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Abstracts

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THE ROLE OF INACCURATE ASSUMPTIONS FOR CHURN PREDICTION IN TELECOMMUNICATIONS

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This topic is dedicated to the accuracy problem in machine learning due to some assumptions. More specifically, a special case of churn prediction in telecommunications is investigated. The source of the mentioned problem is the shift in definition of a churner. A churner is defined as the user who has stopped using some specific services, in the considered case it is telecommunication services from specific operator. The most common exact definition of the churner in telecommunications is the client that has not done any revenue generating actions for 3 months. However, it is common among other authors [1] to change the original definition by reducing the observation period for churned identification – this is motivated by the fact that for the most of churners the inactivity for one month is followed by 3 months inactivity. In many datasets the definition of the churner is not provided at all, thus it makes questionable the relevancy of the actual problem being solved.

In this research we investigate the consequences of the changes of churn definition, a set of standard machine learning methods is applied to the dataset labelled according to different churn definitions. We show that inaccuracies of the achieved prediction are at least of the same order as the differences of performance of different machine learning techniques in other authors' researches [1], thus questioning the scientific value of such comparison without addressing the inaccuracy due to shifts in definitions.

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TECHNIQUES TO IMPROVE PRECISION OF NEURAL NETWORK-BASED BIOSENSORS UNDER EFFECT OF NOISE

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Biosensors are devices for the detection and analysis of chemical compounds based on biochemical processes [1]. One of the most widespread biosensor types are enzyme-based amperometric biosensors [2], where the sample components are enzymatically converted into products, which produce an electric current that is measured. The biosensor is then calibrated using known samples to establish a dependence between component concentrations and electric current. Unknown samples can then be analyzed by measuring their current signal and, by using the inverse of the dependence, determining their composition i.e., solving an inverse biosensor problem. For a single substrate, this can be achieved very easily, but for multiple substrates its solution is complicated by the ill-posedness of the problem [3], especially when the biosensor response is under the effect of noise. This can negatively impact the precision of the device [4].

One promising method to alleviate the ill-posedness of such problems is to use neural networks, which have successfully been used to solve inverse problems in heat conduction, such as determining the initial condition of the heat equation from the temperature profile [5]. Although neural networks have been applied to solve the inverse biosensor problem [6], we were unable to find any reports dealing with biosensor precision under the effects of noise in this case.

Since biosensors are used in various performance-critical applications, such as environmental monitoring and protection, food safety, medicine and so on [2], an investigation of effects of noise is necessary, in order to find ways to mitigate deterioration of biosensor performance. In this talk, we present our findings on noise effects and techniques currently in development to improve biosensor precision, such as neural network training set extension.

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ON THE FUNCTIONAL INDEPENDENCE OF THE RIEMANN ZETA-FUNCTION

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Hölder proved [1] that the Euler gamma-function does not satisfy any algebraic differential equation. Hilbert [2] conjectured that the same property has the Riemann zeta-function

$$\zeta(s)=\sum_{m=1}^{\infty}\frac{1}{m^s}, \quad s=\sigma+it,\ \sigma>1.$$

The Hilbert conjecture was proved by Ostrowski [3]. Voronin obtained [4] the functional independence of the function $\zeta(s)$. He proved that if $F_0, F_1, , \dots, F_m : \mathbb{C}^N \to \mathbb{C}$ are continuous functions, and the equality

$$\sum_{l=0}^{m} s^l F_l\left(\zeta(s), \zeta'(s), \dots, \zeta^{(N-1)}(s)\right)=0$$

is satisfied identically for $s \in \mathbb{C}$, then $F_l \equiv 0$ for all $l = 0, 1, \dots, m$.

In the report, we discuss the joint functional independence of the Riemann zeta-function. Let $N = N_1 + \dots + N_r$.

Theorem 1. *Suppose that $F_0, F_1, \dots, F_m : \mathbb{C}^N \to \mathbb{C}$ are continuous functions, and the equality*

$$\sum_{l=0}^{m}(s_1\cdots s_r)^l F_l\left(\zeta(s_1), \zeta'(s_1), \dots, \zeta^{(N_1-1)}(s_1), \dots, \zeta(s_r), \zeta'(s_r), \dots, \zeta^{(N_r-1)}(s_r)\right)=0$$

is satisfied identically for $s_1, \dots, s_r$. Then $F_l \equiv 0$ for all $l = 0, 1, \dots, m$.

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ON A NONLINEAR BOUNDARY VALUE PROBLEM ARISING IN HEAT TRANSFER

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The nonlinear boundary value problem

$$x'' + \frac{1-\alpha}{t}\, x' + \lambda\, e^{x} = 0, \quad x(\rho) = 0, \quad x(1) = 0 \tag{1}$$

is considered, where $\alpha \in \mathbb{R}$, $\lambda > 0$, and $0 < \rho < 1$. Problem (1) arises in determination of a steady convective flow of a chemically reacting fluid in a tall vertical annulus of radii ρ and 1. We study the existence and multiplicity of positive solutions for (1).

