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On Securities Portfolio Optimization, Preferences, Payoff Matrix Estimation and Uncertain Mixed Decision Making

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Abstract. Securities portfolio optimization has been analysed so far on the assumption that the estimation of the probability distribution concerning future rates of return is possible thanks to historical data. However, sometimes it is desirable to forecast profits by considering factors which are not included in past and present results. The purpose of the paper is to investigate the stocks portfolio optimization in the context of decision making under complete uncertainty, i.e. uncertainty with unknown probabilities, which allows the investor to refer to scenario planning. In the contribution, we propose the use of a decision rule for portfolio optimization under complete uncertainty. The procedure takes into account the decision maker's nature and enables one to select the optimal mixed strategy, which is characteristic of portfolio optimization where variables denoting the share of particular securities are continuous (not binary). The decision process is discussed for two types of decision makers: an active one (who estimates the profit matrix on his own) and a passive one (who uses a profit matrix generated by experts). Additionally, we analyse the impact of the profit matrix estimation (subjectively or objectively) on the decision making process.

Keywords: portfolio optimization, stocks, payoff matrix, optimal mixed solution, uncertainty, decision maker's preferences.

JEL Classification: C44, C61, D81, D84, G11.

Conference topic: Financial Risk Management of Business Development.

Introduction

Securities portfolio optimization is the process of choosing the proportions of various securities to be held in a portfolio, in such a way as to make the portfolio better than any other according to some criterion. The criterion combines considerations of the expected values of the portfolio's rate of return as well as of the return dispersion and possibly other measures of financial risk. In this contribution we limit the research to stocks (shares). The stock portfolio optimization is usually led in the framework of decision making under stochastic uncertainty which concerns decision problems with at least one parameter being a random variable with a known probability distribution (i.e. the rate of return). When making decision under stochastic uncertainty, the decision maker is able to define possible states of nature (scenarios, events) and the probability that a particular scenario will occur.

Nevertheless, the use of historical data in order to estimate the probability distribution may sometimes prove inadequate for stock portfolio optimization when exchange rates times series are not long enough or when the investor (stockbroker, financial expert) is able to predict totally new future events that are not included in historical rates of return. The technical analysis (a security analysis methodology for forecasting the direction of prices on the basis of charts and statistics generated by market activity) and fundamental analysis (analysis attempting to measure the intrinsic value of the security by examining related economic, financial and other qualitative and quantitative company-specific factors) constitute a very helpful tool in forecasting future rates of return, but they also might turn out to be insufficient because they are both carried out on past and present data. In such circumstances the calculation of the likelihood of the occurrence of future possible scenarios may be unenforceable.

Therefore, we would like to investigate the stock portfolio optimization in the context of decision making under complete uncertainty, i.e. uncertainty with unknown probabilities, which allows the investor to refer to scenario planning and to take into consideration his or her nature (preferences) instead of probabilities defined objectively. In the paper, we propose a decision rule for stock portfolio optimization under complete uncertainty. The decision process is discussed for two types of decision makers: an active one and a passive one. The first one estimates the profit matrix (rate of return matrix) on his own. The second one makes the decision on the basis of a profit matrix defined by experts. We are also going to analyse the impact of the profit matrix estimation on the decision making process.

The paper is organized as follows. The first part deals with previous research concerning stock portfolio optimization. Then scenario-based decision making under complete uncertainty is briefly described. In the main part of the contribution, we present two decision rules supporting stock portfolio optimization for passive and active investors, respectively. We also provide a case study. Conclusions are gathered in the last part.

Previous research

As it was mentioned in the introduction, the stock portfolio optimization serves to determine the percentage structure of the portfolio consisting of diverse shares. Modern portfolio theory, fathered by (Markowitz 1952, 1959), assumes that an investor wants to maximize a portfolio's expected return contingent on any given amount of risk, with risk measured by the standard deviation of the portfolio's rate of return. For portfolios that meet this criterion, known as efficient portfolios, achieving a higher expected return requires taking on more risk, so investors are faced with a trade-off between risk and expected return (Andriotto, Teti 2014). This risk-expected return relationship of efficient portfolios is graphically represented by a curve known as the efficient frontier. All efficient portfolios are represented by a point on the efficient frontier. Papers devoted to portfolio optimization, based on the assumption that the expected rates of return related to particular securities are computed by means of historical data, are numerous (Anholcer *et al.* 2004; Gaspars, Sikora 2004; Gaspars 2004, 2005; Gaspars-Wieloch 2008; Haugen 2001; He, Zhou 2011; Jurek 2001; Kitowicz, Sikora 2004; Kolm *et al.* 2014; Konno, Yamazaki 1991; Pirvu, Schulze 2012; Weber *et al.* 2015).

