13th International Conference
MODERN BUILDING MATERIALS, STRUCTURES AND TECHNIQUES

16–17 May 2019, Vilnius, Lithuania
Vilnius Gediminas Technical University

eISSN 2029-9915
eISBN 978-609-476-197-3

https://doi.org/10.3846/mbmst.2019.095

# Non-linear analysis of slab-column connections with openings

Ludmila Kormosova[1], Tomas Augustin[2], Jaroslav Halvonik[3]

*Faculty of Civil Engineering, Slovak University of Technology, Bratislava, Slovakia*

*E-mail: [1]ludmila.kormosova@stuba.sk (corresponding author)*

**Abstract.** This paper is focused on the assessment of punching capacity of the slab-column connections without transverse reinforcement with openings located close to a column. Non-linear analysis and design equations from the relevant codes of practise are used for the prediction of the punching resistance of flat slab specimens supported by the rectangular columns with different ratio *h*/*d* of the column's cross-section. The non-linear models were calibrated using experimental results from the laboratory tests. The main goal of the study is an investigation of the effect of the openings on the punching capacity and as well as testing accuracy of the design equations for the prediction of the punching resistance. Several methods for the reduction of the control perimeter length accounting for presence of the openings were tested in order to find the most general method for the estimation of the punching capacity. However, standard methods introduced in the relevant codes of practise provide very inconsistent results concerning of the model's safety for different position of the openings as well as for different cross-section of the columns. Therefore modification of the methods was proposed and verified using obtained results.

**Keywords:** punching, openings, non-linear analysis, calibration.

## Introduction

Punching is one of the most dangerous form of structural failure in reinforced concrete slabs due to its brittleness. Failure at one local support may lead to the overloading of neighbouring areas and then may spread over the whole structure, thereby resulting in a progressive collapse. The reason of the punching is a concentration of shear stresses at the vicinity of local support like is a column, edge or corner of a wall. The concentration of shear stresses can be also increased by the shape of a supporting member, e.g. columns with ratio $h/b > 3d$, where $d$ is an effective depth of a slab and $h$ and $b$ are dimensions of the column cross-section, or by the presence of the openings placed in the vicinity of a column.

The design equations for prediction of the punching capacity introduced in the modern codes of practice were calibrated using the test results obtained from the experiments carried out on the fragments of flat slabs without openings. Tests on slab specimens with the openings are rare. Therefore, models that take into account an influence of the openings on the punching capacity have limited validity and their justification is further needed.

Some experts have conducted research on punching behaviour of flat slabs with openings. In the sixties, there were (Moe, 1961) and (Hognestad, Elstner, & Hanson, 1964). The more recent research was carried out by (El-Salakawy, Polak & Soliman, 1999) and (Teng, Cheong, & Kuang, 2004). Teng tested 20 slabs with a thickness of 150 mm without transverse reinforcement. Square columns with different dimensions supported slab specimens with different location of the openings. The most recent research represents works of (Borges, Melo, & Gomes, 2013) and (Elshafiey, Hussein, & Abdel-Aziz, 2012). Borges tested 13 RC flat slabs with a thickness of 200 mm. Three specimens were reference slabs without openings, three slabs had one opening and seven slabs with two symmetrically placed openings. Six slabs were also reinforced by transverse reinforcement. EL-Shafiey tested seven slab specimens with a thickness of 150 mm, one reference slab, three slabs with square openings with different dimension located at the face of the column and three slabs with openings at the column corner. Eight slabs with symmetrically placed openings tested (Augustin, Fillo, Halvonik, & Marčiš, 2018). The slabs had thickness of 250 mm, two slabs were reference without openings and six slabs were tested with three different positions of the openings.

The most of the tests were carried out on the slab specimens with non-symmetrical arrangement of the openings, which makes more difficult to evaluate separately an influence of the openings and an influence of non axis-symetric conditions on the punching capacity. Therefore, our non-linear analyses were carried out on the slab specimens with the symmetrically placed openings.

### Calibration of the non-linear models

The fracture-mechanics parameters of the numerical model are based on work (Kadlec & Cervenka, 2015) and (Augustin et al., 2018). For calibration of the numerical model, a specimen without openings was used. After successful calibration, these parameters were used for specimens with the openings.

The five finite elements on the thickness of the slab were chosen, because no changes in the trend (5 – 10 elements) of the load-deflection curve or the ultimate resistance were noticed (Kadlec & Cervenka, 2015) and (Augustin et al., 2018). Also, the combination of tetra and brick elements is not recommended, leading to the underestimation of the ultimate resistance. For this case the extrusion of the brick elements seems to be reasonable.