We will consider the Banach space $C_{[\rho,1]}$ with the norm $\|x\| := \max\limits_{\rho \le t \le 1} \big|x(t)\big|$. Based on the Krasnosels'kiĭ-Guo compression-expansion fixed point theorem, we prove the main result.

THEOREM 2. *Let α be a real number and let ρ be a real number in the interval $(0,1)$. Then, there exists a positive number λ^* such that for every λ in the interval $(0,\lambda^*)$ the problem (1) has two positive solutions $\underline{x}$ and $\overline{x}$ such that $\|\underline{x}\| < 1 < \|\overline{x}\|$.*

We also give a bifurcation analysis for (1). The results obtained are illustrated by numerical examples.

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ON SOME INVERSE PROBLEMS THAT USE NONLOCALITY OF FRACTIONAL DERIVATIVES

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We consider two inverse problems that use nonlocality of fractional derivatives.

First problem consists in determination of a time-dependent source term of a time-fractional diffusion equation from final measurements [1]. We prove the uniqueness of solution of this problem. The proof uses power-type asymptotic expansions of Mittag-Leffler functions that are involved in formulas of Fourier coefficients of solution of direct problem. The power-type asymptotics is related to the strong memory effects of the model caused by the nonlocal time derivative. The proof is not valid in the case of the classical diffusion equation when the formulas of Fourier coefficients of the direct problem contain exponentially decaying kernels.

In the second problem the aim is to find a solution of the time-fractional diffusion equation in case initial and final data are given but only partial boundary data are specified [2]. Proof of the uniqueness uses a technique that is similar to the method applied in the first problem.

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STABILITY OF CONVECTIVE FLOWS CAUSED BY INTERNAL HEATING

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Three problems related to linear stability of convective flows in vertical channels are investigated.

Problem 1. Consider convection in a vertical fluid layer bounded by two infinite parallel planes. Steady convective flow in the vertical direction is generated by internal heat sources. The density of heat sources is a nonlinear function of the temperature [1]. Assume that there exists throughflow with constant velocity U between permeable walls of the layer in the direction perpendicular to the planes. Base flow and temperature satisfy a nonlinear boundary value problem. Linear stability of the base flow is investigated for different values of the parameters of the problem. Numerical results show that stability boundary is determined by the relative importance of internal heating (destabilizing factor) and throughflow (stabilizing factor). Depending on the Prandtl number two instability modes (hydrodynamic mode and buoyant mode) are identified.

Problem 2. Similar problem is considered for the case of a flow in a tall vertical annulus with a radial throughflow between permeable boundaries. Calculations show that the increase of the radial Reynolds number leads to the appearance of a second minimum on a marginal stability curve. It is shown that the rate of stabilization of the base flow is different for inward and outward radial flows.

Problem 3. The effect of a magnetic field on the stability of a convective flow due to internal heat sources is analyzed. We generalize the results in [2] for the case of non-homogeneous distribution of the heat sources. The problem of internal volumetric heating of a flow should be taken into account in blanket designs for thermonuclear reactors. Calculations are performed for two values of the Prandtl number representing two types of liquids that are currently considered for blanket design: (a) liquid metals and (b) Flibe (a molten salt made from the mixture of lithium fluoride and berylium fluoride)).

This research was funded by the Latvian Council of Science project No. lzp-2020/1-0076.

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VOTING-KEMIRA METHOD FOR DETERMINING CRITERIA PRIORITY AND WEIGHTS IN SOLVING MULTI-ATTRIBUTE DECISION MAKING PROBLEMS

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The KEmeny Median Indicator Ranks Accordance (KEMIRA) method [1] is one of the newest Multi-Attribute Decision Making (MADM) methods that is used for problems in which criteria are inherently divided into two or more categories. Increasing computational complexity in the face of large-scale problems as well as the inflexibility of the method in increasing the categories of criteria are among the disadvantages of this method. Preferential voting is a decision-making method based on a linear programming model with weight restrictions [2], which in combination with KEMIRA can significantly eliminate its shortcomings. This talk presents the new Voting-KEMIRA method to achieve this purpose, which is more flexible and has less computational complexity. The new hybrid method is implemented on a real–world problem and compared to the previous method.

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APPROXIMATION BY A DIRICHLET SERIES CONNECTED TO THE PERIODIC HURWITZ ZETA-FUNCTION

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Let $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0\}$ be a periodic sequence of complex numbers with minimal period $q \in \mathbb{N}$, $0 < \alpha \leqslant 1$ fixed parameter and $s = \sigma + it$. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series $\zeta(s, \alpha, \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s}$, and has analytic continuation to the whole complex plane. It is known [1] that the function $\zeta(s, \alpha; \mathfrak{a})$ with transcendental or rational α is universal in the sense that its shifts $\zeta(s + i\tau, \alpha; \mathfrak{a})$ approximate a wide class of analytic functions defined in the strip $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$.