However, the use of past data to estimate the probability distribution of rates of return may be inadequate for stock portfolio optimization if the investor intends to take into account new factors which can influence future results. Some of those factors can be detected thanks to the technical analysis (which attempts to identify patterns that can suggest future activity) and fundamental analysis (which attempts to study everything that can affect the security's value), but the decision maker (or experts) may even predict events which are not included in studies aforementioned (Castellano, Scaccia 2014; Graf, Six 2014; Gurgul, Wójtowicz 2014). That is why, from the author's point of view, it is desirable to examine the problem of stock portfolio optimization for the situation where the expected rates of return (calculated by means of the likelihood resulting from historical prices) is replaced by predicted rates of return (which, apart from considering past and present events, include information concerning the future). The novel approach refers to scenario planning and decision making with unknown probabilities. Additionally, it allows investors to choose a strategy which reflects their preferences and nature.

Scenario-based decision making under complete uncertainty

According to the Knightian definition (Knight 1921), we will assume that decision making under complete uncertainty (DMCU) is characterized by a situation where the decision maker (DM) has to choose the appropriate alternative (decision, strategy) on the basis of some scenarios which probabilities are not known – uncertainty with unknown probabilities (Courtney *et al.* 1997; Dominiak 2006; Groenewald, Pretorius 2011; Render *et al.* 2006; Sikora 2008; Trzaskalik 2008; von Neumann, Morgenstern 1944; Walliser 2008; Williams *et al.* 1997). DMCU is also related to cases where probabilities are known but the DM does not want to make use of that data (Trzaskalik 2008). Note that if the DM does not know the list of possible scenarios, we deal with DMTI (decision making under total ignorance). On the other hand, if the DM knows the set of events and their probabilities of occurrence, then the problem is related to DMSU (decision making under stochastic uncertainty).

Scenario planning, used within the framework of DMU (Pomerol 2001), is a technique for facilitating the process of identifying uncertain and uncontrollable factors which may influence the effects of decisions in the strategic management context. The way how scenarios should be constructed is described e.g. in (Dominiak 2006; Van der Heijden 1996). The result of the choice made under uncertainty with scenario planning depends on two factors: which decision will be selected and which scenario will occur.

DMCU+SP (decision making under complete uncertainty with scenario planning) may be presented by means of a profit matrix (see Table 1) where n denotes the number of decisions (A_1, …, A_j, …, A_n), m signifies the number of mutually exclusive scenarios (S_1, …, S_i, …, S_m) and a_{ij} is the profit resulting from the choice of decision A_j and the occurrence of scenario S_i . We assume that the distribution of payoffs related to a given decision is discrete.

Table 1. Payoff matrix (general case) (Source: prepared by the author)

Scenarios\alternatives	A_1	A_j	A_n
S_1	$a_{1,1}$	$a_{1,j}$	$a_{1,n}$
S_i	$a_{i,1}$	$a_{i,j}$	$a_{i,n}$
S_m	$a_{m,1}$	$a_{m,j}$	$a_{m,n}$