For the fixed crack model coefficient (where 0 – fully rotated crack, 1 – fully fixed crack) using "0", lower value of the ultimate resistance and unrealistic shear crack inclination is obtained (Figure 1). Therefore because of a good match with the experiment, fixed crack was used in the analysis.

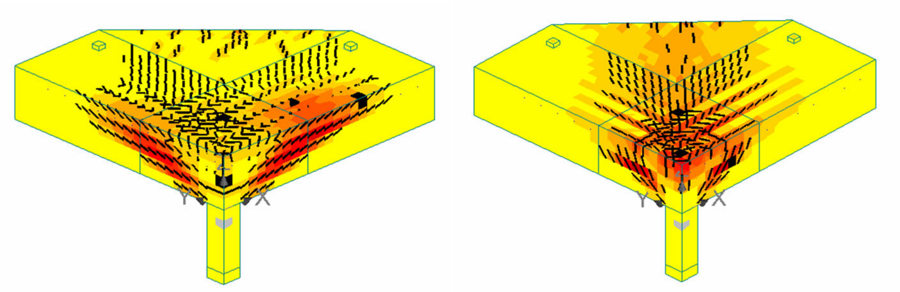

Figure 1. Fully rotated crack (left), fully fixed crack (right)

Atena (V. Cervenka, Jendele, & J. Cervenka, 2018), MC1990 (*fib* MC1990, 1993) and MC 2010 (*fib* MC2010, 2012) have a small difference in concrete tensile strength (10%). However, the difference in fracture energy is significant. This difference is justified in (*fib* Bulletin 70, 2013) and refers to a better match with experiments (Figure 2). Based on these experiments, the MC1990 significantly underestimates the fracture energy for all concrete strengths up to 110 MPa. However, Atena FEM software is based on MC1990 and using the fracture energy leads to good match with experiments (Figure 2, difference only 12%). The fracture energy model introduced in MC2010 is not recommended to be used in Atena because it leads to the ultimate resistance overestimation.

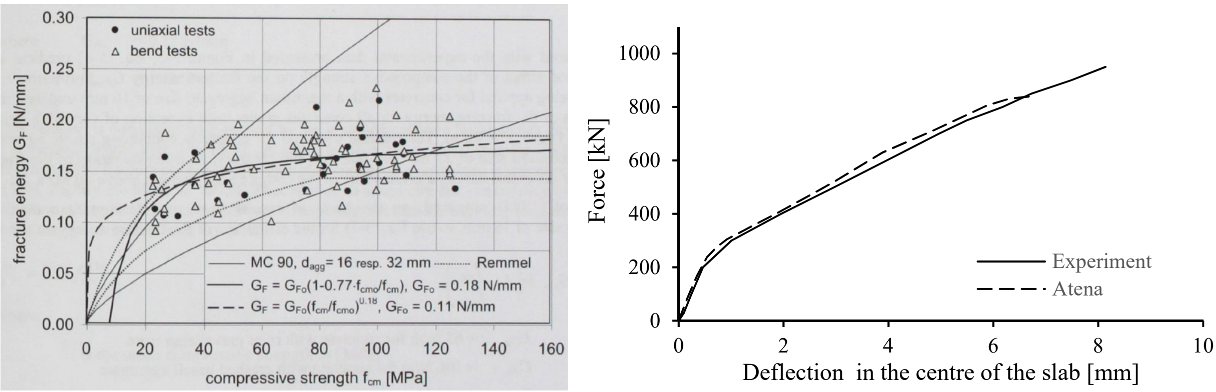


Figure 2. Fracture energy-compressive strength relation (left) (Fib Bulletin 70, 2013),
load-deflection relation S02 (Augustin et al., 2018)

### Non-linear analysis

*Description of the specimens*

The non-linear analyses were carried out on the fragments of flat slabs with a thickness of 200 mm and an effective depth of 159 mm. All specimens are supported by rectangular column with 5 different dimensions of the cross-section; 160×150 mm; 320×150 mm; 480×150 mm; 640×150 mm and 950×150 mm. The longest dimension represents value of 6*d* and the smallest 1*d*. Cylinder concrete strength $f_{cm}$ was assumed 30 MPa and reinforcing steel with yield strength of $f_{ym}$ = 580 MPa. Reinforcement ratio ρ was assumed 1.26% (bars with diameter of ϕ16 mm by 100 mm). No additional reinforcement was used next to the openings. Analysed specimens are without transverse reinforcement. The maximum aggregate size $d_{g,\max}$ was assumed 16 mm.