In the report, we construct a Dirichlet series which is absolutely convergent in D and has good approximation properties. Let $\theta > 0$ be a fixed number, and $v_u(m, \alpha) = \exp\{-((m + \alpha)/u)^\theta\}$ for $u > 0$ and $m \in \mathbb{N}_0$. Then the series $\zeta_u(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m v_u(m,\alpha)}{(m+\alpha)^s}$ is absolutely convergent for $\sigma > \sigma_0$ with arbitrary finite σ_0. Let $\Omega = \prod_{m=0}^{\infty} \gamma_m$, $\gamma_m = \{s \in \mathbb{C} : |s| = 1|\}$ for all $m \in \mathbb{N}$, and m_H be a probability Haar measure on Ω. Define the random element $\zeta(s, \alpha, \omega; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m \omega(m)}{(m+\alpha)^s}$, $\omega = \{\omega(m) : m \in \mathbb{N}_0\} \in \Omega$. Denote by $\mathcal{K}$ the class of compact subsets of D with connected complements, and by $H(K)$, $K \in \mathcal{K}$, the class of continuous functions on K that are analytic in the interior of K. Then the following statement is true.

THEOREM 3. *Suppose that the number α is transcendental, and $u_T \to \infty$, $u_T \ll T^2$ as $T \to \infty$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Then the limit*

$$\lim_{T\to\infty} \frac{1}{T} \text{meas}\left\{\tau \in [0, T] : \sup_{s\in K} |\zeta_{u_T}(s + i\tau, \alpha; \mathfrak{a}) - f(s)| < \varepsilon\right\}$$
$$= m_H \left\{\omega \in \Omega : \sup_{s\in K} |\zeta(s, \alpha, \omega; \mathfrak{a}) - f(s)| < \varepsilon\right\}$$

exists and is positive for all but at most countably many $\varepsilon > 0$.

A similar result is also valid for rational α with different Ω.

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STUDY OF TWO COMBINED HEAT, MASS TRANSFER AND REACTION MODELS WITH APPLICATIONS TO BIOMASS PELLET COMBUSTION

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Extensive research has been devoted to simulating gasification or combustion processes [1],[2] in isotropic heterogeneous materials such as wood, wood briquettes, or pellets. Biomass pellets composed of wood, straw, peat, etc are a promising alternative to fossil fuels in many applications such as home heating. In order to further increase the efficiency while reducing harmful emissions, smart control methods have been proposed, e.g. using external electric and magnetic fields. If heat and mass transfer processes within the pellets are to be taken into account and the controlling device is to be of low computational power, then minimalistic mathematical models are required.

A network model has been previously proposed by the authors [3] however, the results have not been validated against more complete models. We develop an alternative model preserving the topology of the network model, consisting of nodes and channels connecting the nodes, with one-dimensional gas dynamics equations governing the gas flow between the nodes. The main object of research in the developed model is the development of a thermal gasification dynamics model, where the primary task is to simplify a real 3-dimensional model to a 1-dimensional model by discretizing pressure and flow with the help of a graph model validation and simulation. Mass conservation laws are used at the nodes to couple the gas dynamics equations on the various nodes. The resistance to gas flow between the nodes in the two models is described by different parameters – permeability coefficient for the simple network model and channel length and diameter for the alternative model; these can be adjusted to apply both methods to a certain problem. The results of the two models are compared for some representative geometries.

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ADSORPTION PROCESS AND WATER QUALITY CONTROL

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With the increase of urbanization we also have to focus on quality of life. Then, the production volumes increase. During the manufacturing process, potentially toxic elements are used and thus the environment is being polluted. When potentially toxic elements get into the water, they become able to affect the human health. There are strict requirements for water quality, and, in order to reduce the quantities of potentially toxic elements, different technologies are used. One way to reduce the pollution is adsorption. The adsorption models are widely used and described. A few years ago we also started working with the adsorption process. We started with the pore volume and surface diffusion model, presented in [1]. The mathematical model and the questions of approximation were analyzed [2]. Next, the adsorption of the potentially toxic elements on biochar was analyzed and the numerical and experimental results were compared [3]. Then we paid attention to the possibility of controlling the adsorption process, taking into account certain properties of the materials and used the models for new elements [4].

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SOME ASPECTS ON THE VALUE-DISTRIBUTION OF THE EPSTEIN ZETA-FUNCTION

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Let Q be a positive definite quadratic $n \times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}} Q \underline{x}$ for $\underline{x} \in \mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, $s = \sigma + it$, is defined, for $\sigma > \frac{n}{2}$, by the series

$$\zeta(s;Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\underline{x}])^{-s},$$

and can be continued analytically to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. The function $\zeta(s;Q)$ was introduced by P. Epstein in [1], its value-distribution was investigated by various authors. An extensive survey of the results on the function $\zeta(s;Q)$ is given in [4].