There are many classical and extended decision rules designed for one-criterion DMU (Basili *et al.* 2005, 2008; Basili 2006; Basili, Chateauneuf 2011; Ellsberg 2001; Etner *et al.* 2012; Gaspars 2007; Gaspars-Wieloch 2012, 2013, 2014a, 2014b, 2014c, 2014d, 2015a, 2015b, 2015c; Ghirardato *et al.* 2004; Gilboa 2009; Gilboa, Schmeidler 1989; Hayashi 2008; Hurwicz 1952; Ioan, C., Ioan, G. 2011; Marinacci 2002; Piasecki 1990; Savage 1961; Schmeidler 1986; Tversky, Kahneman 1992; Wald 1950) and multicriteria DMU (Aghdaie *et al.* 2013; Ben Amor *et al.* 2007; Dominiak 2006, 2009; Durbach 2014; Eiselt, Marianov 2014; Gaspars-Wieloch 2014e, 2015d, 2015e; Ginevičius, Zubrecovas 2009; Goodwin, Wright 2001; Hopfe *et al.* 2013; Janjic *et al.* 2013; Korhonen 2001; Lee 2012; Liu *et al.* 2011; Mikhaidov, Tsvetinov 2004; Montibeller *et al.* 2006; Ram *et al*. 2010; Ramik *et al.* 2008; Ravindran 2008; Stewart 2005; Suo *et al.* 2012; Triantaphyllou, Lin 1996; Tsaur *et al*. 2002; Urli, Nadeau 2004; Wang, Elhag 2006; Wojewnik, Szapiro 2010; Xu 2000; Yu 2002). However, the majority of the extended rules refer to the probability calculus, which is rather characteristic of DMSU. Existing decision rules differ one from another with respect to the DM's preferences measured, for instance, by the coefficient of pessimism (α) or optimism (β).

DMU procedures are dedicated to searching an optimal pure (the DM chooses and executes only one alternative) or mixed strategy (the DM selects and performs a weighted combination of several accessible alternatives), see e.g. bonds portfolio construction, cultivation of different plants (Officer, Anderson 1968; Puppe, Schlag 2009; Sikora 2008). This paper concern the second case since the stock portfolio optimization consists in defining optimal shares of particular stocks in the portfolio. The sum of those shares is equal to 1 and variables are continuous.

It is worth emphasizing that some rules find application when the DM intends to perform the selected strategy only once (one-shot decisions). Others are recommended for people contemplating realization of the chosen variant many times (multi-shots decisions). This contribution focuses on one-shot decision problems because in the case of stock portfolio optimization the investor usually defines the optimal portfolio structure for a single period and then, after comparing theoretical rates of return to real rates of return, he or she may take into consideration new information and new possible future events.

The decision rules presented in this paper allow the DM to find an optimal mixed strategy. Note that existing one-criterion and multicriteria procedures for mixed strategies rather relate to game theory, i.e. game between players (Czerwiński 1969; Gilboa 2009; Grigorieva 2014; Lozan, Ungureanu 2013; Luce, Raiffa 1957; Voorneveld *et al.* 1999, 2000), than game against nature (which constitutes a neutral opponent). Therefore, the creation of an approach for uncertain mixed decision making with scenario planning (or scenario-based mixed DMCU) seems to be vital and desirable. The method described in this contribution has the following features:

1) It enables the DM to reduce the set of potential scenarios to the set of the most subjectively probable scenarios and to make the final decision on the basis of the selected events (idea presented in (Gaspars-Wieloch 2013, 2014b, 2015b)).
2) It allows the DM to take into consideration his or her preferences, nature and predictions (the rate of return matrix may be estimated intuitively by the investor – case of the active investor, the decision is determined by the level of optimism measured by β – case of the passive investor), see e.g. (Gaspars-Wieloch 2014c; Hurwicz 1952).
3) It takes into account all rates of return connected with potential shares, which is especially important in the case of significant differences between the dispersions of particular predicted incomes (Gaspars-Wieloch 2015c; Ioan, C., Ioan, G. 2011).

Hence, the approach proposed in this article has some similarities with previous decision rules worked out by the author and concerning DMCU, but this one is adapted to portfolio analysis by taking into account the width of particular intervals of rates of return and by formulating an optimization model for searching mixed strategies, i.e. portfolio structures.

Uncertain stock portfolio optimization for passive investors

Passive investors are defined in this paper as decision makers who do not intend to forecast future scenarios concerning particular stocks and to determine the most probable states, but they are interested in receiving final recommendations from experts and analysts. In such circumstances the rate of return matrix (RR matrix) is not supposed to be estimated by investors. They are only asked to:

- define the set of stocks ($A_1, \ldots, A_j, \ldots, A_n$) that they are willing to include in their portfolio and
- declare their preferences measured by the coefficient of optimism $\beta \in [0,1]$ (β tends to 1 for extreme optimists). That parameter may be calculated on the basis of psychological tests (see e.g. Sawicki, Tyszko 2011) or subjectively estimated by the decision makers themselves.