Three different slab specimens were analysed (Figure 3) for each type of a column. The first one (Figure 3a) is the slab SL0-x without openings. The second one is the slab SL1-x with two openings 150×240 mm located at the shorter side of the column (Figure 3b) and the third one is the slab SL2-x with two openings located in the middle of the longer side of the column (Figure 3c). Eight concentrated forces for the whole specimen were used for loading, two forces per one quarter. The forces were introduced in steps until the failure was reached.

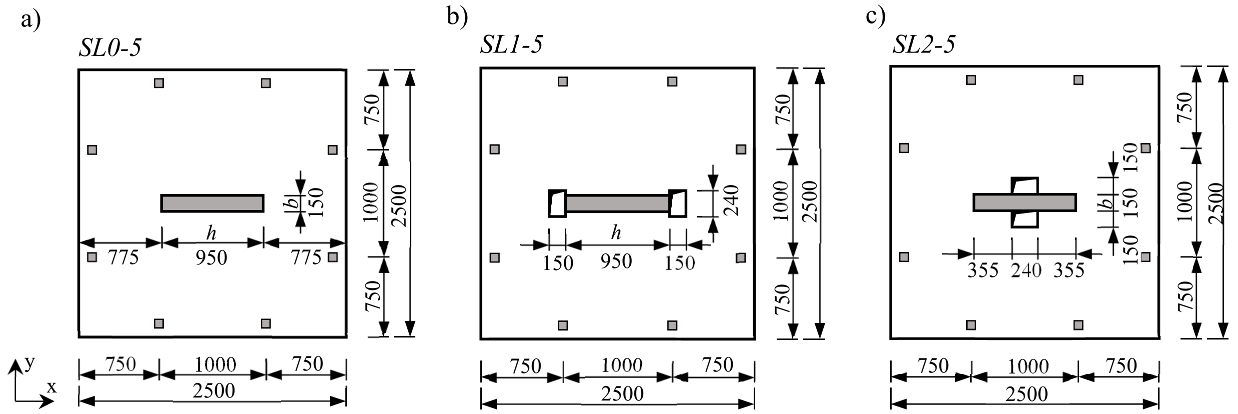


Figure 3. Analysed slab specimens: a) slab without openings; b) openings at column edges; c) openings in the middle of the column

*Obtained results*

Non-linear models of the slab fragments were created and analysed based on the recommendations introduced in previous chapter. Used procedures for non-linear modelling and analyses were calibrated using tests results obtained from the experiments that were carried out at the laboratory in Bratislava. Calculated maximum shear forces and maximum stresses in bending reinforcement are introduced in the Table 1. All analysed specimens failed by punching. Yielding of main reinforcement has been reached only in several bars (no bending failure was observed).

Table 1. Failure loads $V_{R,NLA}$ and stresses in main reinforcement obtained from non-linear analyses

| Slab | 150×160 mm (1) | | 150×320 mm (2) | | 150×480 mm (3) | | 150×640 mm (4) | | 150×950 mm (5) | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $V_{R,NLA}$ | $\sigma_{s,max}$ | $V_{R,NLA}$ | $\sigma_{s,max}$ | $V_{R,NLA}$ | $\sigma_{s,max}$ | $V_{R,NLA}$ | $\sigma_{s,max}$ | $V_{R,NLA}$ | $\sigma_{s,max}$ |
| | [kN] | [MPa] | [ kN ] | [ MPa ] | [ kN ] | [MPa] | [kN] | [ MPa ] | [ kN ] | [ MPa ] |
| SL0-x | 561.3 | 344 | 641.3 | 426 | 721.3 | 529 | 781.3 | 550 | 901.3 | 580 |
| SL1-x | 351.3 | 329 | 461.3 | 353 | 541.3 | 421 | 631.3 | 527 | 761.3 | 547 |
| SL2-x | 341.3 | 409 | 511.3 | 474 | 631.3 | 550 | 681.3 | 580 | 831.3 | 580 |

## Design equations

Two design models were used for prediction of the punching capacity. Current EC2 (2004) model and the model based on the critical shear crack theory (CSCT) with design equations expressed in closed form (CF CSCT model). The first model is empirical and the second physical.