In the talk, some aspects of joint works with A. Laurinčikas on the value-distribution of the function $\zeta(s;Q)$ will be presented. More precisely, the Bohr-Jessen type limit theorems in the sense of the weak convergence of probability measures on the complex plane will be considered, and the explicit forms of the limit measures will be given. We will present the results of continuous (when t in $\zeta(\sigma+it;Q)$ can take arbitrary real values) and discrete (when t in $\zeta(\sigma+it;Q)$ runs over the set $\{kh : k \in \mathbb{N}_0\}$ with fixed $h > 0$) types. For discrete type, we will analyse two cases depending on the arithmetic nature of the number h. The above results are published in [2] and [3]. Moreover, some assumptions on joint limit theorems for a collection of Epstein's zeta-functions will be discussed.

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ASYMPTOTIC ANALYSIS OF NON-NEWTONIAN FLOWS IN THIN TUBE STRUCTURES

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Thin tube structures are finite unions of thin cylinders depending on the small parameter, ratio of the diameter of the cross section to the length of the cylinder. Flows in such domains model blood flow in a network of vessels. The asymptotic expansion of the solution of the steady Stokes and Navier-Stokes equations in these domains with no slip boundary condition was constructed in the papers [1], [2], and the book [3]. However, the blood exhibits a non-Newtonian rheology, when the viscosity depends on the strain rate. In the present talk we consider such rheology. Applying the Banach fixed point theorem we prove the existence and uniqueness of a solution and its regularity. An asymptotic approximation is constructed and justified by an error estimate. The results are published in the set of three papers [4], [5], [6].

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EXISTENCE OF NON-STATIONARY POISEUILLE TYPE SOLUTIONS UNDER MINIMAL REGULARITY ASSUMPTIONS

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Existence and uniqueness of a solution to the non-stationary Navier-Stokes equations having a prescribed flow rate (flux) in the infinite cylinder $\Pi = \{x : (x_1, x_2) \in \sigma, x_3 \in (-\infty, +\infty)\}$, where σ is bounded domain in $\mathbb{R}^2$, are proved. It is assumed that the flow rate $F(t)$ is an element of $L^2(0,T)$ and the initial data $u_0 = (0, 0, u_{0n})$ is an element of $L^2(\sigma)$. The non-stationary Poiseuille solution has the form $u(x,t) = (0, 0, U(x', t))$, $p(x,t) = -q(t)x(n) + p_0(t)$, where $(U(x', t), q(t))$ is a solution of an inverse problem for the heat equation with a specific over-determination condition. Under the above regularity assumptions the solution of the problem does not have the usual for parabolic problems regularity: it is much weaker. The results were obtained jointly with R. Čiegis [1].

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ON THE SPECTRUM STRUCTURE FOR ONE DIFFERENCE EIGENVALUE PROBLEM WITH NONLOCAL BOUNDARY CONDITIONS

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We consider the difference eigenvalue prolem corresponding to the differential one

$$\frac{d^2u}{dx^2} + \lambda u = 0, \tag{1}$$

$$u(0) = \gamma_1 \int_0^1 \alpha(x)u(x)dx, \tag{2}$$

$$u(1) = \gamma_2 \int_0^1 \beta(x)u(x)dx. \tag{3}$$

The main purpose of the present report is to provide some new results on the spectrum structure of the difference eigenvalue problem and to apply these results for the investigation of stability of difference schemes for parabolic equations. The spectrum structure of the difference eigenvalue problem could be interpreted as one of the principal approaches for the investigation of the stability of difference schemes and convergence of iterative methods.

The theoretical results were supplemented by numerical experiment. Several important conclusions were drawn from the numerical experiment. So, the areas of stability of difference schemes for parabolic equation with integral conditions (2), (3) obtained in some papers (see f. e. [1]) change little or remain the same if $\alpha(x)$ and $\beta(x)$ exchange each other. The real area of stability obtained in the numerical experiment may differ significantly.

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MODELING AND SIMULATION OF SEMICONDUCTOR LASERS FOR HIGH EMISSION POWER APPLICATIONS

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Semiconductor diode lasers are small, efficient, and relatively cheap devices used in many modern applications, including optical communications, medicine, metal cutting and welding, car manufacturing, e.t.c. Various high power applications require emission powers exceeding several ten Watts from a single diode and up to a few kiloWatts from a combined laser system.

A comprehensive study of complex dynamics in such devices typically requires consideration of numerically challenging 1(time)+2(space)-dimensional or even (1+3)-dimensional models. In many cases, simultaneous treatment of different but still self-consistently coupled model components defined in differing computational domains and acting on significantly different time and space scales is required. For this reason, the efficiency of numerical algorithms is of the highest priority.