A possible decision rule for stock portfolio optimization in the case of passive investors (SPO-PI) may contain the steps presented below:

– Step 1: Given P – a set of potential stocks (suggested by the investor), and a rate of return matrix (generated by experts) for the considered portfolio optimization problem, choose an appropriate value of parameter β according to the DM's level of optimism (psychological tests or subjective estimation).

– Step 2: Solve the max-min problem in order to calculate the maximized minimum guaranteed income (y^*), Eqs (1)–(4).

$$y \to \max ; \tag{1}$$

$$\sum_{j}^{n} a_{ij} x_j \geq y , \qquad i = 1,...,m ; \tag{2}$$

$$\sum_{j}^{n} x_j = 1 ; \tag{3}$$

$$x_j \geq 0 , \qquad j = 1,...,n , \tag{4}$$

where: a_{ij} stands for the rate of return connected with stock A_j and scenario S_i, x_j denotes the share of stock A_j in the portfolio, n is the number of potential securities, m signifies the number of considered states.

Note that model (1)–(4) represents the Wald's decision rule for mixed strategies and it is designed for extreme pessimists (Wald 1950). The solution of that problem does not constitute the final structure of the portfolio in the case of SPO-PI, but it is just used to compute the rate of return that can be gained even if the worst scenario occurs, i.e. y^*.

– Step 3: Find the maximum revenue (M^*), Eq. (5).

$$M^* = \max_{\substack{i=1,...,m \\ j=1,...,n}} \{a_{ij}\} . \tag{5}$$

– Step 4: Find the set of the most probable scenarios (K) with the aid of s (6)–(10).

$$S_i \in K \Leftrightarrow \left(\underset{a_{ij}(j=1,...,n)}{\exists} a_{ij} \geq r_\beta \right) \vee \left(d_i \geq d_\beta \right) ; \tag{6}$$

$$r_\beta = \beta(M^* - y^*) + y^* ; \tag{7}$$

$$d_\beta = \beta(d_{\max} - d_{\min}) + d_{\min} ; \tag{8}$$

$$d_i = \sum_{j=1}^{n} d_{ij} , \qquad i = 1,...,m ; \tag{9}$$

$$d_{ij} = m - \max\left\{p^j(a_{ij})\right\}, \qquad i = 1,...,m; \quad j = 1,...,n , \tag{10}$$

where: r_β belongs to [y^*, M^*] and denotes the expected level of revenue dependent on β, d_{ij} denotes the number of rates of return related to stock A_j which are worse than a_{ij}, d_i is the total number of "dominance cases" related to state S_i, d_{max} and d_{min} are the highest and the lowest number of "dominance cases" respectively and $p^j(a_{ij})$ is the position of rate of return a_{ij} in the non-increasing sequence of all rates connected with stock A_j (when a_{ij} has the same value than other rates of return concerning a given security, then it is recommended to choose the farthest position of this rate of return in the sequence – see Eq. (10)). Constraint (7) shows the relationship between the coefficient of optimism and the expected (desired) revenue. Scenario S_i may belong to K if and only if it contains at least one rate of return not lower than r_β or its number of "dominance cases" is sufficiently close to d_{max} (Eq. (6)). Scenario with d_{max} and with at least one payoff equal to M^* might be treated as the best state of nature, but in many decision problems such an event does not exist.

As we can see, the goal of step 4 is to reduce the original set of scenarios to a smaller set containing the states of nature which reflect the DM's nature and predictions.

– Step 5: Solve the mixed portfolio optimization problem using Eqs (11)–(16). The optimal solution represents the mixed strategy reflecting the declared DM's nature.

$$\sum_{i \in K} \max\{g_i, 0\} \to \min ; \tag{11}$$

$$\sum_{j}^{n} a_{ij} x_j = r_\beta - g_i, \qquad i \in K; \tag{12}$$

$$\sum_{j}^{n} x_j = 1; \tag{13}$$

$$x_j \geq 0, \qquad j = 1,...,n; \tag{14}$$

$$\sum_{j=1}^{n} \sigma_j x_j \leq \beta(\sigma_{\max} - 1.5\sigma_{\min}) + 1.5\sigma_{\min}; \tag{15}$$