*EC2 (2004) model*

The EC2 (2004) model is based on a formula proposed by (Zsutty, 1968) in 1968. Zsutty statistically evaluated the relation between the shear strength and the amount of the main reinforcement, expressed by the reinforcement ratio ρ and the concrete compressive strength on cylinders $f_c$ [MPa]. After some refinements, the final Eq. (1) was published in Model Code 1990.

$$V_{Rd,c} = \frac{C_{Rk,c}}{\gamma_C} k\left(100 * \rho f_{ck}\right)^{1/3} u_1 d \ , \tag{1}$$

where: $C_{Rk,c}$ – is a empirical factor $C_{Rk,c}$ = 0.18 MPa; $f_{ck}$ – characteristic concrete compressive cylinder strength; ρ – reinforcement ratio, $\rho = (\rho_x \rho_y)^{0.5}$; $d$ – effective depth, the average value of the effective depths in two orthogonal directions $d_x$ and $d_y$; $k$ – a factor which takes into account the size effect $k = 1+(200\text{ [mm]}/d)^{0.5} \leq 2.0$; $\gamma_C$ – partial safety factor; $u_1$ – is the length of basic control perimeter at distance $2d$ from the face of a column.

*CF CSCT model*

The model is based on the critical shear crack theory (Muttoni & Ruiz, 2008). Muttoni and Ruiz (2016) expressed punching resistance in closed-form (2) without direct calculation of the slab rotation ψ in order to simplify the design of the flat slabs for punching. The design expression looks very similar to the EC2 (2004) formula now. The CF CSCT model was evaluated using power-law failure criterion instead of hyperbolic criterion used in the MC2010 model.

The model was chosen for the evaluation because used failure criterion is very similar with the criterion introduced by Muttoni and Ruiz (2008) from safety point of view. Failure criterion in the Model Code 2010 was updated in 5% fractile and therefore provides too conservative results.

$$V_{Rd,c} = \frac{k_b}{\gamma_C}\left(100 * \rho f_{ck} \frac{d_{dg}}{a_v}\right)^{1/3} b_0 d_v \leq 0.6 * \frac{\sqrt{f_{ck}}}{\gamma_C} b_0 d_v ; \tag{2}$$

$$k_b = \sqrt{8 * \mu \frac{d}{b_0}} , \tag{3}$$

where: $d_v$ – an effective depth for shear, usually $d_v = d$; $d_{dg}$ – is a coefficient that takes into account the type of concrete and its aggregate properties, i.e., 32 [mm] for concrete of a normal weight; $a_v$ – shear span, the geometric average of the shear spans in both orthogonal directions and not less than $2.5d$; $b_0$ – a control perimeter at distance $0.5d_v$ from the face of a column; μ – a parameter accounting for the shear force and bending moment in the region of the shear, for an internal column without an unbalanced moment μ = 8; in the case with the openings μ should be multiplied by $k_o \leq 1.0$; $k_o$ – a factor accounting for presence of the openings, ratio of the reduced control perimeter length due to openings and perimeter length evaluated for the same slab ignoring the openings.