In the present lecture, we shall give a brief overview of the governing PDE models: a dynamic (1+2)D model for optical fields and carrier density, a static quasi-3D model for current flow, and another static quasi-3D heat transfer model. The coupling of the models will be explained [1], and a short introduction of the main numerical algorithms and challenges will be performed [1; 2].

Our studies will be illustrated by modeling and simulations of 2^n laser devices coupled through the optical cross-feedback bypassing a specially constructed external cavity [3]. This laser configuration has the potential of generating a combined beam of superb quality and kiloWatt-range emission power, needed, e.g., for metal cutting and welding applications. Besides increasing the number of laser diodes and, thus, required CPU time, the biggest challenge, in this case, is the construction of an efficient model for the laser cross-coupling. It should be precise enough, account for all essential properties of the external cavity, and should not slow down the simulations of the laser diodes themself at the same time.

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SPATIO-TEMPORAL DIVERGENCE IN THE FRACTIONAL LOGISTIC MAP OF MATRICES

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Complex spatio-temporal behaviour of the fractional logistic map of matrices is considered in this talk. The scalar fractional logistic map is a discrete counterpart of the fractional logistic differential equation and can be regarded as a generalization of the classical logistic map [1]:

$$x^{(k)} = x^{(0)} + \sum_{j=1}^{k} G_{j-1} \left(a x^{(k-j)} \left(1 - x^{(k-j)} \right) - x^{(k-j)} \right), \tag{1}$$

where $x^{(0)}$, $x^{(0)} \in \mathbb{R}$; $k \in \mathbb{N}$; $G_0 = 1$;

$$G_j = \left(1 - \frac{1-\beta}{j} \right) G_{j-1}, \tag{2}$$

and the parameter β describes the order of the fractional derivative ($0 < \beta \leq 1$); the parameter of the classical logistic map $0 < a < 4$; the initial condition $0 < x^{(0)} < 1$. Necessary conditions for finite-time stabilization of unstable orbits in the scalar fractional logistic map are derived in [1].

The scalar variable $x^{(k)}$ in the classical logistic map can be replaced by a square matrix $X^{(k)}$. It is shown in [2] that such a replacement can yield the effect of explosive divergence in the logistic map of matrices.

The main objective of this talk is to provide the insight into the dynamics of the fractional logistic map of matrices [3]. In particular, it will be demonstrated how and why the investigated model does incorporate the effect of finite-time stabilization from the scalar fraction logistic map, and the effect of explosive divergence from the logistic map of matrices. Potential applications of the spatio-temporal divergence in the fractional logistic map of matrices will be also discussed.

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REMARKABLE THREE-DIMENSIONAL AUTONOMOUS SYSTEMS

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The Duffing differential equation

$$x'' + \delta x' + \alpha x + \beta x^3 = h \cos \omega t \tag{1}$$

is known as the minimal scalar equation exhibiting chaotic behavior. It can be written in the form of the three-dimensional system

$$\begin{cases} x' = y, \\ y' = -\delta y - \alpha x - \beta x^3 - h \cos z, \\ z' = \omega. \end{cases} \tag{2}$$

It was observed that the similar system

$$\begin{cases} x' = y, \\ y' = by + x - x^3 - kz, \\ z' = w(y - z) \end{cases} \tag{3}$$

can also be chaotic.

We trace the emergency of chaos from periodic solutions in this system varying the parameter w.

After the detailed analysis how this happened, we consider three-dimensional systems of the form

$$\begin{cases} x' = f_1(x, z) - x, \\ y' = f_2(x, y, z) - y, \\ z' = f_3(x, z) - z, \end{cases}$$

which are shown to have complicated behavior of solutions.

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A SINGLE FAR - FIELD PATTERN DETERMINES THE SHAPE OF SCATTERING SCREENS

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The scattering of acoustic waves from thin objects called screens leads to the mathematical description of Helmholtz equation with certain boundary conditions. The aim of this study is the inverse problem to find the shape of scattering screen from far field measurements. We consider the scattering by two dimensional flat screens in the three dimensional space and one dimensional possibly curved screens in the plane. Our results state that one measurement of the scattering data determines the shape of the screens uniquely. These result are obtained by combining the connection of the Hilbert transformed to the Mellin transform.