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(a_{ij} - \overline{a_j}\right)^2}, \qquad j = 1,...,n, \tag{16}$$

where: K denotes the set of the most probable events, g_i is the deviation from r_β of the rate of return achieved by the DM if scenario S_i occurs. Symbol σ_j signifies the standard deviation of all estimated rates of return connected with stock A_j, σ_{min} and σ_{max} are the standard deviation of the stocks with the smallest and the highest dispersion of rates of return, respectively. Note that when calculating the standard deviation, all rates of return are taken into consideration (not only those related to set K) and all of them have the same probability since the goal of this measure is to compare the dispersion of rates of return of particular securities (Eq. (16)).

When β tends to 1, then the DM is an optimist and expects a very high income, close to the rate of return characterized by formula (5), i.e. M^*. On the other hand, when β tends to 0, the decision maker is less demanding and wants to gain at least y^*. Parameter β has an impact on the set of the most likely scenarios. One may make the following assumption – higher the value of this parameter is, less probable, in the DM's opinion, states of nature offering low rates of return are. Therefore, such scenarios are not considered in the optimization model.

Additionally, parameter β may be used in order to determine a linear combination of stocks with a suitable dispersion of rates of return (Eq. (15)). The model recommends stocks with a small dispersion for pessimists. However condition (15) is optional and can be formulated in a different way. Here we suggest using value $1.5\sigma_{min}$ as the upper bound for radical pessimist ($0(\sigma_{max}-1.5\ \sigma_{min})+1.5\sigma_{min} = 1.5\sigma_{min}$) since with such a constraint the extreme pessimist has the possibility to generate a portfolio containing more than one stock, which would not be the case if value $1.0\sigma_{min}$ was used. Of course value $1.5\sigma_{min}$ may be replaced by another value belonging to $[\sigma_{min}, \sigma_{max})$, depending on the rate of return dispersions of a given problem.

Both sides of condition (12) represent the true rate of return obtained if the shares of a particular portfolio equal $x_1, x_2, \ldots, x_n$ and scenario S_i takes place. The aim of the optimization model (Eq. (11)) is to minimize, within set K, the sum of all deviations between the true rates of return and the expected (desired) ones. Note that only positive deviations are treated as unwanted since then the expected revenue exceeds the true income, i.e. the true rate of return is lower than the desired one. Negative deviations do not need to be minimized because they occur when the investor gains more than he wishes (it is a desirable situation).

One of the main questions is how to select the most probable scenarios on the basis of the coefficient of optimism declared by the decision maker. In this paper we assume that set K should contain all scenarios that satisfy conditions (6)–(10), but it is only a suggestion. Similar assumptions have been made in (Gaspars-Wieloch 2013, 2015b, 2015d, 2015e).

It is worth underlining that in the case of SPO-PI, optimal strategies suggested for radical pessimists ($\beta = 0$) are totally the same as strategies recommended by Wald (see model (1)–(4)) since then the DM prefers analyzing all possible scenarios to taking into consideration only the most probable ones.

Uncertain stock portfolio optimization for active investors

In the case of an active investor the procedure has, partially, totally different steps since he (or she) controls each stage of the decision making process. This time, the RR-matrix is estimated by the decision maker, which means that it is not forecasted in an objective way. Furthermore, the predicted rates of return contain some information about the DM's nature. This time, investors (even if they consider the same set of potential stocks) make the decision on the basis of different matrices. Therefore, the choice of the most probable scenarios should not be made by means of the coefficient of optimism because the level of optimism is already included in the matrix. Thus, the set of the most probable states may be defined by the decision maker without any additional computations. The investor knows very well the RR-matrix because it was generated by the DM and he is able to select, subjectively, events which are, in his opinion, the most feasible. The estimation of parameter r^* (desired level of revenue) should be also performed subjectively by the investor.