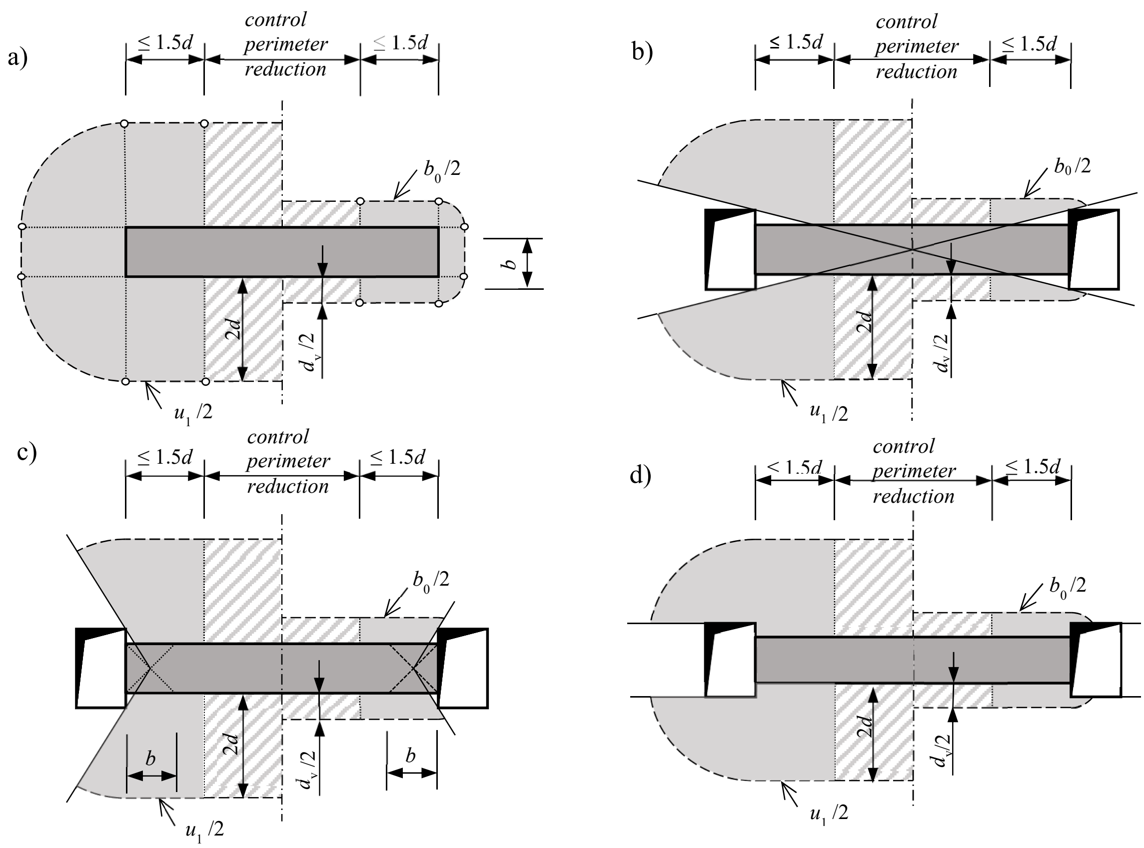


Figure 4. Reduction of the control perimeters due to a) large loaded area; b, c, d) due to opennings b) standard method; c) Teng's proposal; d) Augustin's proposal, on the left the EC2 (2004) model, on the right the CF CSCT model

*Models accounting for the openings and large loaded area*

An influence of the openings on the punching capacity can be taken into account by the reduction of the control perimeter length according to Figure 4b (right) in the case of CSCT model and Figure 4b (left) for the EC2 (2004) model. The reduced control perimeter length is obtained by subtraction of the part that is enclosed by lines radiating from the centroid of the loaded area (standard method). In the case of the elongated columns, two ways for the length reduction were used. The difference is in the origin of the radiating lines. The first is the standard method where centroid of the loaded area is the origin, while the second is a matter of discussion, e.g. (Teng et al., 2004) proposes to assume origin in the centroid of the end portion of the column, see Figure 4c. The last method represents proposal (Augustin et al., 2018) for reduction of the control perimeter length by the application of lines parallel with edges of the openings instead of lines radiating from the centre of the loaded area see Figure 4d. An influence of the large loaded area that is represented by a column cross-section can be accounted for by the reduction of the control perimeter according to Figure 4a.

### Evaluation of the results and discussion

Punching capacities $V_{model}$ obtained from the design equations were calculated with partial safety factor $\gamma_C$ equal unity, where applicable and instead of the characteristic value of concrete strength $f_{ck}$ the mean value $f_{cm}$ has been used. Punching capacities of the slab fragments were assessed using 6 differently reduced control perimeter lengths. The first two were evaluated with lines radiating from the centroid of the loaded area (standard method), one with and second without additional reduction according to Figure 4a in the case of elongated columns with ratio $h/d > 3.0$. Another two with lines radiating according to Teng's proposal and finally, last two by lines parallel with edges of the openings, Augustin's proposal.

The model's safety was investigated using ratio $V_{R,NLA}/V_{model}$. Value of the ratio > 1.0, the results are on the safe side and vice versa < 1.0 the model is unsafe.

Obtained results are shown in Figures 5a,b in the case of the EC2 model and Figures 5c,d in the case of the CF CSCT model. Firstly, we want to point out on the results obtained for the slabs without openings. In the case of the EC2 (2004) model the ratio $V_{R,NLA}/V_{model}$ is nearly constant for any dimension of a column with values slightly above 1.0, if full length of the control perimeter is assumed. In the case of the CF CSCT model the maximum value is 1.20 for ratio $h/d$ =1.0 and for $h/d \geq 2.0$ is again nearly constant with a value of 1.02. Opposite, if the reduction of the control perimeter is applied according to Figure 4a, the ratio $V_{R,NLA}/V_{model}$ increases with increasing of the $h/d$ ratio. The similar results were observed also for specimens with the openings. This observation indicates that the concentration of shear stresses at edges of the elongated columns does not significantly influence punching capacity if $h/d \leq 6.0$ and $h/b \leq 6.3$.