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THE FUČÍK SPECTRUM FOR THE SECOND ORDER DIFFERENTIAL OPERATORS WITH THE ROBIN AND INTEGRAL BOUNDARY CONDITIONS

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We investigate the structure of the FučÍk spectrum for the following boundary value problem

$$\begin{cases} u''(x) + \alpha u^+(x) - \beta u^-(x) = 0, \quad x \in (0,1), \\ u(0) \cdot \sin c = u'(0) \cdot \cos c, \quad \displaystyle\int_0^1 u(x)\,\mathrm{d}x = \gamma \cdot u'(0), \end{cases} \tag{1}$$

where $\alpha, \beta, \gamma \in \mathbb{R}$ and $-\frac{\pi}{2} < c \leq \frac{\pi}{2}$. By the FučÍk spectrum we mean the set of all pairs $(\alpha, \beta) \in \mathbb{R}^2$ such that the problem (1) has a non-trivial solution u. We provide the implicit description of the FučÍk spectrum in the first quadrant at two different levels: the implicit description with inequality type conditions (that allow to control the nodal properties of corresponding non-trivial solutions) and the compact implicit description (which is more suitable for simple implementation in computer algebra systems and numerical computing packages). Moreover, we show the qualitative difference between the FučÍk spectra in the case of homogeneous ($\gamma = 0$) and non-homogeneous ($\gamma \neq 0$) integral boundary conditions. Finally, in two special cases when the Robin boundary condition is reduced to the Neumann condition ($c = 0$) and to the Dirichlet condition ($c = \frac{\pi}{2}$), we prove that the FučÍk spectrum consists of two continuous curves in the first quadrant since we provide the parametrization of both FučÍk curves.

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ON JOINT UNIVERSALITY OF DIRICHLET *L*-FUNCTIONS

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Let χ be a Dirichlet character modulo $q \in \mathbb{N}$, i. e., χ is a non-vanishing group homomorphism from the group $(\mathbb{Z}/q\mathbb{Z})^*$ of prime residue modulo q to $\mathbb{C}$. The Dirichlet L-function $L(s,\chi)$, $s = \sigma + it$, is defined, for $\sigma > 1$, by

$$L(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s} = \prod_p \left(1 - \frac{\chi(p)}{p^s}\right)^{-1},$$

and has the meromorphic continuation to the whole $\mathbb{C}$.

The joint universality of Dirichlet L-functions with pairwise non-equivalent Dirichlet characters was obtained by S.M. Voronin (1975). Ł. Pańkowski [1] used the generalized shifts $(L(s + i\alpha_1\tau^{a_1}\log^{b_1}\tau, \chi_1), \dots, L(s + i\alpha_r\tau^{a_r}\log^{b_r}\tau, \chi_r))$. Our report is devoted to approximation of analytic functions by more general shifts. Suppose that, for $j = 1, \dots, r$, $\gamma_j(\tau)$ is an increasing real continuously differentiable function on $[T_0, \infty)$ with monotonic derivative $\gamma_j'(\tau) = \delta_j(\tau)(1 + o(1))$ such that $\delta_1(\tau) = o(\delta_2(\tau))$, $\dots \delta_{r-1}(\tau) = o(\delta_r(\tau))$ and $\gamma_j(2\tau)\max_{\tau \leqslant u \leqslant 2\tau}(\gamma_j'(u))^{-1} \ll \tau$.

Let $\mathcal{K}$ be the class of compact subsets of the strip $\{s \in \mathbb{C} : 1/2 < \sigma < 1\}$ with connected complements, and $H_0(K)$, $K \in \mathcal{K}$, the class of continuous non-vanishing functions on K that are analytic in the interior of K. Then we have

THEOREM 4. *Suppose that $\gamma_1(\tau), \dots, \gamma_r(\tau)$ satisfy the above hypotheses. For $j = 1, \dots, r$, let $K_j \in \mathcal{K}$ and $f_j \in H_0(K_j)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T - T_0} \operatorname{meas}\left\{\tau \in [T_0, T] : \sup_{1\leqslant j\leqslant r}\sup_{s\in K_j} |L(s + i\gamma_j(\tau), \chi_j) - f_j(s)| < \varepsilon\right\} > 0.$$

Moreover, "lim inf" can be replaced by "lim" for all but at most countably many $\varepsilon > 0$.

Generalizations for some compositions $F(L(s,\chi_1), \dots, L(s,\chi_r))$ are possible as well.

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ARTIFICIAL NEURAL NETWORKS FOR SOLVING ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

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In recent years, deep neural networks have shown the impressive results in solving different tasks in computer vision, natural language processing, game theory, etc. Deep Learning has transformed how categorization, pattern recognition, and regression tasks are performed today across various application domains.

The use of artificial neural networks to solve ordinary differential equation problems has started in the 1990s [1]. Various algorithms have been proposed since that time for solving ordinary and partial differential equations on regular and irregular domains [2]. The search and selection of suitable neural network architecture is a difficult task.

In this talk, we consider Physics-Informed Neural Networks (PINN) [3] that encode the differential model equations as a component of the neural network itself.