A possible procedure for stock portfolio optimization in the case of active investors (SPO-AI) may contain the following steps:

– Step 1: Suggest P – a set of potential stocks and generate the RR-matrix (performed intuitively by the investor).
– Step 2: Solve the max-min problem in order to calculate the maximized minimum guaranteed income (y^*), Eqs (1)–(4).
– Step 3: Find the maximum revenue (M^*), Eq. (5).
– Step 4: Find K – the set of the most probable scenarios (performed intuitively by the investor).
– Step 5: Solve the mixed portfolio optimization problem using Eqs (17)–(21). The optimal solution represents the mixed strategy reflecting the DM's nature.

$$\sum_{i \in K} \max\{g_i, 0\} \rightarrow \min ; \quad (17)$$

$$\sum_{j}^{n} a_{ij} x_j = r^* - g_i , \qquad i \in K ; \quad (18)$$

$$\sum_{j}^{n} x_j = 1 ; \quad (19)$$

$$x_j \geq 0 , \qquad j = 1,\ldots,n ; \quad (20)$$

$$\sum_{j=1}^{n} \sigma_j x_j \leq s , \quad (21)$$

where: r^* belongs to $[y^*, M^*]$ and signifies the desired level of revenue. Symbol s denotes the accepted level of dispersion of rates of return, s belongs to $[\sigma_{min}, \sigma_{max}]$.

As it can be observed, the active investor does not need to declare his or her coefficient of optimism (pessimism) since the information concerning the DM's nature is included in estimated rates of return.

Case study

Let us illustrate both procedures. We assume that D1 is a passive investor and D2 – an active one. D1 and D2 are both optimists. However they will declare their preferences in a different way. The analysis is going to be made on the assumption that both decision makers have pointed the same set of potential stocks: P = {A1, A2, A3, A4, A5, A6}, e.g. Hilton Food (HFG), Vectura (VEC), TalkTalk (TALK), CineWorld (CINE), Game Digital (GMD), Lonmin (LMI) from LSE (London Stock Exchange.

SPO-PI (D1):

– Step 1: Experts generate a RR-matrix (Table 2), D1 declares his level of optimism: β = 0.8. The way the matrix is estimated is beyond the scope of this article. Here we just show how objectively estimated rates of return can be used for particular investors.
– Step 2: The maximized minimum guaranteed income is calculated (Eqs (1)–(4)): y^* = 1.4453% (x_1 = 0.0876, x_2 = 0.3066, x_3 = 0.6058, x_4 = x_5 = x_6 = 0), which means that even if the worst scenario occurs, the generated portfolio guarantees at least 1.44% per quarter (y^* is treated as the revenue for an radical pessimist).
– Step 3: The maximum revenue is found: M^*= 8% (Eq. (5)). That rate of return is assigned to an extreme optimist.

Table 2. Quarterly rate of return matrix (generated by experts for investor D1) – in percentage (Source: prepared by the author)

Scenarios\stocks	A1	A2	A3	A4	A5	A6
S1	2	7	–1	0	5	3
S2	–1	5	0	3	8	1
S3	4	0	3	5	2	7
S4	–4	–6	6	6	–5	–6
S5	6	3	0	–5	–1	4

– Step 4: The set of the most probable scenarios is determined (Eqs (6)–(10) and Table 3): $r_\beta = 0.8(8 - 1.4453) + 1.4453 = 6.689\%$, $d_{min} = 8$, $d_{max} = 16$, $d_\beta = 0.8(16 - 8) + 8 = 14.4$. Hence, $K = \{S1, S2, S3\}$ since S1, S2 and S3 has at least one rate of return not lower than r_β and for S3 index d_i is not lower than d_β.

Table 3. “Dominance cases” (Source: prepared by the author)

Scenarios\stocks	A1	A2	A3	A4	A5	A6	d_i
S1	2	4	0	1	3	2	12
S2	1	3	1	2	4	1	12
S3	3	1	3	3	2	4	16
S4	0	0	4	4	0	0	8
S5	4	2	1	0	1	3	11

– Step 5: The standard deviation for each stock is computed: $\sigma_1 = 3.975$ pp, $\sigma_2 = 5.070$ pp, $\sigma_3 = 2.881$ pp, $\sigma_4 = 4.438$ pp, $\sigma_5 = 5.070$ pp, $\sigma_6 = 4.868$ pp. The mixed portfolio optimization problem is solved (Eqs (11)–(16)). The optimal solution is: $g_1 = 2.28$ pp, $g_2 = 0.00$ pp, $g_3 = 4.02$ pp, $x_1 = 0$, $x_2 = 0$, $x_3 = 0.057$, $x_4 = 0$, $x_5 = 0.82$, $x_6 = 0.122$) and it guarantees a true rate of return belonging to [2.667%, 6.689%] on condition that one of the scenarios from set K occurs (S1, S2 or S3). Note that this interval contains values much higher than y^*.

