemble of Evolino RNN, is reliable forecasting tool useful for investors in currency market. Its prediction is stochastically informative potential distribution, which is helpful for portfolio selection. Shape and parameters of distribution influence decision making in currency market.

Testing in Forex market in real time confirmed, that assessment of portfolio riskiness, finding most orthogonal elements of portfolio, influence better results for trading in real market. Orthogonality and optimization influence the better selection of investment portfolio.

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