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ASYMPTOTIC ANALYSIS OF STURM–LIOUVILLE PROBLEM WITH DIRICHLET AND NONLOCAL TWO-POINT BOUNDARY CONDITIONS

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Consider the following one-dimensional Sturm–Liouville equation

$$-u''(t) + q(t)u(t) = \lambda u(t), \quad t \in (0,1), \tag{1}$$

where the real-valued function $q \in C[0,1]$.

We investigate Sturm–Liouville Problem (SLP) which consist of equation (1) on $[0,1]$ with one classical (local) Dirichlet type Boundary Condition (BC)

$$u(0) = 0, \tag{2}$$

another two-point Nonlocal Boundary Condition (NBC)

$$u'(1) = \gamma u(\xi), \qquad \xi \in (0,1], \qquad \text{(Case 1)} \qquad (3_1)$$

$$u'(1) = \gamma u'(\xi), \qquad \xi \in [0,1), \qquad \text{(Case 2)} \qquad (3_2)$$

$$u(1) = \gamma u(\xi), \qquad \xi \in (0,1), \qquad \text{(Case 3)} \qquad (3_3)$$

where $\gamma \in \mathbb{R}$.

We obtain asymptotic formulas for eigenvalues and eigenfunctions of the one–dimensional Sturm–Liouville equation with one classical Dirichlet type boundary condition and two-point nonlocal boundary condition

We analyze the characteristic equation of the boundary value problem for eigenvalues and derive asymptotic formulas of arbitrary order. We apply the obtained results to the problem with two-point nonlocal boundary condition.

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ALTERNATING DIRECTION METHOD FOR POISSON EQUATION WITH NONLOCAL CONDITIONS

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We consider the nonlocal boundary value problem for two-dimensional Poisson equation in rectangular domain

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y), \quad (x,y) \in \Omega = \{0 < x < 1, \quad 0 < y < 1\}, \tag{1}$$

with the following Dirichlet and integral boundary conditions

$$\begin{aligned} &\int_0^{\xi_1} u(x,y)dx = \mu_1(y), \quad \int_{\xi_2}^1 u(x,y)dx = \mu_2(y), \\ &u(x,0) = \mu_3(x), \quad u(x,1) = \mu_4(x), \end{aligned} \tag{2}$$

where $0 < \xi_1 < 1$, $0 < \xi_2 < 1$.

The corresponding difference scheme for this problem under the condition that the desired solution belongs to the Sobolev space $W_2^s(1 < s \leq 3)$ has been investigated in [1]. Our main goal is to study the alternating direction method (ADI) for solving a system of difference equations. Proof of the convergence of this method is based on the structure of the spectrum of the one-dimensional eigenvalue problem

$$\begin{aligned} &h^{-2}(u_{i-1} - 2u_i + u_{i+1}) + \lambda u_i = 0, \quad i = 1, 2, \ldots, N-1, \\ &l_1(u_i) = 0, \quad l_2(u_i) = 0, \end{aligned} \tag{3}$$

where $l_k(u_i)$, $k = 1, 2$ are expressions obtained from integral conditions (2) by trapezoidal rule or the Simpson rule.

We proved that in the both cases (the trapezoidal and the Simpson rules) the spectrum of problem (3) consists only of positive eigenvalues $\lambda \in (0, 4/h^2]$. It follows that the convergence of the ADI method does not depend on whether the eigenvectors are linearly independent or not.

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PARALLEL 3D ADI SCHEME FOR HYBRID DIMENSION MODEL OF HEAT CONDUCTION PROBLEM

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The model describes the heat equation in 3D domains, and this problem is reduced to a hybrid dimension problem, keeping the initial dimension only in some parts and reducing it to one-dimensional equation within the domains in some distance from the bases regions. Such mathematical models are typical in industrial installations such as pipelines. Our aim is to add two additional improvements into this methodology. First, the economical ADI type finite volume scheme is constructed to solve the non-classical heat conduction problem. Special interface conditions are defined between 3D and 1D parts. It is proved that the ADI scheme is unconditionally stable. Second, the parallel factorization algorithm is proposed to solve the obtained systems of discrete equations. Due to both modifications the run-time of computations is reduced essentially. Results of computational experiments confirm the theoretical error analysis and scalability estimates of the parallel algorithm.

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FUČÍK SPECTRUM FOR THE PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

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Let us consider the problem

$$x'' = -\mu x^+ + \lambda x^-, \tag{1}$$

$$x(0) = 0, \tag{2}$$

with nonlocal two-point boundary condition of Samarskii-Bitsadze type

$$x(1) = \gamma x(\xi), \quad \gamma \in \mathbb{R}, \tag{3}$$

in the case where $\xi = \frac{m}{n} \in (0,1)$. We suppose that m and n $(0 < m < n)$ are positive coprime integer numbers. Some properties of Fučík type spectrum will be presented for arbitrary rational values of parameter ξ. The analytical description and visualization of the spectrum for some values of parameters γ and ξ will be given.