If we take look on the specimens with the openings placed at the column edges, we can find that standard method with lines radiating from the centroid of a column provides inconsistent and many times conservative solution for both models. Inconsistent, because model's safety expressed by the ratio $V_{R,NLA}/V_{model}$ is not constant for different column's dimensions. In the case of the EC2 (2004) model the ratio $V_{R,NLA}/V_{model}$ decreases with increasing of $h/d$, from the value of 1.72 for $h/d$ = 1.0 to 1.06 for $h/d$ = 6.0. If reduction of the control perimeter length is assumed according to Figure 4a, the results are different, because for $h/d > 3.0$ the ratio $V_{R,NLA}/V_{model}$ increases, with the maximum value of 1.45 for $h/d$ = 6.0. In the case of the CF CSCT model, the results are similar with the EC2 (2004) model, the ratio $V_{R,NLA}/V_{model}$ decreases with increasing of $h/d$, from the value of 2.06 for $h/d$ = 1.0 to 1.09 for $h/d$ = 6.0. If reduction of the control perimeter length according to Figure 4a is assumed, the $V_{R,NLA}/V_{model}$ ratio increases again for $h/d > 3.0$ up to the maximum value of 1.95 for $h/d$ = 6.0.

Teng's proposal provides very inconsistent results, similar with the standard method, and with many times higher model's safety, e.g. for $h/d$ = 6.0 the $V_{R,NLA}/V_{model}$ ratio reached a value of 2.25 for both design models, if the reduction of control perimeter according to Figure 4a is assumed.

The most consistent results provides Augustin's proposal with lines parallel with edges of the openings, especially in connection with the CF CSCT model. The relation between the $V_{R,NLA}/V_{model}$ ratio and $h/d$ ratio is similar with the slabs without openings and the level of safety is slightly above values obtained for these slabs. In the case of the EC2 (2004) model it can be found similarity with the results obtained on the slabs without openings, however $V_{R,NLA}/V_{model}$ ratio is below values obtained for these slabs. Particularly, in the case of the column with $h/d$ = 1.0 is $V_{R,NLA}/V_{model}$ ratio deep below one, only 0.78.

High inconsistency of the standard method and as well as Teng's proposal was found out also in the case of slabs with openings placed in the middle of the column's longer side. In the case of the EC2 (2004) model the standard method provides very conservative solutions with the maximum value of the $V_{R,NLA}/V_{model}$ ratio 2.38 for $h/d$ = 3. In the case of the CF CSCT model the level of safety changes with $h/d$ ratio, the maximum value is 2.15 for $h/d$ = 2 and the minimum 1.48 for $h/d$ = 6. The reduction of the control perimeter according to Figure 4a did not influence results much.

In the case of Teng's proposal and the EC2 (2004) model the results are very different in comparison with standard method. The $V_{R,NLA}/V_{model}$ ratio decreases from value of 1.75 for $h/d$ = 1 to value of 0.93 for $h/d$ = 4 and then increases up to the value of 1.03 for $h/d$ = 6. In the case of the reduced control perimeter (Figure 4a) the relation is similar, however the results are a bit better, for $h/d$ = 4 the value is 1.00 and for $h/d$ = 6 is 1.21. In the case of case of the CF CSCT model the relation $V_{R,NLA}/V_{model}$ and $h/d$ ratio is different from the EC2 (2004) model. The maximum value of $V_{R,NLA}/V_{model}$ ratio is 2.07 for $h/d$ = 1, then decreases to the value of 1.28 for $h/d$ = 3 and then increases up to the value of 1.45 for $h/d$ = 6. The reduction of the control perimeter according to Figure 4a did not influence results much too.

The best results provides again Augustin's proposal, particularly in the case of the CF CSCT model, where the value of $V_{R,NLA}/V_{model}$ ratio is fluctuating between 1.28 for $h/d = 1$ and 1.14 for $h/d = 6$ and the course is very similar with the results obtained for the slabs without openings. In the case of the EC2 (2004) model the results are very similar with slabs without openings for $h/d \geq 2$. Only for $h/d = 1$ the $V_{R,NLA}/V_{model}$ ratio fell below one with a value of 0.76.