SPO-AI (D2):

– Step 1: Investor D2 generates the RR-matrix (Table 4). In his opinion only 4 scenarios are possible. Note that rates of return estimated by D2 are significantly higher than rates of return estimated by experts for the portfolio optimization problem solved by D1 since the first ones (D2) are defined in a subjective way and the second ones (D1) were set objectively. Thus, we are not allowed to apply the same procedure for D2 because if we use SPO-PI for D2 the DM’s optimistic nature will be doubly taken into consideration (in the subjective matrix and by applying parameter β to select K, r_β and a suitable level of rate of return dispersion), which will lead to overestimated values.

Table 4. Quarterly rate of return matrix (generated by investor D2) – in percentage (Source: prepared by the author)

Scenarios\stocks	A1	A2	A3	A4	A5	A6
S1	4	9	2	2	6	4
S2	2	6	2	1	7	2
S3	6	2	5	5	2	8
S4	–1	–5	7	8	–1	–2

– Step 2: $y^* = 3.77\%$.

– Step 3: $M^* = 9\%$.

– Step 4: Investor D2 states that $K = \{S1, S3\}$.

– Step 5: Investor D2 defines $r^* = 7.2\%$ (note that 7.2% belongs to [3.77%, 9%]) and $s = 4.5$ pp (4.5 pp belongs to $[\sigma_{min}, \sigma_{max}]$ = [2.45 pp, 6.06 pp]). The mixed portfolio optimization problem is solved. The optimal portfolio structure is: $x_1 = 0$, $x_2 = 0.133$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0$, $x_6 = 0.867$ and $g_1 = 2.53$ pp, $g_3 = 0.00$ pp, which signifies that the true rate of return belongs to [4.66%, 7.2%] on condition that one of the scenarios from set K occurs (S1 or S3).

As we can observe, final portfolios are different for particular decision makers. Intervals of true rates of return may also differ significantly even if both investors are optimists. It is normal and justifiable since D1 and D2 are characterized by different preferences, commitment and predictions.

Conclusions

In the paper we propose a procedure supporting stock portfolio optimization when the probability distribution concerning future rates of return is not known (complete uncertainty). The decision rule takes into consideration the decision maker’s nature (pessimists, optimists, moderate decision makers) and commitment (passive and active decision makers). It is worth emphasizing that the research does not focus on how rates of return should be estimated (it is assumed that this step is performed by experts or individual investors), but it concentrates on decision making on the basis of a given rate of return matrix (estimated objectively or subjectively).

We conclude that even if two investors intend to construct a portfolio based on the same set of potential stocks and even if they have a similar nature, the final structure of their portfolios may be significantly different, depending on the degree of their commitment and knowledge about the future (Graf, Six 2014). We also demonstrate that:

a) when the RR (rate of return) matrix is estimated objectively (by experts), parameter β may be used to select the most probable scenarios, determine the desired level of revenue and calculate the upper bound of dispersion;
b) when the RR matrix is estimated subjectively (by the investor), parameter β cannot be used for the above purposes since the information about the DM's nature has been already included in the subjective matrix.

The new decision rule has at least four advantages. Firstly, it recommends different portfolios depending on the DM's level of optimism, commitment and predictions. Secondly, it takes into account the dispersion of all estimated rates of return (not only those related to scenarios belonging to K) connected with particular stocks. Thirdly, it does not treat nature as a conscious opponent who is altering strategies depending on the outcomes (the set of the most probable scenarios is selected before solving the optimization problem). The status of a given state of nature does not vary depending on the stock, but is fixed for all securities. Such a way of reasoning may be a good answer to the critical analysis of classical decision rules carried out by (Officer, Anderson 1968) who claim that the nature is not a conscious opponent and it cannot change the "strategy" (i.e. the scenario) to minimize or maximize, for each choice of the decision maker, his or her outcome. Fourthly, the reduction of the original set of potential scenarios to K (the set of the most probable states) allows the DM to obtain a higher minimal guaranteed income than y^* (the minimal guaranteed income according to the Wald's rule).

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