Fučík type problems are of interest over the last number of years, however problems of this type with nonlocal boundary conditions still lack attention. Thus, the purpose of this report is to present the investigation results on the spectrum of Fučík type problem (1) with boundary conditions (2)–(3).

Some of the results are the logical continuation and generalization of previous authors' investigations. This investigation continues and generalize the results obtained for problem (1) – (3) with $\xi = \frac{1}{2}$ [1], $\xi = \frac{1}{3}$ [2], $\xi = \frac{1}{4}$ [3] and $\xi = \frac{1}{n}$ [4].

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SPLINE COLLOCATION FOR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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We consider a class of fractional weakly singular integro-differential equations

$$\begin{aligned}(D^{\alpha_p}_{Cap}y)(t) &+ \sum_{i=0}^{p-1} d_i(t)(D^{\alpha_i}_{Cap}y)(t)\\ &+ \sum_{i=0}^{q}\int_0^t (t-s)^{-\kappa_i}K_i(t,s)(D^{\theta_i}_{Cap}y)(s)ds = f(t), \quad 0\le t\le b, \end{aligned}\tag{1}$$

subject to the conditions

$$\sum_{j=0}^{n_0}\beta_{ij0}\,y^{(j)}(0) + \sum_{k=1}^{l}\sum_{j=0}^{n_1}\beta_{ijk}\,y^{(j)}(b_k) + \beta_i\int_0^{\bar b_i} y(s)ds = \gamma_i\,, \quad i=0,\dots,n-1, \quad n:=\lceil\alpha_p\rceil. \tag{2}$$

Here D^{δ}_{Cap} is the Caputo differential operator of order $\delta > 0$ and by $\lceil\delta\rceil$ we denote the smallest integer greater or equal to a real number δ. We assume that

$$\begin{aligned}&0\le\alpha_0<\alpha_1<\dots<\alpha_p\le n, \quad p\in\mathbb{N}:=\{1,2,\dots\}, \quad n=\lceil\alpha_p\rceil,\\ &0\le\theta_j<\alpha_p, \quad 0\le\kappa_j<1, \quad j=0,\dots,q, \quad q\in\mathbb{N}_0:=\{0\}\cup\mathbb{N},\\ &0<b_1<\dots<b_l\le b, \quad l\in\mathbb{N}, \quad 0<\bar b_i\le b, \quad i=0,\dots,n-1,\\ &n_0,n_1\in\mathbb{N}_0, \quad n_0<n,\ n_1<n, \quad \beta_{ij0},\beta_{ijk},\beta_i,\gamma_i\in(-\infty,\infty),\\ &d_i\in C[0,b]\ (i=0,\dots,p-1), \quad f\in C[0,b],\\ &K_j\in C(\Delta), \quad j=0,\dots,q, \quad \Delta:=\{(t,s):0\le s\le t\le b\},\\ &C(\Omega)\text{ is the set of continuous functions on }\Omega.\end{aligned}$$

Following [1], we reformulate (1)–(2) as a Volterra integral equation of the second kind with respect to the fractional derivative $D^{\alpha_p}_{Cap}y$ of the solution y for the original problem. We then regularize the solution by a suitable smoothing transformation and solve the transformed integral equation by a piecewise polynomial collocation method on a mildly graded or uniform grid. We show the convergence of the proposed algorithm and present global superconvergence results for a class of specific collocation parameters. Finally, we complement the theoretical results with some numerical examples.

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FLOQUET MULTIPLIERS OF PERIODIC SOLUTIONS FOR A TWO DEGREE OF FREEDOM HAMILTONIAN SYSTEM

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We consider a two degree of freedom Duffing oscillator with no damping and no forcing written in the following form

$$\begin{cases} x_1' &= y_1, \\ x_2' &= y_2, \\ y_1' &= \alpha x_1 + \beta x_1(x_1^2 + x_2^2), \\ y_2' &= \alpha x_2 + \beta x_2(x_1^2 + x_2^2), \end{cases} \tag{3}$$

where α and β ($\beta \neq 0$) are real parameters. The system (3) is a Hamiltonian system with a Hamiltonian function

$$H(x_1, x_2, y_1, y_2) = \frac{y_1^2 + y_2^2}{2} - \frac{\alpha}{2}(x_1^2 + x_2^2) - \frac{\beta}{4}(x_1^2 + x_2^2)^2. \tag{4}$$

We discuss the existence and stability of periodic solutions of the system (3) in the form

$$p(t) = \Big(r\cos(\omega t), r\sin(\omega t), -r\omega\sin(\omega t), r\omega\cos(\omega t)\Big)^T \tag{5}$$

depending on the parameters α, β.

For linearized system with respect to the periodic solution $p(t)$, we calculate the monodromy matrix and investigate the dynamics of the Floquet multipliers of $p(t)$ depending on the parameters α, β.

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