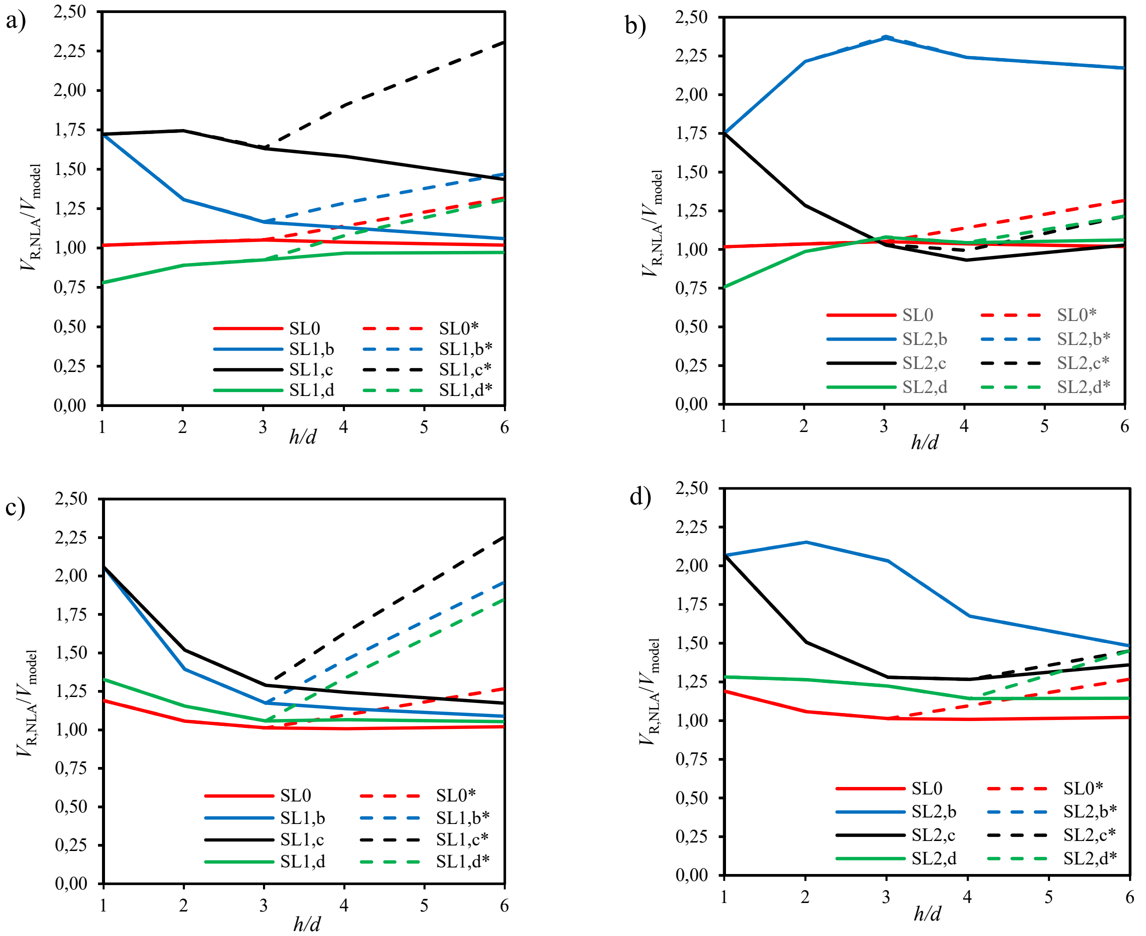


Figure 5. Relation between $V_{R,NLA}/V_{model}$ and $h/d$ ratio: a) the EC2 (2004) model with openings placed at column edges; b) the EC2 (2004) model with openings placed in the middle of column; c) the CF CSCT model with openings placed at column edges; d) the CF CSCT model with openings placed in the middle of column; * reduction of the control perimeters due to large loaded area

## Conclusions

The study is focused on the punching capacity of the slab-column connections with two symmetrically placed openings at two different positions supported by the column with dimension $h$ varying from $1 \leq h/d \leq 6$. The parametric study works with results obtained from non-linear analyses (NLA) of the slab fragments that were carried out using software Atena. Results of the real experiments were used for calibration of the non-linear models. Obtained punching resistances from (NLA) were used for safety verification of the design equations for prediction of the punching capacity. Based on the results of performed parametric study following conclusion can be drawn.

1. In the case of the slabs without openings it was observed that the concentration of shear stresses at edges of the elongated columns has a small influence on the punching capacity assessed by two design models for the tested range of the $h/d$ ratio. The level of safety expressed by the $V_{R,NLA}/V_{model}$ ratio was even for the full range of $h/d$, with values ranging from 1.02 to 1.05 in the case of the EC2 (2004) model and from 1.19 to 1.02 for CF CSCT model.
2. Three different approaches for the reduction of the control perimeter length due to the openings were tested. Standard method as well as Teng's proposal provide very inconsistent results for both applied design models, e.g. standard method with the EC2 (2004) model and $h/d = 3.0$ in the case of the openings at column edges gives $V_{R,NLA}/V_{model} = 2.38$, while for openings placed in the middle of the column a value of 1.15. Similar results were obtained also with the CF CSCT model.

3. Much better results were obtained with Augustin's proposal. In the case of the CF CSCT model the relation between $V_{R,NLA}/V_{model}$ and $h/d$ ratio is fairly even independently of the opening position and it is always above curve obtained for the opening free slabs. In the case of the EC2 (2004) model the course of the relation $V_{R,NLA}/V_{model}$ and $h/d$ ratio is fairly even too, but in the case of the openings placed at column edges the model's safety is permanently below values obtained for slab without openings. Therefore some corrections of the proposal are needed.
4. Further reduction of the control perimeter length due to the column dimension for $h > 3d$, see Figure 4a, significantly increases model's safety, particularly for slabs with openings located next to the column edges, however too much over the target value. Therefore our proposal is to increase an effective length of the column perimeter measured from the column's edge from the value of $1.5d$ to at least $2.0d$.
5. Current methods that allow account for the openings close to the column for the punching capacity evaluation provide very inconsistent results from the safety level point of view for different position of the openings as well as for different dimensions of the columns. The best chance to develop one consistent method has Augustin's proposal with subtracting control perimeter length that is enclosed by lines parallel with the edges of the openings.

**Acknowledgements**

This work was supported by the Slovak Research and Development Agency under the contract No. APVV-15-0658 and Scientific Grant Agency of the Ministry of Education, science, research and sport of the Slovak Republic and the Slovak Academy of Sciences No 1/0240/19.

**References**

Augustin, T., Fillo, Ľ., Halvonik, J., & Marčiš, M. (2018). Punching resistance of flat slabs with openings – Experimental investigation. *Solid State Phenomena: 24th Concrete Days 2017*, *272*, 41-46. Litomyšl, Czech Republic. https://doi.org/10.4028/www.scientific.net/SSP.272.41

Borges, L. L. J., Melo, G. S., & Gomes, R. B. (2013). Punching shear of reinforced concrete flat plates with openings. *ACI Structural Journal*, *110*(4), 547-556. https://doi.org/10.14359/51685740

Cervenka, V., Jendele, L., & Cervenka, J. (2018) *ATENA Program Documentation, Part 1, Theory*. Prague: s.n.

El-Salakawy, E. F., Polak, M. A., & Soliman, M. H. (1999). Reinforced concrete slab-column edge connections with openings. *ACI Structural Journal*, *96*(1), 79-87. https://doi.org/10.14359/598

Elshafiey, T. M., Hussein, M., & Abdel-Aziz, M. (2012). Behaviour of flat slabs with openings adjacent to column. In *Structural Faults & Repair-2012 14th International Conference* (vol. 2).

European Commitee for Standardization. (2004). *Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*.

European Commitee for Standardization. (2018). *Background documents to prEN 1992-1-1* (CEN TC250/SC2), pp. 53-78.

Fédération Internationale du Béton (fib). (2012). *Bulletin n. 65/66 Model Code 2010. Final draft* (vol. 1, 2. fib). Lausanne.

Fédération Internationale du Béton (fib). (2013). *Bulletin 70 Code-type Models for Structural Behaviour of Concrete. Background of MC2010*. Lausanne.

Hognestad, E., Elstner, R. C., & Hanson, J. A. (1964). Shear strength of reinforced structural lightweight aggregate concrete slabs. *ACI Journal Proceedings*, *61*(6), 643-656. https://doi.org/10.14359/7800

Kadlec, L., & Cervenka, V. (2015, 18-20 May). Uncertainty of numnerical models for punching. In *Concrete – Innovation and Design, fib Symposium.* Copenhagen.

Moe, J. (1961). Shearing strength of reinforced concrete slabs and footings under concentrate loads. *Development Department Bulletin* D47. Portland Cement Association.

Muttoni, A., & Ruiz, M. F. (2008). Shear strength of members without transverse reinforcement as function of critical shear crack width. *ACI Structural Journal*, *105*(2), 163-172. https://doi.org/10.14359/19731

Muttoni, A., & Ruiz, M. F. (2016). *Proposal for punching shear provisions based on fib-MC2010*. Copenhagen: s.n.

Muttoni, A., Ruiz, M. F., & Simões, J. T. (2018). The theoretical principles of the critical shear crack theory for punching shear failures and derivation of consistent closed-form design expressions. *Structural Concrete*, *19*(1), 174-190. https://doi.org/10.1002/suco.201700088

Teng, S., Cheong, H. K., & Kuang, K. L. (2004). Punching shear strength of slabs with openings and supported on rectangular columns. *ACI Structural Journal*, *101*(5), 678-687. https://doi.org/10.14359/13390

Zsutty, T. (1968). Beam shear strength prediction by analysis of existing data. *ACI Journal Proceedings*, *65*(11), 943-951. https://doi.org/10.14359/7526